Last Name: ______________________

First Name: ______________________

ID Number: ______________________

Lab day/time: ______________________

Lecture time: ______________________

I have neither given nor received aid on this examination, nor have I concealed any violation of the Honor Code.

Signature: ______________________

EECS 206 Exam 3, 2006-4-20

DO NOT TURN THIS PAGE OVER UNTIL TOLD TO BEGIN!

- This is a 2 hour exam.
- It is closed book, closed notes, closed computer.
- You may use two 8.5”x11” pieces of paper, both sides, and a calculator.
- There are 28 problems for a total of 150 points. The questions are not necessarily in order of increasing difficulty.
- This exam has 6 pages. Make sure your copy is complete.
- Read difference equations carefully: $y[n-1]$ etc. might be on either side of equality.
- Continuing to write *anything* after the ending time is announced will be considered an honor code violation.
  Fill out your name etc. above now, and do not wait until the end to circle your answers!
- Clearly circle your final answers on this copy of the exam, not elsewhere.

1. (5 points)
Determine the z-transform of the signal $(-1)^n\, u[n] + (-2)^n\, u[n]$.

a) $\dfrac{1}{2+3z^{-1}}$ b) $\dfrac{1}{(1+z^{-1})(1+2z^{-1})}$ c) $\dfrac{2z^2+3z}{z^2+3z+2}$ d) $\dfrac{2z+3}{z^2+3z+2}$ e) $\dfrac{z^2+3z+2}{2z^2+3z}$ f) $\dfrac{2+2z^{-1}}{1+2z^{-1}}$

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2. (5 points)
Determine the z-transform of the signal $\{\underline{1}, 2\} + u[n]$.

a) $1+2z+\dfrac{1}{1+z^{-1}}$ c) $1+2z+\dfrac{z}{1-z}$ e) $\dfrac{2+z+2z^2}{1+z}$

b) $1+2z^{-1}+\dfrac{1}{1+z}$ d) $\dfrac{2+z^{-1}-2z^{-2}}{1-z^{-1}}$ f) $\dfrac{2+z^{-1}+2z^{-2}}{1+z^{-1}}$

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3. (5 points)
Determine the z-transform of the signal $\{\underline{1}, 2, 2, 2, 2, 2, \ldots\}$.

a) $\dfrac{3-z^{-1}}{1-z^{-1}}$ b) $\dfrac{1+z^{-1}}{1-2z^{-1}}$ c) $\dfrac{2+z^{-1}}{1-2z^{-1}}$ d) $\dfrac{1+z^{-1}}{1-z^{-1}}$ e) $\dfrac{2+z^{-1}}{1-z^{-1}}$ f) $\dfrac{2+z^{-1}}{1+z^{-1}}$

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4. (5 points)
Determine the inverse z-transform of $\dfrac{2z-3}{z(z-1)}$.

a) $\{\underline{2}, -3\}$ c) $\{\underline{2}, -3\} * u[n-1]$ e) $3\,\delta[n-1] - u[n]$

b) $\{\underline{2}, -3\} * u[n]$ d) $3\,\delta[n] - u[n]$ f) $3\,\delta[n-1] + u[n-1]$

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5. (5 points)
Given $\{\underline{1}, -3\} \rightarrow$ $\boxed{\text{LTI } h[n]}$ $\rightarrow \{\underline{1}, -5, 6\}$, determine $h[1]$.

a) -2 b) -1 c) 0 d) 1 e) 2 f) insufficient information

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6. (5 points)
Given $\{\underline{1}, 1, -6\} \rightarrow$ $\boxed{\text{LTI}}$ $\rightarrow \{\underline{1}, 1, 1\}$. Which of the following best describes the system? (“S” = BIBO stable)

a) FIR and S b) FIR and not S c) IIR and S d) IIR and not S e) noncausal f) nonlinear

