Distributed Controllers for Human-Robot Locomotion: A Scalable Approach Based on Decomposition and Hybrid Zero Dynamics

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Abstract—This paper presents a formal foundation, based on decomposition, hybrid zero dynamics (HZD), and a scalable optimization, to develop distributed control algorithms for hybrid models of collaborative human-robot locomotion. The proposed approach considers a centralized controller and then decomposes the dynamics and feedback laws with a parameterization to synthesize local controllers. The Jacobian matrix of the Poincaré map with local controllers is studied and compared to that with centralized ones. An optimization problem is then set up to tune the parameters of the local controllers for asymptotic stability. The proposed approach can significantly reduce the number of controller parameters to be optimized for the synthesis of distributed controllers. The analytical results are numerically evaluated with simulations of a multi-domain hybrid model with 19 degrees of freedom for stable amputee locomotion with a powered knee-ankle prosthetic leg.

Index Terms—Distributed Control, Stability of Hybrid Systems, Robotics.

I. INTRODUCTION

Cooperative human-robot locomotion is a complex problem as by nature, the human and prosthesis act as individual subsystems—this motivates the development of decentralized/distributed controllers to effectively coordinate the action of subsystems. State-of-the-art feedback controllers for autonomous legged locomotion are mainly tailored to centralized techniques that have no provision to consider decentralized walking models. Although decentralized controllers have recently been developed for powered prostheses, existing techniques mainly focus on the development of linear and time-varying local controllers, see e.g., [1], [2]. These techniques require different parameters at different time periods to be tuned to account for the nonlinear dynamics in locomotion. This results in clinicians spending significant time in tuning control parameters [3] for each individual. These roadblocks motivate the development of a class of nonlinear distributed controllers that consider the independent nature of the human and prosthesis systems and thus facilitate a reduction in the number of parameters.

Hybrid systems theory has become a powerful approach for modeling legged locomotion [4]–[10]. Existing nonlinear control approaches that address the hybrid nature of locomotion are tailored to centralized techniques, such as hybrid reduction [11], [12], controlled symmetries [6], transverse linearization [7], [13], and Hybrid Zero Dynamics (HZD) [14]. The use of distributed/decentralized control is extensively observed in large-scale systems, e.g., power systems [15] and network systems [16]. Moreover, the wealth of research on decentralization is tailored to the stabilization of equilibrium points in ordinary differential equations (ODEs) but not periodic orbits (i.e., gaits) of hybrid models of locomotion [17].

Recently, in [18], distributed controllers were extended to exoskeletons but does not consider the interaction forces. References [19]–[21] presented nonlinear local controllers for powered prostheses based on the availability of force measurements to capture the strong interaction between the subsystems. In [22], [23], we introduced a nonlinear decentralized control scheme that does not require any force measurements but still consider the interaction amongst the subsystems. Here, the decentralized controllers are synthesized by setting up a large-scale optimization problem based on an iterative sequence of bilinear matrix inequalities (BMIs) for a bipedal model with point feet. The BMI algorithm develops local controllers from scratch by optimizing for a significant amount of controller parameters. This may result in a computational bottleneck for multi-domain locomotion models with nontrivial feet and high degrees of freedom (DOFs). In our most recent work [24], we explored an alternative approach, based on decomposition, towards decentralization for quadrupedal locomotion.
This approach presumes the existence of an orbit stabilized by centralized controllers from which local controllers are synthesized with fewer design parameters. In particular, we showed through numerical simulations that decomposition is a feasible approach but did not provide any formal foundations and considered the problem only for symmetric quadrupedal systems.

The overarching goal of this paper is to present a systematic and scalable approach, based on decomposition, HZD, and optimization, to develop nonlinear distributed controllers that exponentially stabilize collaborative human-robot locomotion with asymmetric models. The contributions and key objectives of this paper are as follows: (1) We propose a decomposition approach for the synthesis of distributed controllers. The integration of decomposition and optimization allows for a significant reduction of the parametric space as we only need to modify and optimize the already-existing centralized HZD controller rather than searching for local controllers from scratch. This leads to a small-scale optimization problem. (2) We study the properties of the Jacobian of the Poincaré map of the centralized and distributed controllers and show that there exists an upper bound on the norm of the difference of Jacobian matrices that significantly simplifies the search for optimal local control parameters. (3) We numerically validate the analytical foundation by designing distributed controllers for multi-domain and collaborative human-robot locomotion with a powered knee-ankle prosthetic leg and nontrivial feet, having a total of 19 DOFs (see Fig. 1). We show that the proposed approach can effectively synthesize local controllers for this comprehensive model of locomotion. In particular, we reduce the number of local controller parameters to be tuned and optimized to almost 5.2% of that in [22].

