Part II: Interdependent Markets
Outline

• Agents for “simple” interdependent markets
  • Simultaneous & Sequential
• Agents for more complex interdependent markets
  • Simulated markets: TAC games
  • Real-world market: Quibids
• Mike: Empirical game-theoretic analysis (EGTA), to predict equilibrium outcomes in these markets
Interdependent markets

Since 1994:
93 auctions, $60+ billion

2011:
$68.6 billion in sales.
100 million active users.

2011:
$85 billion spent in online advertising
worldwide (24% increase from 2010).

2011:
582,000 tons
$5.7 billion/year
Key bidder challenges

Bidders have preferences over outcomes, but outcomes depend on the actions of others.

Bidders have interdependent preferences, but markets are often independent.

Complements

39 simultaneous auctions
125,000 transactions/day
$5.38 billion/year
Key bidder challenges

Bidders have preferences over outcomes, but outcomes depend on the actions of others.

Bidders have interdependent preferences, but markets are often independent.

Substitutes

39 simultaneous auctions
125,000 transactions/day
$5.38 billion/year
Traditional economic solutions

...outcomes depend on the actions of others.

- Dominant strategy: strategy $S$ is best, no matter what others do.
- Equilibrium strategy profile: assignment of strategies to players s.t. each player’s strategy is a best response to its opponents’ strategies.

Dominant strategies often don’t exist.
Finding equilibria can be hard in practice. [Daskalakis 2008]

...interdependent preferences, but must bid in independent auctions.

- Use interdependent (combinatorial) auctions.
- Compute equilibria in stylized theoretical models.

Independent auctions are inevitable.
Bidder complexity remains in practice even if theoretical model abstracts it away.
AI approach

Build autonomous agents that bid in complex auctions (usually without explicitly computing equilibria).
AI approach

Build autonomous agents that bid in complex auctions (usually without explicitly computing equilibria).

Just Do It!

Build autonomous agents that bid in complex auctions (usually without explicitly computing equilibria).
AI approach

Just Do It!

But how?

Build autonomous agents that bid in complex auctions (usually without explicitly computing equilibria).
Agent architecture

Two-phased architecture: 1. Prediction and 2. Optimization (PO)
PO bidding heuristics seem to work well in practice. But do they have any theoretical justification?

Two-phased architecture: 1. Prediction and 2. Optimization (PO)
- Consider “simpler” models (not simple enough to solve analytically)
- Develop and test multiple PO bidding heuristics for these models
  - Empirically search for equilibria among these PO heuristics
  - Use PO heuristics that comprise found equilibria to guide bidding
- Argue that PO bidding heuristics are sufficient
“Simpler” models

Simultaneous and Sequential Auctions

Bidder 1

Bidder 2

Bidder 3

Bidder 4

FOR SALE
“Simpler” models

Simultaneous Auctions
“Simpler” models

Sequential Auctions

Bidder 1

Bidder 2

Bidder 3

Bidder 4

FOR SALE

A D G J
PO Heuristics for Second-price, Sealed-bid, Simultaneous Auctions
Simultaneous auctions

Bidder 1

Bidder 2

Bidder 3

Bidder 4

FOR SALE

A

B

Tuesday, July 24, 12
Simultaneous auctions

Bidder 1

Bidder 2

Bidder 3

Bidder 4

FOR SALE

A

B

$0

$0

$0

$0
Simultaneous auctions

Bidder 1

Bidder 2

Bidder 3

Bidder 4

FOR SALE

A

B
Simultaneous auctions

Bidder 1
-A
Bidder 2
-A
Bidder 3
-A
Bidder 4
-A

FOR SALE

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Simultaneous auctions

FOR SALE

Tuesday, July 24, 12
Simultaneous auctions

Bidder 1
- $6
A

Bidder 2
- $0
A 6
B 4

Bidder 3
- $0
A 2
B 7

Bidder 4
- $7
A 4
B 8

SOLD
Simultaneous auctions

Bidder 1

Bidder 2

Bidder 3

Bidder 4

Values

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Values

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<td>B</td>
<td>12</td>
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<tr>
<td>AB</td>
<td>15</td>
<td>8</td>
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</tbody>
</table>
Simultaneous auctions

