

absolutely summable impulse response: $\sum |h[n]|$ is finite. **EX:** $\sum_{n=0}^{\infty} (\frac{3}{4})^n = \frac{1}{1-\frac{3}{4}} = 4$ but $\sum_{n=1}^{\infty} \frac{1}{n} \rightarrow \infty$.

summable Necessary and sufficient for **BIBO stability** of an **LTI** system. Also see **poles**.

affine system $y[n] = ax[n] + b$ for constants a and $b \neq 0$. This is a linear-plus-constant function.

Affine systems are *not* **linear systems**, although they do look like linear systems.

aliasing **EX:** $x(t) = \cos(2\pi 6t)$ **sampled** at 10 Hz: $t = \frac{n}{10} \rightarrow x[n] = \cos(1.2\pi n) = \cos(0.8\pi n)$.
Ideal **interpolation** (for a **sinusoid**) $n = 10t \rightarrow x(t) = \cos(2\pi 4t)$. 6 Hz aliased to 4.
Can avoid aliasing by **sampling** faster than **Nyquist** rate, or use an **antialias** filter.

amplitude of a **sinusoid**: $|A|$ in sinusoidal signal $A \cos(\omega t + \theta)$. Amplitude is always ≥ 0 .

antialias filter Analog lowpass filter that ensures that the **sampling** rate exceeds **Nyquist** rate, ensuring that **aliasing** does not occur during interpolation of the sampled signal.

argument of a complex number: another name for **phase**. Matlab: `angle(1+j)=0.7854=π/4`.

ARMA Auto-Regressive Moving-Average **difference equation**. $\#y[n] > 1$ and $\#x[n] > 1$.

average power Continuous time: Average power= $MS(x) = \frac{1}{T} \int_0^T |x(t)|^2 dt$ for $x(t)$ period= T .

power Discrete time: Average power= $MS(x) = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$ for $x[n]$ period= N .

band-pass filter (BPF) Filter with **frequency response** $H(e^{j\omega})$ small at **DC** $\omega = 0$ and small at $\omega = \pi$, but large at some intermediate band $a < \omega < b$. BPFs reduce high-frequency noise.

bandwidth Maximum **frequency** in a cont.-time signal $x(t)$. Sometimes defined as twice this.

beat signal **EX:** $\cos(99t) + \cos(101t) = 2 \cos(1t) \cos(100t)$ like varying **amplitude** $\cos(100t)$.

This beating sound can be used to tune a piano; also models a tone on AM radio.

BIBO stable Bounded-Input-Bounded-Output system: $x[n] \rightarrow \overline{\text{BIBO}} \rightarrow y[n]$. Means:

If $|x[n]| \leq M$ for some constant M , then $|y[n]| \leq N$ for some constant N .

An **LTI** system is BIBO stable iff all of its **poles** are inside the unit circle.

cascade connection of systems: $x[n] \rightarrow \overline{\mathbf{g}[n]} \rightarrow \overline{\mathbf{h}[n]} \rightarrow y[n]$. Overall **impulse response**:
 $y[n] = h[n] * (g[n] * x[n]) = (h[n] * g[n]) * x[n] \rightarrow$ **convolve** their **impulse responses**.

causal: signal: $x[n] = 0$ for $n < 0$; causal **system** has a causal **impulse response** $h[n]$.

comb filter $y[n] = x[n] + x[n-1] + \dots + x[n-M+1]$ eliminates all $A_k \cos(\frac{2\pi}{M}kn + \theta_k)$ in $x[n]$.

transfer function= $H(z) = (z - e^{j\frac{2\pi}{M}}) \dots (z - e^{j\frac{2\pi}{M}(M-1)})/z^{M-1}$ has **zeros** at $e^{j\frac{2\pi}{M}k}$, which are the M^{th} **roots of unity**. Eliminates all **harmonics** of periodic function.

complex conjugate of complex number $z = x + jy$ is $z^* = x - jy$; of $z = Me^{j\theta}$ is $z^* = Me^{-j\theta}$.

