ASSIGNED: Feb. 10, 2006. **READ:** Part 3d of Official Lecture Notes (available on-line).
DUE DATE: Feb. 17, 2006. **TOPICS:** Discrete-time signals, sinusoids, and line spectra.

Show work on separate sheets of paper. Include all hand and Matlab plots and code.

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[15] 1. Compute the periods of the discrete-time sinusoids: (Answers better be integers!)
[5] (a) $\cos(\frac{5}{3}\pi n+1)$. [5] (b) $\cos(0.56\pi n+1)$. [5] (c) $\cos(3n+1)$.

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[20] 2. By hand, plot the line spectra of the following signals for $|\omega|\leq 3\pi$:

[10] (a) $x[n]=5+2\cos(\frac{\pi}{6}n+1)+4\cos(\frac{\pi}{3}n-2)$

[10] (b) $x[n]=\{\ldots 3,-1,\underline{3},-1,3,-1\ldots\}$. **Hints:** $(-1)^n=\cos(\pi n)$; what is $M(x)$?

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[20] 3. *Line spectra of discrete-time signals obtained by sampling continuous-time signals:*
$x(t)=\cos(2\pi 300t)+2\cos(2\pi 600t)$ is sampled at 1 kHz (thus $x[n]=x(t=n/1000)$).

[5] (a) By hand, plot the line spectrum of $x(t)$ for $|f|\leq 1500$ Hertz.

[5] (b) By hand, plot the line spectrum of $x[n]$ for $|\omega|\leq 3\pi$ (3 periods of spectrum).

[5] (c) Give a *second* continuous-time signal $y(t)$ such that $y[n]=x[n]$ after sampling.

[5] (d) $x[n]\rightarrow$ |IDEAL INTERPOLATOR| $\rightarrow \hat{x}(t)$. Does $\hat{x}(t)=x(t)$?

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[25] 4. Periodic **ramp** x[n] = Compute all constants $\{x_k,c_0,c_1,\theta_1,a_0,a_1,b_1\}$

$$x[n]=\begin{cases}1, & n=\ldots\text{-}3,0,3,6\ldots;\\ 2, & n=\ldots\text{-}2,1,4,7\ldots;\\ 3, & n=\ldots\text{-}1,2,5,8\ldots;\end{cases}=\begin{cases}x_0+x_1e^{j\frac{2\pi}{3}n}+x_2e^{j\frac{4\pi}{3}n}\\ c_0+c_1\cos(\frac{2\pi}{3}n+\theta_1)\\ a_0+a_1\cos(\frac{2\pi}{3}n)+b_1\sin(\frac{2\pi}{3}n)\end{cases}$$

Hint: Compute x_0,c_0,a_0 directly, then solve 2 equations in 2 unknowns for a_1,b_1.
Note: Isn't there an easier way to compute Fourier coefficients? Yes–the DFT!

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[20] 5. *Voice scramblers in continuous-time and discrete-time:*
Voice scramblers were used extensively for radio communication in World War II. They worked by swapping the upper and lower sidebands of the signal spectum. To see a simple illustration of the concept used, run the following Matlab code:

```
load handel;X=y(27001:35192);FX=fft(X);FY=fftshift(FX);Y=real(ifft(FY));
subplot(321),plot(X);subplot(322),plot(fftshift(abs(FX)));sound(X)
subplot(323),plot(Y);subplot(324),plot(fftshift(abs(FY)));sound(Y)
```

Note: The plots of `X` and `Y` look the same. But *listen* to them.

[5] (a) Listen to the original signal X and its scrambled version Y.
Can you understand the content of X by listening to Y?

[5] (b) Examine the magnitude spectra FX and FY of X and Y.
Describe in words how their spectra are related to each other.

[5] (c) How can you recover the original X from the scrambled Y?

[5] (d) In discrete-time, there is another, even simpler relation between X and Y. See if you can figure out what it is.

"An optimist is an accordion player with a beeper"-Ted Koppel