**Bidder 1**
- Value: $6
- Value: $0

**Bidder 2**
- Value: $6
- Value: $0

**Bidder 3**
- Value: $6
- Value: $0

**Bidder 4**
- Value: $7

**Results**
- Bidder 1: A = 10, B = 5
- Bidder 2: A = 6, B = 4
- Bidder 3: A = 2, B = 7
- Bidder 4: A = 4, B = 8

**SOLD**
Simultaneous auctions

IPV: Independent private values drawn from a commonly known prior.
A suite of PO heuristics

DOWNLOAD
market state information from server

PREDICT
market prices

OPTIMIZE:
i.e., make decisions

UPLOAD
decisions to server
OPTIMIZE: i.e., make decisions
## Price Predictions

<table>
<thead>
<tr>
<th>Point Predictions</th>
<th>Distributional Predictions</th>
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<tbody>
<tr>
<td><strong>A</strong></td>
<td>$1</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>$2</td>
</tr>
</tbody>
</table>

The table on the left shows point predictions for two categories, A and B, with corresponding prices of $1 and $2. The right side illustrates distributional predictions, with marginal and joint distributions depicted graphically.
Taxonomy of PO Heuristics

- **given point prices**
  - StraightMV
  - TargetBidder
    - TargetPrice
    - TargetMV
    - TargetMV*

- **given price distributions**
  - collapse distributions
    - EVM
  - exploit distributions
    - AverageMU
    - Bid Evaluator
    - Sample Average
      - BE(TMU)
      - BE(TMU*)
      - SAA
      - SAA*

- **bidding heuristic**
  - LocalBid
Taxonomy of PO Heuristics

- **bidding heuristic**
  - **given price distributions**
    - **collapse distributions**
      - **exploit distributions**
        - **LocalBid**
  - **given point prices**
    - **StraightMU**
    - **TargetBidder**
      - **TargetPrice**
      - **TargetMV**
      - **TargetMV***
    - **EVM**
    - **AverageMU**
    - **Bid Evaluator**
      - **Sample Average**
    - **TargetMU**
    - **TargetMU***
    - **StraightMU**
    - **BE(TMU)**
    - **BE(TMU)***
    - **SAA**
    - **SAA***

---

**PO Heuristics**

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Point Price Predictions

Model

- a set of goods $\mathcal{X}$
- a valuation function $v: 2^\mathcal{X} \rightarrow \mathbb{R}$
- a per-good price prediction $p: \mathcal{X} \rightarrow \mathbb{R}$

Assumption: Linear Prices

- a per-bundle price prediction: $P(X) = \sum_{x \in X} p(x)$, for $X \subseteq \mathcal{X}$
Point Price Predictions

Deterministic Bidding Problem

\[ \text{DET}(p) = \max_{b \in \mathbb{R}^X} (\nu(\text{Winnings}(p,b)) - P(\text{Winnings}(p,b))) \]

Winner Determination Rule

\( x \in \text{Winnings}(p,b) \) if and only if \( b(x) \geq p(x) \)
Naive Idea

For each good, bid (up to) its independent value.
Complementary Goods

FOR SALE

Values

<table>
<thead>
<tr>
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Price Predictions

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<tbody>
<tr>
<td>Bid</td>
<td>$5</td>
<td>$4</td>
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Bidder 1

$0

Tuesday, July 24, 12
Complementary Goods

Values

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Bidder 1

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Bid

FOR SALE

Tuesday, July 24, 12
Bidder 1 loses both but wishes it had won both (since 10 > 9)
Substitutable Goods
Substitutable Goods

Bidder 1

<table>
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<tr>
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$0

Price Predictions

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<tbody>
<tr>
<td>Predicted Price</td>
<td>$25</td>
<td>$15</td>
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FOR SALE

A

B

Tuesday, July 24, 12
Substitutable Goods

Bidder 1

Values

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Bid

Price Predictions

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<td>1</td>
<td>$25</td>
<td>$15</td>
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FOR SALE

A

B

$0

- $40
Substitutable Goods

Bidder 1 wins both but wishes it hadn’t (since 35 < 40)
For each good, bid (up to) its marginal (or dependent) value, relative to its winnings $X$: i.e.,

$$MV(x, X) = v(X \cup \{x\}) - v(X \setminus \{x\}).$$
Better Idea

For each good, bid (up to) its marginal (or dependent) value, relative to its winnings $X$: i.e.,

$$ MV(x, X) = v(X \cup \{x\}) - v(X \setminus \{x\}). $$

Fails for simultaneous auctions, since $MV(x, \emptyset)$ is independent value.
For each good, bid (up to) its marginal (or dependent) value, relative to its potential winnings, given price predictions $p$: i.e., $MV(x,p)$. 
Definitions