Properties: $|z|^2 = zz^*$; $Re[z] = \frac{1}{2}(z + z^*)$; $Im[z] = \frac{1}{2j}(z - z^*)$; $e^{j2\angle z} = \frac{z}{z^*}$.

complex exponential Continuous time: $x(t) = e^{j\omega t}$. Discrete time: $x[n] = e^{j\omega n}$ where ω =**frequency**.
Fourier series are expansions of periodic signals in terms of these functions.

complex plane Plot of **Imaginary part** of complex number vs. **real part** of complex number.
You can visualize $z = 3 + j4$ at Cartesian coordinates $(3, 4)$ in the complex plane.

conjugate symmetry If $x(t)$ is real-valued, then $x_{-k} = x_k^*$ in its **Fourier series** expansion, where x_k^* is **complex conjugate** of x_k . **DFT**: If $x[n]$ is real-valued, then $X_{N-k} = X_k^*$

convolution $y[n] = h[n] * x[n] = \sum_{i=0}^n h[i]x[n-i] = \sum_{i=0}^n h[n-i]x[i]$ if $h[n]$ and $x[n]$ **causal**.
 $x[n] \rightarrow \underline{\text{LTI}} \rightarrow y[n] = h[n] * x[n]$ where $h[n]$ =**impulse response** of **LTI** system.

correlation Continuous time: $C(x, y) = \int x(t)y(t)^* dt$. Discrete time: $C(x, y) = \sum x[n]y[n]^*$.

correlation coefficient $C_N(x, y) = \frac{C(x, y)}{\sqrt{C(x, x)C(y, y)}}$ where $C(x, y)$ =**correlation** and $C(x, x) = E(x)$ =**energy**.

$|C_N(x, y)| \leq 1$ by Cauchy-Schwarz, so $C_N(x, y)$ =(the similarity of $x[n]$ and $y[n]$).

DC term in **Fourier series** is constant x_0 or a_0 (continuous time) or X_0 (discrete time).
DC means Direct Current, which does not vary with time, vs. AC, which is a **sinusoid**.

delay $x[n]$ delayed by $D > 0$ is $x[n - D]$: D seconds later; shift $x[n]$ graph right by $D > 0$.

DFT N-point DFT of $\{x[n]\}$ is $X_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nk}$ for $k = 0, 1 \dots N - 1$. Use **fft**.
Discrete Fourier Transform computes coefficients X_k of the discrete-time Fourier series
 $x[n] = X_0 + X_1e^{j\frac{2\pi}{N}n} + X_2e^{j\frac{4\pi}{N}n} + \dots + X_{N-1}e^{j2\pi\frac{N-1}{N}n}$ where $x[n]$ has period=N.

difference equation $y[n] + a_1y[n-1] + \dots + a_Ny[n-N] = b_0x[n] + b_1x[n-1] + \dots + b_Mx[n-M]$.
ARMA; describes an **LTI** system; this is how it is actually implemented on a chip.

duration Continuous time: If signal **support**=[a, b] \rightarrow duration=($b - a$)=how long it lasts.
Discrete time: If signal **support**=[a, b] \rightarrow duration=($b - a + 1$). Note the "+1".

energy Continuous: $E(x) = \int_a^b |x(t)|^2 dt$. Discrete: $E(x) = \sum_{n=a}^b |x[n]|^2$ if **support**=[a, b].

Euler's formula: $e^{j\theta} = \cos \theta + j \sin \theta$. **Polar** \Leftrightarrow **rectangular**: $Me^{j\theta} = M \cos \theta + jM \sin \theta$.

even function $x(t) = x(-t)$: $x(t)$ is symmetric about the vertical axis $t = 0$. **EX**: $\cos(t), t^2, t^4$.

A **periodic** even function has **Fourier series** having $b_k = 0$ and x_k real numbers.

fft Matlab command that computes **DFT**. Use: **fft(X,N)/N**. See also **fftshift**.

fftshift Matlab command that swaps first and second halves of a vector. Used with **fft**.
Displays the **line spectrum** with negative **frequencies** to left of positive ones.

filter **LTI** system with **frequency response** performing some task, e.g., **noise** reduction.

FIR Finite Impulse Response system: its **impulse response** $h[n]$ has finite **duration**.
FIR systems are: also **MA** systems; always **BIBO stable**; all **poles** are at **origin**.

Fourier series $x(t) = x_0 + x_1e^{j\frac{2\pi}{T}t} + x_2e^{j\frac{4\pi}{T}t} + x_3e^{j\frac{6\pi}{T}t} + \dots + x_1^*e^{-j\frac{2\pi}{T}t} + x_2^*e^{-j\frac{4\pi}{T}t} + x_3^*e^{-j\frac{6\pi}{T}t} + \dots$
where $x_k = \frac{1}{T} \int_0^T x(t)e^{-j\frac{2\pi}{T}kt} dt$ for integers k and $x(t)$ is real-valued with period=T.

