ASSIGNED: Feb. 17, 2006. **READ:** Part 3e of Official Lecture Notes (available on-line).
DUE DATE: Feb. 24, 2006. **TOPICS:** Fourier series of discrete-time signals (DFT).

Show work on separate sheets of paper. Include all hand and Matlab plots and code.

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[50] 1. *DFT=computation of Fourier series coefficients for discrete-time signals:*
The periodic discrete-time signal $x[n] = \{\ldots 3, 1, 4, 2, \underline{3}, 1, 4, 2, 3, 1, 4, 2 \ldots\}$.

- [20] (a) **By hand**, compute the 4-point DFT of $x[n]$. Check your answer: `fft(X,4)/4`
  This is the only time you will have to compute a DFT by hand. 4 X_k@[5]each.
- [05] (b) Plug your answer to (a) into: $x[n] = X_0 + X_1 e^{j\frac{\pi}{2}n} + X_2 e^{j\pi n} + X_3 e^{j\frac{3\pi}{2}n}$
- [05] (c) By hand, plot the line spectrum of $x[n]$ for $|\omega| \leq 3\pi$.
- [10] (d) Plug $n = 0, 1, 2, 3$ into your answer to (b) and confirm you get 1 period of $x[n]$.
- [05] (e) Compute the average power of $x[n]$ directly from 1 period of $x[n]$.
- [05] (f) Compute the average power of $x[n]$ using Parseval's theorem and X_k.
  Also, confirm your answers to (e) and (f) agree (they DO, don't they?)

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[20] 2. Compute the 8-point DFTs of the following signals (1 period of each is shown):
(a) $\{7, 7, 7, 7, 7, 7, 7, 7\}$ (b) $\{0, 0, 0, 1, 0, 0, 0, 0\}$ (c) $\cos(\frac{3}{4}\pi n)$ (d) $\cos(\frac{3}{4}\pi n + 1)$
Do **not** compute these by hand! **Think** and write the answer down by inspection.

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[30] 3. *Using the DFT for signal compression:*
MP3 digital audio compression and JPEG digital image compression do this in part:
The basic idea of signal compression is that the Fourier series expansions of signals often have many Fourier coefficients that are small, and can thus be discarded (set=0) without affecting the signal much. Compare #3 of HW#6 to #3 of HW#4.

```
load train;N=length(y);F=fft(y)/N;F=F.*[abs(F)>?]; %"?"=your threshold
Y=N*real(ifft(F));NMSE=sum((y-Y).^ 2)/sum(y.^ 2);C=sum(abs(F)>?)/N;%kept
```

These 2 lines of Matlab code use the DFT to compress a train whistle signal. They:
(1) load the train whistle $x[n]$=`train.mat`; (2) compute its DFT $\{X_k\}$;
(3) set $X_k = 0$ for all values of $|X_k| <$`threshold`, where you set `"?"=threshold`;
(4) compute the compressed train whistle signal $y[n]$ using remaining nonzero X_k;
Only nonzero X_k are used to synthesize the compressed signal→need less storage;
Compute Normalized Mean Square Error `NMSE` and Compression or sparsity ratio `C`.

- [05] (a) Add lines of Matlab code to: (1) plot $x[n]$ and its compressed version $y[n]$;
  (2) plot $|X_k|$ before and after sparsification (note "underbrush" is cleared out).
  Use `subplot` and various Matlab labelling commands to **label** your plots.
- [20] (b) Try 5 different values of `threshold` and run to compute the resulting NMSEs.
- [05] (c) Determine the value of `threshold` that makes $NMSE \leq 0.0018$ (rounded off).

"Football embodies the worst of America–violence punctuated by committee meetings"–
George Will