**Surplus:** $\sigma(X,p) = v(X) - P(X)$

**Optimal Surplus:** $\sigma^*(p) = \max_{X \subseteq \chi} \sigma(X,p)$

**Optimal Bundle:** $X^* \in \arg \max_{X \subseteq \chi} \sigma(X,p)$

**Marginal Value at Prices:**
$$MV(x,p) = \sigma^*(p[p(x) \leftarrow 0]) - \sigma^*(p[p(x) \leftarrow \infty])$$
Complementary Goods

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<tr>
<td>$5</td>
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Bidder 1

Bid: $0

FOR SALE

A

B
Complementary Goods

\[ MV(A, p) = (v(AB) - p(B)) - (v(B) - p(B))^+ = 6 - 0 = 6 \]
\[ MV(B, p) = (v(AB) - p(A)) - (v(A) - p(A))^+ = 5 - 0 = 5 \]
Complementary Goods

\[ MV(A, p) = (v(AB) - p(B)) - (v(B) - p(B))^+ = 6 - 0 = 6 \]

\[ MV(B, p) = (v(AB) - p(A)) - (v(A) - p(A))^+ = 5 - 0 = 5 \]
Complementary Goods

\[ MV(A,p) = (v(AB) - p(B)) - (v(B) - p(B))^+ = 6 - 0 = 6 \]
\[ MV(B,p) = (v(AB) - p(A)) - (v(A) - p(A))^+ = 5 - 0 = 5 \]
Complementary Goods

\[ \text{MV}(A,p) = (v(AB) - p(B)) - (v(B) - p(B))^+ = 6 - 0 = 6 \]
\[ \text{MV}(B,p) = (v(AB) - p(A)) - (v(A) - p(A))^+ = 5 - 0 = 5 \]

Bidder 1 wins both goods and is happy! (Since 10 > 9)
Substitutable Goods

Values

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Bidder I

Bid

Price Predictions

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FOR SALE

A

B
Substitutable Goods

\[ MV(A, p) = v(A) - (v(B) - p(B))^+ = 30 - 10 = 20 \]

\[ MV(B, p) = v(B) - (v(A) - p(A))^+ = 25 - 5 = 20 \]
Substitutable Goods

\[ MV(A,p) = v(A) - (v(B) - p(B))^+ = 30 - 10 = 20 \]

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Substitutable Goods

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Substitutable Goods

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MV(A,p) = v(A) - (v(B) - p(B))^+ = 30 - 10 = 20
\]

\[
MV(B,p) = v(B) - (v(A) - p(A))^+ = 25 - 5 = 20
\]

Bidder 1 wins only B, and is happy!
(since 25 > 15)
Not So Fast ...

Values

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Bidder 1

Bid

$0

Price Predictions

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FOR SALE

A

B
Not So Fast ...

MV at prices for each good: $2 - (2 - 1) = 1$
Not So Fast ...

MV at prices for each good: $2 - (2 - 1) = 1$
Not So Fast ...

MV at prices for each good: 2 - (2 - 1) = 1
Not So Fast ...

MV at prices for each good: $2 - (2 - 1) = 1$

Bidder wins all the goods, but wishes it had only won one. (since $l < N$)
MV Characterization Theorem

For all goods $x$ in $\mathcal{X}$,

- $x$ is in all optimal bundles iff $MV(x, p) > p(x)$
- $x$ is in no optimal bundles iff $MV(x, p) < p(x)$
- otherwise, $MV(x, p) = p(x)$
Taxonomy of PO Heuristics

TreeNode: given point prices
  - TreeNode: StraightMV
  - TreeNode: TargetBidder
    - TreeNode: TargetPrice
    - TreeNode: TargetMV
    - TreeNode: TargetMV*
  - TreeNode: bidding heuristic
    - TreeNode: EVM
    - TreeNode: AverageMU
    - TreeNode: Bid Evaluator
      - TreeNode: BE(TMU)
      - TreeNode: BE(TMU*)
    - TreeNode: exploit distributions
      - TreeNode: Sample Average
        - TreeNode: SAA
        - TreeNode: SAA*
  - TreeNode: given price distributions
    - TreeNode: collapse distributions
MV-based Strategies