Fourier series $x(t) = a_0 + a_1 \cos(\frac{2\pi}{T}t) + a_2 \cos(\frac{4\pi}{T}t) + \dots + b_1 \sin(\frac{2\pi}{T}t) + b_2 \cos(\frac{4\pi}{T}t) + \dots$ where $a_k = \frac{2}{T} \int_0^T x(t) \cos(\frac{2\pi}{T}kt) dt$ and $b_k = \frac{2}{T} \int_0^T x(t) \sin(\frac{2\pi}{T}kt) dt$ and $x(t)$ has period= T .
(trig form) BUT: note $a_0 = \frac{1}{T} \int_0^T x(t) dt = M(x)$ =**DC** term is a special case: note $\frac{1}{T}$, not $\frac{2}{T}$.

frequency of a sinusoid: ω in signal $A \cos(\omega t + \theta)$. Units of ω : $\frac{\text{RAD}}{\text{SEC}}$. Units of $f = \frac{\omega}{2\pi}$: Hertz.

frequency response $H(e^{j\omega}) = H(z)|_{z=e^{j\omega}}$ where $H(z)$ is **transfer function**. Sometimes write: $H(\omega)$.

response $\cos(\omega_0 n) \rightarrow \overline{\text{LTI}} \rightarrow A \cos(\omega_0 n + \theta)$ where $H(e^{j\omega_0}) = Ae^{j\theta}$. A =**gain** of system.

$x[n] = \sum X_k e^{j\frac{2\pi}{N}kn} \rightarrow \overline{\text{LTI}} \rightarrow y[n] = \sum Y_k e^{j\frac{2\pi}{N}kn}$ where $Y_k = H(e^{j\frac{2\pi}{N}k})X_k$.

fundamental Sinusoid $c_1 \cos(\frac{2\pi}{T}t + \theta_1)$ at **frequency** $\frac{1}{T}$ Hertz in **Fourier series** of **periodic** $x(t)$.

gain $\text{Gain} = |H(e^{j\omega})|$ =**magnitude of frequency response**. Describes **filtering** effect.

harmonic Sinusoid $c_k \cos(\frac{2\pi}{T}kt + \theta_k)$ at **frequency** $\frac{k}{T}$ Hz in **Fourier series** of **periodic** $x(t)$.

high-pass Filter with **frequency response** $H(e^{j\omega})$ small at **DC** $\omega = 0$ and large at $\omega = \pi$.
filter (HPF) HPFs enhance edges in images and signals, but also amplify high-frequency noise.

histogram Plot of #times a signal value lies in a given range vs. range of signal values (bins).
Can be used to compute good approximations to **mean**, **mean square**, **rms**, etc.

IIR Infinite Impulse Response system: **impulse response** $h[n]$ has infinite **duration**.

imaginary part of a complex number: $\text{Im}[x+jy] = y$; $\text{Im}[Me^{j\theta}] = M \sin \theta$. Matlab: $\text{imag}(3+4j)=4$.
Note that the imaginary part of $3 + j4$ is 4, **NOT** $j4$! A **VERY** common mistake!

impulse response Discrete time: $\delta[n] = 0$ unless $n = 0$; $\delta[0] = 1$. Continuous time: wait for EECS 306.

impulse response $h[n]$: Response to an impulse: $\delta[n] \rightarrow \overline{\text{LTI}} \rightarrow h[n]$. $y[n] = b_0x[n] + \dots + b_Mx[n-M]$,
 $x[n] \rightarrow \overline{\text{LTI}} \rightarrow y[n] = h[n] * x[n]$; see **convolution**. then $h[n] = \{b_0, b_1 \dots b_M\}$.

interference A signal nature (60 Hz) or humans (jamming) adds to a desired signal that obscures the desired signal. Unlike added **noise**, interference is usually known approximately.

interpolation The act of reconstructing $x(t)$ from its **samples** $x[n] = x(t = n\Delta)$. Also: D-to-A.
Zero-order hold: $\hat{x}(t) = x[n]$ for $\{n\Delta < t < (n+1)\Delta\}$; linear; exact using a **sinc**.
Exact interpolation of discrete **Fourier series**: set $n = t/\Delta$ in all the **sinusoids**.

inverse z-xform Get $x[n]$ from its **z-xform** $X(z) = \mathcal{Z}\{x[n]\}$. **EX:** $X(z) = \frac{z}{z-3} \rightarrow x[n] = 3^n u[n]$.
Do this by inspection or **partial fraction expansion** of **rational function** $X(z)$.