7. (5 points)
For the system $y[n] = y[n-2] + x[n] + x[n-2]$, determine the response to the input signal $\cos\left(\frac{\pi}{4}n\right)$.
a) $\cos\left(\frac{\pi}{4}n\right)$ b) $\sin\left(\frac{\pi}{4}n\right)$ c) $-\cos\left(\frac{\pi}{4}n\right)$ d) $-\sin\left(\frac{\pi}{4}n\right)$ e) 0 f) $\cos\left(\frac{\pi}{4}n - \pi/4\right)$

8. (5 points)
For the system $y[n] = y[n-2] + x[n] + x[n-2]$, which of the following signals is eliminated?
a) $\sin\left(\frac{\pi}{4}n\right)$ b) $\sin\left(\frac{\pi}{2}n\right)$ c) $\sin\left(\frac{3\pi}{4}n\right)$ d) $\sin(\pi n)$ e) $u[n]$ f) $(-1)^n\, u[n]$

9. (5 points)
For the system $y[n] = y[n-2] + x[n] + x[n-2]$, the output signal $\cos\left(\frac{3\pi}{4}n\right)$ is produced by which input signal?
a) $\cos\left(\frac{3\pi}{4}n - \frac{\pi}{2}\right)$ b) $\cos\left(\frac{3\pi}{4}n\right)$ c) $\cos\left(\frac{3\pi}{4}n + \frac{\pi}{2}\right)$ d) $\sqrt{2}\cos\left(\frac{3\pi}{4}n - \frac{\pi}{2}\right)$ e) $\sqrt{2}\cos\left(\frac{3\pi}{4}n\right)$ f) $\sqrt{2}\cos\left(\frac{3\pi}{4}n + \frac{\pi}{2}\right)$

10. (5 points)
The system $y[n] = y[n-2] + x[n] + x[n-2]$ is (C = causal, S = BIBO stable):
a) C and S b) C and not S c) not C and S d) not C and not S e) nonlinear f) FIR

11. (5 points)
For the system $y[n] = y[n-2] + x[n] + x[n-2]$, determine the frequency response.
a) $\dfrac{1 + \mathrm{e}^{-j2\omega}}{1 - \mathrm{e}^{-j2\omega}}$ b) $\dfrac{1 - \mathrm{e}^{-j2\omega}}{1 + \mathrm{e}^{-j2\omega}}$ c) $\dfrac{1 + \mathrm{e}^{j2\omega}}{1 - \mathrm{e}^{j2\omega}}$ d) $\dfrac{1 - \mathrm{e}^{j2\omega}}{1 + \mathrm{e}^{j2\omega}}$ e) $\tan(\omega)$ f) 1

12. (5 points)
The signal $\cos\left(\frac{\pi}{4}n\right)$ is the input to the system $y[n] = y[n-2] + x[n] + x[n-2]$.
Determine the average power of the output signal.
a) 0 b) 1/4 c) 1/2 d) 1 e) 2 f) 4

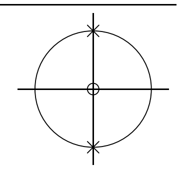
13. (5 points)
For the system $y[n] = y[n-2] + 4\,x[n-1]$, determine its system function.

a) $\frac{z^2-1}{z}$ b) $\frac{z}{z^2-1}$ c) $\frac{z^2+1}{z}$ d) $4\frac{z^2-1}{z}$ e) $4\frac{z}{z^2-1}$ f) $4\frac{z}{z^2+1}$

14. (5 points)
For the system $y[n] = y[n-2] + 4\,x[n-1]$, determine the impulse response.

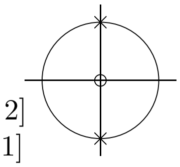
a) $2^n\,u[n] - (-2)^n\,u[n]$
c) $2\,u[n] - 2(-1)^n\,u[n-1]$
e) $-2\,u[n] + 2(-1)^n\,u[n]$
b) $2^n\,u[n] + (-2)^n\,u[n]$
d) $2\,u[n] + 2(-1)^n\,u[n-1]$
f) $2\,u[n] - 2(-1)^n\,u[n]$

15. (5 points)
An LTI system has the following pole-zero plot. Determine its system function.