**StraightMV**: bid marginal values at prices on all goods

**Target Heuristics**: choose an optimal bundle and bid only on the goods in that bundle

- **TargetPrice**: bids predicted prices on goods in bundle
- **TargetMV**: bids marginal values at prices on goods in bundle
- **TargetMV\*: bids target marginal values at prices goods in bundle
MV-Based Strategies, cont.
Target MV of $x$ at prices $p$ given target $X^*$

$$\text{MV}(x,p,X^*) = \tilde{\sigma}^*(\tilde{p}[p(x)\leftarrow 0]) - \tilde{\sigma}^*(\tilde{p}[p(x)\leftarrow \infty])$$

where $\tilde{p} = p[p(y)\leftarrow \infty, \forall y \notin X^*]$
MV-Based Strategies, cont.

Target MV of $x$ at prices $p$ given target $X^*$

$$MV(x, p, X^*) = \tilde{\sigma}^*(\tilde{p}[p(x) \leftarrow 0]) - \tilde{\sigma}^*(\tilde{p}[p(x) \leftarrow \infty])$$

where $\tilde{p} = p[p(y) \leftarrow \infty, \forall y \not\in X^*]$

**Marginal Value at Prices:**

$$MV(x, p) = \sigma^*(p[p(x) \leftarrow 0]) - \sigma^*(p[p(x) \leftarrow \infty])$$
MV-Based Strategies, cont.

Target MV of $x$ at prices $p$ given target $X^*$

$$MV(x,p,X^*) = \tilde{\sigma}^*(\tilde{p}[p(x)\leftarrow 0]) - \tilde{\sigma}^*(\tilde{p}[p(x)\leftarrow \infty])$$

where $\tilde{p} = p[p(y)\leftarrow \infty, \forall y \notin X^*]

**Lemma**

$$MV(x,p,X^*) \geq MV(x,p), \forall x \in X^*$$

Marginal Value at Prices:

$$MV(x,p) = \sigma^*(p[p(x)\leftarrow 0]) - \sigma^*(p[p(x)\leftarrow \infty])$$
MV-Based Strategies, cont.

Target MV of $x$ at prices $p$ given target $X^*$

$$MV(x, p, X^*) = \tilde{\sigma}^*(\tilde{p}[p(x)\leftarrow 0]) - \tilde{\sigma}^*(\tilde{p}[p(x)\leftarrow \infty])$$

where $\tilde{p} = p[p(y)\leftarrow \infty], \forall y \notin X^*$

Lemma

$$MV(x, p, X^*) \geq MV(x, p), \forall x \in X^*$$

Proof

$$\sigma^*(p[p(x)\leftarrow 0]) = \sigma^*(\tilde{p}[\tilde{p}(x)\leftarrow 0]) = \sigma(X^*, p) + p(x)$$

$$\sigma^*(\tilde{p}[\tilde{p}(x)\leftarrow \infty]) \leq \sigma^*(p[p(x)\leftarrow \infty])$$
Which heuristic is best?

Diminishing Marginal Values (DMV)

Winning more of some good can only decrease the MVs of other goods. Winning less of some good can only increase the MVs of other goods.

Observation:
Assuming DMV, TargetMV* dominates TargetMV dominates Target Price.

But, in general, none of the MV-based heuristics dominates any other

- StraightMV hedges, so could easily win too much
- Target Heuristics don’t hedge, so could easily lose too much
Taxonomy of PO Heuristics

- **Given point prices**
  - StraightMV
  - TargetBidder
    - TargetPrice
    - TargetMV
    - TargetMV*

- **Given price distributions**
  - Bidding heuristic
    - Collapse distributions
      - EVM
      - AverageMU
    - Exploit distributions
      - Bid Evaluator
      - Sample Average
        - TargetMU
        - TargetMU*
        - StraightMU
        - BE(TMU)
        - BE(TMU*)
        - SAA
        - SAA*
Distributional Price Predictions

Model

- a set of goods $\mathcal{X}$
- a valuation function $v: 2^\mathcal{X} \rightarrow \mathbb{R}$
- a range of possible (assume discretized) prices $\mathcal{P}$
- a per-good distributional price prediction $f: \mathcal{P} \rightarrow [0, 1]$ s.t. $\sum_{p \in \mathcal{P}} f(p) = 1$