line spectrum Plot of the x_k in **Fourier series** of a signal vs. **frequency** ω . Also for X_k in **DFT**.
 x_k is depicted in plot by a vertical line of height $|x_k|$ at $\omega = \frac{2\pi}{T}k$ where T =**period**.

linear combination of two signals $x_1[n]$ and $x_2[n]$ is the signal $(ax_1[n] + bx_2[n])$ for constants a and b .
Lin. system: Response to linear combination is linear combination of responses.

linear system This means that if $x_1[n] \rightarrow \overline{\text{LINEAR}} \rightarrow y_1[n]$ and $x_2[n] \rightarrow \overline{\text{LINEAR}} \rightarrow y_2[n]$
then $(ax_1[n] + bx_2[n]) \rightarrow \overline{\text{LINEAR}} \rightarrow (ay_1[n] + by_2[n])$ for any constants a and b .
Often works: if doubling input $x[n]$ doubles output $y[n]$, system is likely to be linear.

low-pass filter (LPF) Filter with frequency response $H(e^{j\omega})$ large at **DC** $\omega = 0$ and small at $\omega = \pi$.
LPFs smooth edges in images and signals, but they do reduce high-frequency noise.

LTI system is both **Linear** and **Time-Invariant**. So what? See **IMPULSE RESPONSE** and **FREQUENCY RESPONSE**.

MA Moving-Average **difference equation:** $y[n] = b_0x[n] + b_1x[n-1] + \dots + b_Mx[n-M]$.
MA systems are: also **FIR** systems; always **BIBO stable**; all **poles** are at **origin**.

magnitude of a complex number: $|Me^{j\theta}| = M$; $|x + jy| = \sqrt{x^2 + y^2}$. Matlab: `abs(3+4j)=5`.

Matlab Computer program used universally in communications, control, and signal processing.
Matlab has been known to drive people crazy (look at what happened to me :)).

mean Cont.: $M(x) = \frac{1}{T} \int_0^T x(t)dt$ for $x(t)$ period= T . $M(x) = x_0 = a_0$ in **Fourier series**.
Disc.: $M(x) = \frac{1}{N} \sum_{n=0}^{N-1} x[n]$ for $x[n]$ period= N . $M(x) = X_0$ in the **DFT** of $x[n]$.

mean square Cont.: $MS(x) = \frac{1}{T} \int_0^T |x(t)|^2 dt = M(x^2) \neq (M(x))^2$ for $x(t)$ periodic with period= T .
Disc.: $MS(x) = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$ for $x[n]$ period= N . Mean square=**average power**.

mean square error **Fourier series** Periodic $x(t) = x_0 + x_1e^{j\frac{2\pi}{T}t} + \dots + x_1^*e^{-j\frac{2\pi}{T}t} + \dots$ (infinite series).
 $\hat{x}(t) = x_0 + x_1e^{j\frac{2\pi}{T}t} + \dots + x_Me^{j\frac{2\pi}{T}Mt} + x_1^*e^{-j\frac{2\pi}{T}t} + \dots + x_M^*e^{-j\frac{2\pi}{T}Mt}$ (finite series).
MSE= $MS(x(t) - \hat{x}(t))$ is the **mean square error** in approximating $x(t)$ with $\hat{x}(t)$.

noise An unknown signal that nature adds to a desired signal that obscures desired signal.
A **lowpass filter** often helps reduce noise, while (mostly) keeping the desired signal.

notch filter $y[n] = x[n] - 2\cos(\omega_o)x[n-1] + x[n-2]$ eliminates $A\cos(\omega_on + \theta)$ component in $x[n]$.
transfer function= $(z - e^{j\omega_o})(z - e^{-j\omega_o})/z^2$ has: **zeros** at $e^{\pm j\omega_o}$; **poles** at **origin**.

Nyquist sampling **Sampling** a continuous-time signal $x(t)$ at twice its **bandwidth** (minimum rate).
The minimum sampling rate for which $x(t)$ can be **interpolated** from its samples.

odd function $x(t) = -x(-t)$: $x(t)$ is antisymmetric about the vertical axis $t = 0$. **EX:** $\sin(t), t, t^3$.
A **periodic odd function** has Fourier series having $a_k = 0$ and x_k pure imaginary.

order of this **MA** (also **FIR**) system is M : $y[n] = b_0x[n] + b_1x[n-1] + \dots + b_Mx[n-M]$.

origin of the **complex plane** is at Cartesian coordinates $(0, 0)$. An elegant name for $0+j0$.

orthogonal complex exponentials signals $x(t)$ and $y(t)$ have **correlation** $C(x, y) = 0$; $MS(x + y) = MS(x) + MS(y)$.
that are **harmonics** yields **Fourier series**.