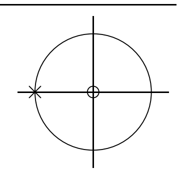
a) $\frac{z}{1+z^2}$ b) $\frac{z}{1-z^2}$ c) $\frac{1}{1+z^2}$ d) $\frac{1}{1-z^2}$ e) $\frac{1-z^2}{z}$ f) $\frac{1+z^2}{z}$

16. (5 points)
An LTI system has the following pole-zero plot. Determine its difference equation.



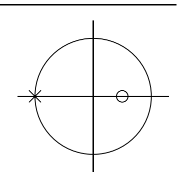
a) $y[n] - y[n-1] = x[n-2]$
c) $y[n] = -y[n-1] + x[n-2]$
e) $y[n] = y[n-1] - x[n-2]$
b) $y[n] - y[n-2] = x[n-1]$
d) $y[n] = -y[n-2] + x[n-1]$
f) $y[n] = y[n-2] - x[n-1]$

17. (5 points)
$u[n] \rightarrow \boxed{\text{LTI}} \rightarrow y[n]$ where the system has the following pole-zero plot. Determine $y[2]$.



a) -2 b) -1 c) 0 d) 1 e) 2 f) 3

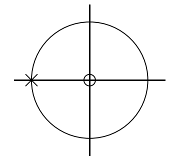
18. (5 points)
An LTI system with impulse response $h[n] = \delta[n-1] + u[n]$ is in series with a system that has the following pole-zero plot. Determine the number of poles of the cascaded system.



a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

19. (5 points)

An LTI system has impulse response $h[n]$ and the following pole-zero plot. Determine $h[3]/h[0]$.



a) -2 b) -1 c) 0 d) 1 e) 2 f) 3

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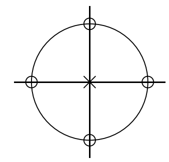
20. (5 points)

The LTI system with the following pole-zero plot will eliminate which of the input signals below?



a) $\cos(n/4)$ b) $\cos(n/2)$ c) $\cos(n)$ d) $\cos(n/2 + \pi/2)$ e) $\sin(n/2)$ f) $(-1)^n$

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21. (5 points)

Which of these filters $h[n]$ eliminates a 5 Hz component in a signal sampled at 30 Hz?

a) $\{\underline{1}, 1, 1\}$ b) $\{\underline{1}, 0, 1\}$ c) $\left\{\underline{1}, \sqrt{2}, 1\right\}$ d) $\left\{\underline{1}, -\sqrt{2}, 1\right\}$ e) $\{\underline{1}, -2, 1\}$ f) $\{\underline{-1}, 1, -1\}$

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22. (5 points)

An LTI system has impulse response $h[n] = u[n] + 2^n u[n]$. Determine its difference equation.

a) $y[n] - 3y[n-1] + 2y[n-2] = -2x[n] + 3x[n-1]$

b) $y[n] + 3y[n-1] - 2y[n-2] = -2x[n] - 3x[n-1]$

c) $y[n] - 3y[n-1] + 2y[n-2] = 2x[n] - 3x[n-1]$

d) $y[n] + 3y[n-1] - 2y[n-2] = -2x[n] + 3x[n-1]$

e) $y[n] - 2y[n-1] + 3y[n-2] = 3x[n] - 2x[n-1]$

f) $y[n] + 2y[n-1] - 3y[n-2] = -3x[n] + 2x[n-1]$

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23. (5 points)

An LTI system has frequency response $\dfrac{3}{e^{j\omega}(-5 + 2\cos\omega)}$. Determine its difference equation.

a) $y[n] = 3y[n-2] + 5x[n-1] - x[n]$

b) $y[n] = 3y[n-2] - 5x[n-1] + x[n]$

c) $y[n] = 3y[n-2] - 5x[n-1] + x[n]$

d) $y[n] = 3y[n-2] - 5x[n-1] + x[n-2]$

e) $y[n] = 5y[n-1] - y[n-2] + 3x[n-2]$

f) $y[n] = 5y[n-1] + y[n-2] + 3x[n-2]$

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24. (5 points)

The output of a digital voice scrambler is related to the input by $y[n] = (-1)^n x[n]$.