Stochastic Bidding Problem

$$\text{BID}(p) = \max_{b \in \mathbb{R}^\mathcal{X}} \mathbb{E}_{p \sim f} [v(\text{Winnings}(p, b)) - P(\text{Winnings}(p, b))]$$

$$= \max_{b \in \mathbb{R}^\mathcal{X}} \mathbb{E}_{p \sim f} [u(p, b)]$$
Expected Value Method

1. Calculate $\bar{p} = \mathbb{E}_{p \sim f}[p]$.
2. Approximate solution to $\text{BID}(p)$ by $\text{DET}(\bar{p})$.

EVM-Based Heuristics

- MV-based heuristics, where MVs are calculated using $\bar{p}$, or an estimate of $\bar{p}$. We call these **MU heuristics**.
MV-based Strategies

**StraightMU**: bid marginal values at *expected* prices on all goods

**Target Heuristics**: choose an optimal bundle and bid only on the goods in that bundle

- **TargetEP**: bids predicted *expected* prices on all goods
- **TargetMU**: bids marginal values at *expected* prices on all goods
- **TargetMU***: bids *target* marginal values at *expected* prices on all goods
Taxonomy of PO Heuristics

given point prices
- StraightMV
- TargetBider
  - TargetPrice
  - TargetMV
  - TargetMV*

bidding heuristic
collapse distributions
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given price distributions
exploit distributions
- LocalBid
Taxonomy of PO Heuristics

- **given point prices**
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- **given price distributions**
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      - BE(TMU*)
      - SAA
      - SAA*

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AUTONOMOUS BIDDING AGENTS
Strategies and Lessons from the Trading Agent Competition

Michael P. Wellman, Amy Greenwald, and Peter Stone
“Collapsing” Heuristics

<table>
<thead>
<tr>
<th>Values</th>
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<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>AB</td>
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</tbody>
</table>

Bidder 1

Optimal Bid

Distributional Price Predictions

A

B

50
100

.5

5

1.0

FOR SALE

A

B
“Collapsing” Heuristics

Values

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<tr>
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Expected Prices

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<tr>
<td>1.0</td>
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Bidder 1

Optimal Bid

FOR SALE

A

B

Tuesday, July 24, 12
“Collapsing” Heuristics

Expected Prices

A  1.0  75
B  1.0  50

FOR SALE

Bidder 1

Values

A  0
B  0
AB  75

Optimal Bid

A  0
B  0

Tuesday, July 24, 12
“Collapsing” Heuristics

Bidder I loses both but wishes it had won both (stay tuned)
“Exploiting” Heuristics

Exploiting Heuristics

Values

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>0</th>
<th>0</th>
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Bidder 1

Optimal Bid

Distributional Price Predictions

<table>
<thead>
<tr>
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<th>100</th>
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<tbody>
<tr>
<td>A</td>
<td>.5</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1.0</td>
<td>5</td>
</tr>
</tbody>
</table>

FOR SALE

A

B
“Exploiting” Heuristics

Value(0,0) = 0
“Exploiting” Heuristics

Optimal Bid

FOR SALE

Value of Stochastic Information

Value(0,0) = 0
Value(50,5) = .5(20) + .5(-5) = 7.5
Algorithm 2 BidEvaluator\((f, S, K)\)

\[
\begin{align*}
\text{bestval} &\leftarrow -\infty \\
\text{sample } S \text{ scenarios } p_1, \ldots, p_S \text{ from } f \\
\text{for all } k = 1 \text{ to } K \text{ do} \\
&\quad b \leftarrow \text{Heuristic}(f) \{\text{candidate bid}\} \\
&\quad \text{currval} \leftarrow 0 \\
&\quad \text{for } i = 1 \text{ to } S \text{ do} \\
&\quad\quad X = \text{Winnings}(b, p_i) \\
&\quad\quad \text{surplus} = v(X) - P(X) \\
&\quad\quad \text{currval} \leftarrow \text{currval} + \frac{1}{S} \text{ surplus} \\
&\quad \text{end for} \\
&\quad \text{if } \text{currval} > \text{bestval} \text{ then} \\
&\quad\quad \text{bestval} \leftarrow \text{currval} \\
&\quad\quad \text{bestsol} \leftarrow b \\
&\quad \text{end if} \\
\text{end for} \\
\text{return } \text{bestsol}
\end{align*}
\]

StraightMU, TargetMU, etc. approximately \( \mathbb{E}_{p \sim f} [u(p,b)] \)
Sample Average Approximation

- sample $S$ scenarios $p_1, \ldots, p_S \sim f$
- $SAA(p_1, \ldots, p_S) = \max_{b \in \mathbb{R}^X} \sum_{i=1}^{S} (v(Winnings(p_i, b)) - P(Winnings(p_i, b)))$

Theorem [e.g., Shapiro and Homem-de-Mello 2001]:
The probability that an optimal solution to $SAA(p_1, \ldots, p_S)$ is an optimal solution to $BID(f) \rightarrow 1$ exponentially fast as $S \rightarrow \infty$. 