Parseval's theorem Cont.: $\frac{1}{T} \int_0^T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |x_k|^2 = a_0^2 + \frac{1}{2} \sum_{k=1}^{\infty} (a_k^2 + b_k^2)$ (see **Fourier series**).

Discrete time: $\frac{1}{N} \int_0^T |x[n]|^2 = \sum_{k=0}^{N-1} |X_k|^2$ where $\{X_k\}$ are the **N-point DFT** of $x[n]$.

States that **average power** can be computed in either the time or Fourier domains.

partial fraction expansion of **RATIONAL FUNCTION** $H(z) = \frac{N(z)}{D(z)} = \frac{A_1}{z-p_1} + \dots + \frac{A_N}{z-p_N}$. Need: degree $D(z) >$ degree $N(z)$.

$\{p_1 \dots p_N\}$ are the **poles** of the **transfer function** $H(z)$; $\{A_n\}$ are its **residues**.

Used to compute the **inverse z-transform** of a **rational function** by inspection.

periodic function $x(t)$ has period= $T \rightarrow x(t) = x(t \pm T) = x(t \pm 2T) = x(t \pm 3T) = \dots$ for ALL time t .

$A \cos(\omega t + \theta)$ has period $T = \frac{2\pi}{\omega}$; $A \cos(\omega n + \theta)$ is not periodic unless $\omega = 2\pi \frac{\text{RATIONAL NUMBER}}$.

phase of complex number: $\angle(Me^{j\theta}) = \theta$; $\angle(x + jy) = \tan^{-1}(\frac{y}{x})$. Use **angle(1+j)= $\pi/4$** .

phase of a sinusoid: θ in the **sinusoid** $A \cos(\omega t + \theta)$. Add π if $M < 0, A < 0$ or $x < 0$ above.

phasor Complex number $Me^{j\theta}$ used to represent sinusoid $M \cos(\omega t + \theta)$. So why do this?

$A \cos(\omega t + \theta) + B \cos(\omega t + \phi) = C \cos(\omega t + \psi) \Leftrightarrow Ae^{j\theta} + Be^{j\phi} = Ce^{j\psi}$ (much easier!).

polar form of complex number: $Me^{j\theta}$; M =**magnitude**=distance from **origin** of **complex plane**

at angle θ =**phase** from real axis. $z=x+jy \rightarrow M=\sqrt{x^2+y^2}$; $\theta = \tan^{-1}(\frac{y}{x})$ if $x > 0$.

poles of a **rational-function transfer-function** $H(z) = \frac{N(z)}{D(z)}$ are the roots of $D(z) = 0$.

EX: $H(z) = \frac{z^2-3z+2}{z^2-7z+12} \rightarrow z^2-7z+12=0 \rightarrow \{\text{poles}\} = \{3, 4\}$. roots($[1 \ -7 \ 12]$)= $3, 4$.

An **LTI** system is **BIBO stable** if and only if all poles are inside the **unit circle**.

pole-zero diagram Plot of **poles** using **X's** and **zeros** using **O's** of **LTI** system in the **complex plane**.

If all **X's** are inside the **unit circle**, then system is **BIBO stable**. Matlab: `zplane(N,D)`.

quantization of $x[n]$ represents each value of $x[n]$ with a finite #bits= N , so $x[n]$ can take 2^N values.

Produces slight roundoff error, which is neglected in EECS 206 since $N=16$ or 32 .

rational function of z is the ratio of two polynomials in z , so it has a **partial fraction expansion**.

Transfer functions in EECS 206 are rational functions with finite #**poles** & **zeros**.

residues The constants $\{A_n\}$ in the **partial fraction expansion** of a **rational function**.

real part of a complex number: $Re[x+jy] = x$; $Re[Me^{j\theta}] = M \cos \theta$. Matlab: `real(3+4j)=3`.

rectangular form of complex number: $x + jy$. Coordinates (x, y) in **complex plane**. cf. **polar form**.

x =**real part**; y =**imaginary part**. $z=Me^{j\theta} \rightarrow x=M \cos \theta$; $y=M \sin \theta \Leftrightarrow P \rightarrow R$ key.

rms $\sqrt{\frac{1}{T} \int_0^T |x(t)|^2 dt} = \sqrt{MS(x)} = \sqrt{M(x^2)}$. **EX:** $\text{rms}[A \cos(\omega t + \theta)] = \frac{A}{\sqrt{2}}$ if $\omega \neq 0$.