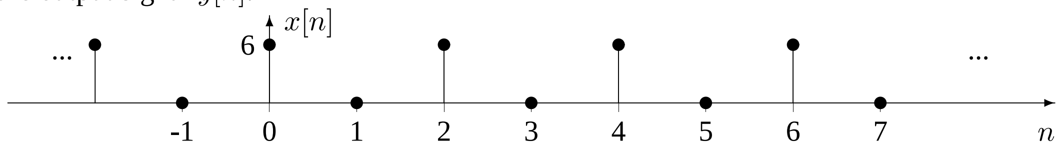
Using the definition of the z-transform, express $Y(z)$ in terms of $X(z)$.

a) $X\left(z^{-1}\right)$ b) $1/X(z)$ c) $-X(z)$ d) $X(-z)$ e) $\frac{1}{1+z^{-1}} X(z)$ f) $-X\left(-z^{-1}\right)$

25. (10 points)
Carefully draw the pole-zero plot of the system $y[n] + \frac{1}{4}\, y[n-2] = x[n] - 0.64\, x[n-2]$.

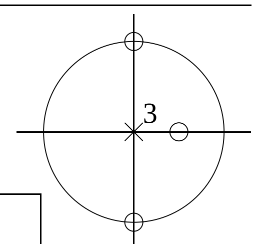
26. (5 points)
The filter $y[n] + \frac{1}{4}\, y[n-2] = x[n-1] - 0.64\, x[n-2]$ would be best described as:
a) lowpass b) highpass c) bandpass d) notch e) FIR f) unstable

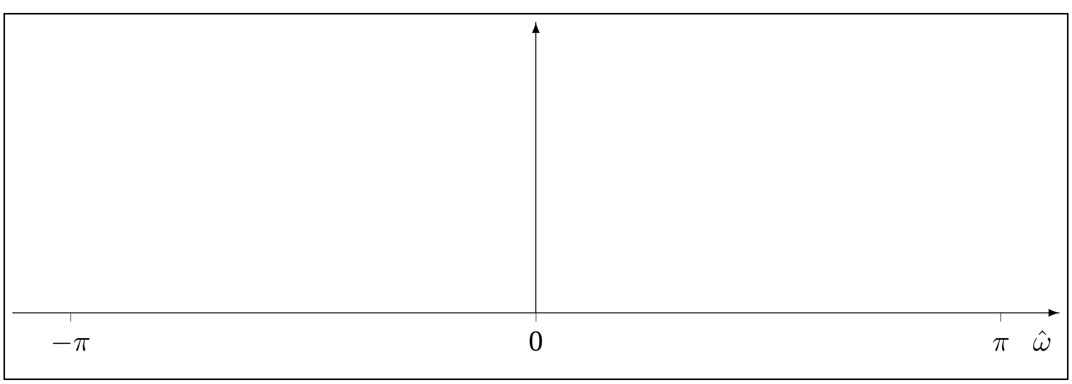
27. (5 points)
The following periodic signal is the input to an LTI system described by $y[n] = \frac{1}{2}\, y[n-1] + x[n]$.
Determine the output signal $y[n]$.



a) $3 + 3\cos(\pi n)$
b) $3 + 2\cos(\pi n)$
c) $2 + 3\cos(\pi n)$
d) $2 + 6\cos(\pi n)$
e) $6 + 2\cos(\pi n)$
f) $6 + 3\cos(\pi n)$

28. (10 points)
Sketch the (relative) magnitude response of the filter that has the following pole-zero plot.





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