$\mathbb{E}_{p \sim f} [u(p, b)]$ 

approximately $\mathbb{E}_{p \sim f} [u(p, b)]$
Another MV-Based Heuristic

**StraightMV:** bid marginal values at prices on all goods

\[ MV(x, p) = \sigma^*(p[p(x) \leftarrow 0]) - \sigma^*(p[p(x) \leftarrow \infty]) \]

**StraightMV, EVM-style (StraightMU):**
bid marginal values at expected prices on all goods

\[ MV(x, \bar{p}) = \sigma^*(\bar{p}[\bar{p}(x) \leftarrow 0]) - \sigma^*(\bar{p}[\bar{p}(x) \leftarrow \infty]) \]

**ExpectedMV:**
bid expected marginal values at prices on all goods

\[ MV(x, p) = \mathbb{E}_{p \sim f}[\sigma^*(p[p(x) \leftarrow 0]) - \sigma^*(p[p(x) \leftarrow \infty])] \]

**SampleAverageMV (AverageMU):**
bid sample average marginal values at prices on all goods
Algorithm 1 SampleAverageMV($f, S$)

\[
\begin{align*}
    b &\leftarrow 0 \\
    \text{for } i = 1 \text{ to } S \text{ do} \\
    &\quad \text{sample } p \text{ from } f \\
    &\quad b \leftarrow b + \frac{1}{S} \text{StraightMV}(p) \\
    \text{end for} \\
    \text{return } b
\end{align*}
\]
Local Bid Search

**Key Idea:**
search for a **locally consistent** set of expected MVs at prices

---

**Algorithm 3 LocalBid\((f, T)\)**

Initialize \(b \leftarrow \text{Heuristic}(f)\)

for \(t = 1\) to \(T\) do

    for \(j = 1\) to \(m\) do

        \(b_j \leftarrow \mathbb{E}_{p \sim f} [v(\text{Winnings}(p, b) \cup \{j\}) - v(\text{Winnings}(p, b) \setminus \{j\})] \)

    end for

end for

return \(b\)
Local Bid Search

Values

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>2</td>
<td>2</td>
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</tbody>
</table>

Price Predictions

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$1</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>$1</td>
<td></td>
</tr>
</tbody>
</table>

Local Bid

FOR SALE

Bidder 1

A

B

Thursday, July 24, 12
Local Bid Search

Update $b_A$:

$$v(\emptyset \cup \{A\}) - v(\emptyset \setminus \{A\}) = 2 - 0 = 2$$

![Diagram showing values and price predictions for bidder 1, with local bid search and FOR SALE sign.]
Local Bid Search
Local Bid Search

Update $b_B$:

$$v(\{A\} \cup \{B\}) - v(\{A\} \setminus \{B\}) = 2 - 2 = 0$$
Local Bid Search

Bidder 1

Values

A  
B  
AB  

Price Predictions

A $1  
B $1  

Local Bid

converged!

FOR SALE
Local Bid Search

Bidder 1 wins only A and is happy! (since 2 > 1)
Scheduling Valuations
[Reeves et al., 2005]

LocalBid

Baselines

Price Predictions

CDFs of Highest Opponent Bids

Bidder 1

Optimization

LocalBid Profit vs. Optimal Profit

AverageMU Profit vs. Optimal Profit

BidEval Profit vs. Optimal Profit

<table>
<thead>
<tr>
<th>LocalBid Profit vs. Optimal Profit</th>
<th>Baselines</th>
<th>[Wellman, et al. 2011]</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.7% OPT</td>
<td>66.1% OPT</td>
<td>94.5% OPT</td>
</tr>
<tr>
<td>98.8% OPT</td>
<td>85.9% OPT</td>
<td></td>
</tr>
</tbody>
</table>