M roots of unity The M solutions to $z^M = 1$, which are $z = e^{j\frac{2\pi}{M}k}$ for $k = 1 \dots M$. *Unity* means "1."

The **zeros** of a **comb filter** are the M^{th} roots of unity, excluding the root $z = 1$.

running Correlation between a signal and **delayed** versions of another signal, regarded as **correlation** a function of the **delay**: $RC[n] = C(x[i], y[i - n])$. Used for time delay estimation.

sampling Act of constructing $x[n] = x(t = n\Delta)$ from values of $x(t)$ at integer multiples of Δ .

sawtooth Periodic $x(t) = at$ for $0 < t < b$ and $x(t) = 0$ for $b < t < c$ (like chain of triangles).

sinc $\text{sinc}(x) = \frac{\sin \pi x}{\pi x}$. Used in ideal interpolator, and also in $h[n]$ for ideal lowpass filter.

sinusoid A function of time of the form $x(t) = A \cos(\omega t + \theta) = (\frac{A}{2} e^{j\theta}) e^{j\omega t} + (\frac{A}{2} e^{-j\theta}) e^{-j\omega t}$.

spectrum See **line spectrum**. In EECS 206, all spectra are line spectra. Not so in EECS 306.

square Periodic $x(t) = a$ for $c < t < d$ and $x(t) = b$ for $d < t < e$ (like chain of squares).

wave $x(t)$ has **period**= $e-c$ and “duty cycle” $\frac{d-c}{e-c}$ if $a > b$, so that a=“on” and b=“off.”

stem plot Plot of discrete-time signal using chain of vertical lines topped with circles. **stem(X)**.

step function $u(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases}$ and $u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n \leq -1 \end{cases}$. Note that $u[0] = 1$.

step response $s[n]$: Response of a system to a **step function**: $u[n] \rightarrow \overline{\text{LTI}} \rightarrow s[n]$

super- of two signals $x_1[n]$ and $x_2[n]$ is the signal $(ax_1[n] + bx_2[n])$ for constants a and b .
position Linear system: Response to a superposition is the superposition of the responses. Another term for **linear combination**, but applied specifically to system inputs.

support of $x(t)$ is interval $[a, b]$ means that $x(t) = 0$ for all $t > a$ and $t < b$. Same for $x[n]$.

system function Another name for **transfer function** $H(z)$. Term used mostly in DSP.

Time- If $x[n] \rightarrow \overline{\text{TI}} \rightarrow y[n]$ then $x[n - D] \rightarrow \overline{\text{TI}} \rightarrow y[n - D]$ for any constant **delay** D .

Invariance A system is **TI** if there are no “ n ”s outside brackets. **EX**: $y[n] = nx[n]$ is *not* TI.

time scale $x(at)$ is $x(t)$ compressed if $a > 1$; expanded if $0 < a < 1$; time-reversed if $a < 0$.

transfer $H(z) = \mathcal{Z}\{h[n]\}$ =**z-transform** of **impulse response** of an **LTI** system. So what?

function Relates: **IMPULSE RESPONSE**, **FREQUENCY RESPONSE**, **DIFFERENCE EQUATION**, **POLE-ZERO DIAGRAM** descriptions.

triangle Periodic $x(t) = a - |t|$ for $|t| < a$ and $x(t) = 0$ rest of period (chain of triangles).

unit The circle $\{z : |z| = 1\} = \{z : z = e^{j\omega}\}$ in the **complex plane**, centered at **origin**.

circle An **LTI** system is **BIBO stable** if and only if all **poles** are inside the unit circle.

zeros of a **rational-function transfer-function** $H(z) = \frac{N(z)}{D(z)}$ are the roots of $N(z) = 0$.

EX: $H(z) = \frac{z^2 - 3z + 2}{z^2 - 7z + 12} \rightarrow z^2 - 3z + 2 = 0 \rightarrow \{\text{zeros}\} = \{1, 2\}$. **roots**($[1 \ -3 \ 2]$)= $1, 2$.

z-xform of $x[n]$ is $X(z) = \mathcal{Z}\{x[n]\} = x[0] + x[1]z^{-1} + x[2]z^{-2} + \dots$ **EX**: $\mathcal{Z}\{3, 1, 4\} = \frac{3z^2 + z + 4}{z^2}$.