Optimization
Taxonomy of PO Heuristics

bidding heuristic

given point prices
- StraightMV
- TargetBidder
  - TargetPrice
  - TargetMV
  - TargetMV*

given price distributions
- collapse distributions
  - EVM
  - AverageMU
  - Bid Evaluator
  - TargetMU
  - TargetMU*
  - StraightMU
  - BE(TMU)
  - BE(TMU*)

exploit distributions
- LocalBid
  - Sample Average
  - SAA
  - SAA*
PREDICT market prices
Where do price predictions come from?
[Wellman et al., 2008] Self-confirming price predictions
Self-confirming price predictions

FOR SALE

A
B

Values

Bidder 1

Bidder 2

Bidder 3

Bidder 4

Price Predictions

LocalBid

LocalBid

LocalBid

LocalBid

A
B
AB

A
B
AB

A
B
AB

A
B
AB

-F$0

-F$0

-F$0

-F$0

65

65

Tuesday, July 24, 12
<table>
<thead>
<tr>
<th>Values</th>
<th>Bidder 1</th>
<th>Price Predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
</tr>
</tbody>
</table>

**Values**

- A: 10
- B: 10
- AB: 10

**LocalBid**

<table>
<thead>
<tr>
<th>Values</th>
<th>Bidder 2</th>
<th>Price Predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
</tr>
</tbody>
</table>

**Values**

- A
- B
- AB

**LocalBid**

<table>
<thead>
<tr>
<th>Values</th>
<th>Bidder 3</th>
<th>Price Predictions</th>
</tr>
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<tr>
<td></td>
<td></td>
<td>B</td>
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</tbody>
</table>

**Values**

- A
- B
- AB

**LocalBid**

<table>
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<tr>
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<th>Price Predictions</th>
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**Values**

- A
- B
- AB

**LocalBid**

**Self-confirming price predictions**

FOR SALE

A

B

---

66

Tuesday, July 24, 12

66
Self-confirming price predictions

Bidder 1

Bidder 2

Bidder 3

Bidder 4

LocalBid

Values

0

A

10

B

10

AB

10

$0

Price Predictions

A

B

Values

0

A

4

B

6

AB

6

$0

Price Predictions

A

4

B

5

Values

0

A

12

B

7

AB

15

$0

Price Predictions

A

13

B

6

Values

0

A

8

B

8

AB

9

$0

Price Predictions

A

2

B

8

FOR SALE

A

B
Self-confirming price predictions

Bidder 1
- $0

Bidder 2
- $0

Bidder 3
- $0

Bidder 4
- $0

LocalBid

Values

<table>
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<th>AB</th>
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<tbody>
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Price Predictions

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</thead>
<tbody>
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<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
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LocalBid

Values

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Price Predictions

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<tbody>
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<tr>
<td>B</td>
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LocalBid

Values

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Price Predictions

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<td>A</td>
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LocalBid

Values

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Price Predictions

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Self-confirming price predictions

FOR SALE

Bidder 1
Values
A 0
B 10
AB 10

Price Predictions
A
B

Bidder 2
Values
A 0
B 6
AB 6

Price Predictions
A
B

Bidder 3
Values
A 12
B 7
AB 15

Price Predictions
A
B

Bidder 4
Values
A 8
B 8
AB 9

Price Predictions
A
B

Good 1
Good 2

LocalBid

Tuesday, July 24, 12
Self-confirming price predictions

Bidder 1

Values

<table>
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<tr>
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<th>AB</th>
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<tr>
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Price Predictions

- $0

Bidder 2

Values

<table>
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Price Predictions

- $0

Bidder 3

Values

<table>
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<td>15</td>
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</table>

Price Predictions

- $0

Bidder 4

Values

<table>
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<tr>
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Price Predictions

- $0

LocalBid

FOR SALE
Self-confirming price predictions
Bidder 1

Values

<table>
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<tr>
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Price Predictions

A

B

LocalBid

$-0$

Bidder 2

Values

<table>
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<th></th>
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Price Predictions

A

B

LocalBid

$-0$

Bidder 3

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Price Predictions

A

B

LocalBid

$-0$

Bidder 4

Values

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<td>8</td>
<td>9</td>
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</table>

Price Predictions

A

B

LocalBid

$-0$

Self-confirming price predictions

FOR SALE

LocalBid

LocalBid

LocalBid

Price Predictions

A

B

A

B

Price Predictions

A

B

A

B

Price Predictions

A

B

A

B

Price Predictions

A

B

A

B

1

2

3

4

5

6

7

8

9

10

0.2

0.4

0.6

0.8

1

Good 1

Good 2

A

B

Self-confirming price predictions

Tuesday, July 24, 12
### Bidder 1
- **Value**
  - A: 0
  - B: 10
  - AB: 10
- **Price Predictions**
  - A
  - B
- **LocalBid**: -$0

### Bidder 2
- **Value**
  - A: 4
  - B: 6
  - AB: 6
- **Price Predictions**
  - A
  - B
- **LocalBid**: -$0

### Bidder 3
- **Value**
  - A: 12
  - B: 7
  - AB: 15
- **Price Predictions**
  - A
  - B
- **LocalBid**: -$0

### Bidder 4
- **Value**
  - A: 8
  - B: 8
  - AB: 9
- **Price Predictions**
  - A
  - B
- **LocalBid**: -$0

---

**Self-confirming price predictions**

**FOR SALE**
Self-Confirming Price Predictions

Algorithm 4 Self-Confirming Price Prediction Search

Initialize $f$

for $t = 1$ to $T$ do

$f' \leftarrow$ outcome of $G$ simulations of Heuristic($f$)

if $D(f, f') < \tau$ then

return $f$

end if

$f \leftarrow \kappa_t f' + (1 - \kappa_t) f$

end for

return $f$

e.g., LocalBid
# Self-Confirming Results

<table>
<thead>
<tr>
<th>Point Strategy</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>StraightMV</td>
<td>0.0116</td>
</tr>
<tr>
<td>TargetMV</td>
<td>0.0660</td>
</tr>
<tr>
<td>TargetMV*</td>
<td>0.0553</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Distribution Strategy</th>
<th>Error</th>
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</thead>
<tbody>
<tr>
<td>StraightMU</td>
<td>0.00450</td>
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<tr>
<td>TargetMU</td>
<td>0.00879</td>
</tr>
<tr>
<td>TargetMU*</td>
<td>0.00788</td>
</tr>
<tr>
<td>AverageMU</td>
<td>0.00516</td>
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</tbody>
</table>
BNE Characterization

IPV implies that agent $i$'s expected utility is conditionally independent of other agents' valuations given their bids.

Therefore, it suffices to model other agents' bids.
BNE Characterization

IPV implies that agent $i$'s expected utility is conditionally independent of other agents' valuations given their bids.

Therefore, it suffices to model other agents' bids.

The distribution of highest other-agent bids (HOAB) is a sufficient statistic for the distribution of other agents' bids in common sealed-bid auctions (e.g., 1st and 2nd-price auctions, modulo ties).

Therefore, it often suffices to model only HOAB.
IPV implies that agent i's expected utility is conditionally independent of other agents' valuations given their bids.

Therefore, it suffices to model other agents' bids.

The distribution of highest other-agent bids (HOAB) is a sufficient statistic for the distribution of other agents' bids in common sealed-bid auctions (e.g., 1st and 2nd auctions, modulo ties).

Therefore, it often suffices to model HOAB.
BNE Characterization

Theorem

Assuming condition 1 (IPV), and condition 2, a best-response to other agents’ strategies takes the form of an optimal PO heuristic, where P predicts the HOABs induced by those strategies.
BNE Characterization

Corollary

Any profile of such optimal PO heuristics constitutes a BNE. Moreover, any BNE can be characterized in this way (or as mixtures thereof).
Sequential Auctions
Assuming condition 1 (IPV), and condition 2, a best-response to other agents’ strategies takes the form of an optimal policy $\pi$ in the full-history MDP that represents $\Gamma$ with $T = Induced(\pi)$. 
Any profile of such optimal PO heuristics constitutes a BNE. Moreover, any BNE can be characterized in this way (or as mixtures thereof).
Outline

• Agents for “simple” interdependent markets
  • Simultaneous & Sequential

• Agents for more complex interdependent markets
  • Simulated markets: TAC games
  • Real-world market: Quibids

• Mike: Empirical game-theoretic analysis (EGTA), to predict equilibrium outcomes in these markets
Outline

• Agents for “simple” interdependent markets
  • Simultaneous & Sequential

• Agents for more complex interdependent markets
  • Simulated markets: TAC games
  • Real-world market: Quibids

• Mike: Empirical game-theoretic analysis (EGTA), to predict equilibrium outcomes in these markets