II. HYBRID MODEL OF LOCOMOTION

In this section, we consider single-domain hybrid models of locomotion to simplify the development of distributed control algorithms. The result can, however, be extended to multi-domain hybrid models of locomotion. We assume that the generalized coordinates of the mechanical system can be given by \( q \in \mathbb{Q} \subseteq \mathbb{R}^{n_q} \). The state vector is also chosen as \( x := \text{col}(q, \dot{q}) \in \mathcal{X} \subseteq \mathbb{R}^n \), in which \( \text{col} \) denotes the column operator, \( n = 2n_q \), and \( \mathcal{X} := T\mathbb{Q} := \mathbb{Q} \times \mathbb{R}^{n_q} \) represents the state manifold. The control inputs (i.e., torques) to the system are represented by \( u \in \mathcal{U} \subseteq \mathbb{R}^m \). The guard of the hybrid system is represented by the \((n - 1)\)-dimensional manifold \( \mathcal{S} \) on which the state trajectory undergoes an abrupt change according to impact dynamics [25]. The open-loop hybrid model of locomotion can be then expressed as follows:

\[
\Sigma^{\text{col}} : \begin{cases} \dot{x} = f(x) + g(x)u, & x \in \mathcal{X} \\ x^+ = \Delta(x^-), & x^- \in \mathcal{X} \cap \mathcal{S}, \end{cases}
\]

where \( x^- \) and \( x^+ \) denote the state of the system right before and right after the discrete-time transition, respectively, \( \Delta : \mathcal{X} \to \mathcal{X} \) is a smooth (i.e., \( C^\infty \)) reset map, and \( f \) and \( g \) are smooth functions. The equations of motion can be described by

\[
D(q) \ddot{q} + H(q, \dot{q}) = B(q)u,
\]

in which \( D(q) \in \mathbb{R}^{n_q \times n_q} \) is the mass-inertia matrix, \( H(q, \dot{q}) \in \mathbb{R}^{n_q} \) represents the Coriolis, centrifugal, and gravitational terms, and \( B(q) \in \mathbb{R}^{n_q \times m} \) is the input matrix.

Assumption 1: (Periodic Orbit and Phasing Variable): There exists a periodic solution (i.e., gait) to the hybrid model (1) for some nominal control inputs and some fundamental period \( T^* > 0 \). The corresponding periodic orbit is represented by \( O \subseteq \mathcal{X} \). To define the desired evolution of the state and control inputs on \( O \), we make use of a time-based phasing variable as \( \tau := \frac{t - t^*}{T^*} \), where \( t^* \) represents the time sample right after the discrete-time event. The evolution of the state and control inputs on \( O \) are further denoted by \( x^*(\tau) := \text{col}(q^*(\tau), \dot{q}^*(\tau)) \) and \( u^*(\tau) \) for \( 0 \leq \tau < 1 \).

We next present an HZD-based centralized controller to exponentially stabilize the desired orbit \( O \) for the hybrid model of locomotion. For this purpose, we make use of the following time-varying holonomic outputs (i.e., virtual constraints) to be imposed

\[
y(\tau, \dot{\tau}) := C(q - q^*(\tau)), \quad (3)
\]

for some full-rank output matrix \( C \) with the property \( \text{dim}(y) \leq \text{dim}(u) \). We are interested in the desired output dynamics \( \dot{y} + k_y y + k_p y = 0 \) for some positive gains \( k_y \) and \( k_p \). Employing the standard input-output (I-O) linearization [26] yields the following centralized controller

\[
u = \Gamma(\tau, x) := -(L_y L_f y)^\top \left( L_f^2 y + \frac{\partial^2 y}{\partial \tau^2} (\dot{\tau})^2 + k_d \dot{y} + k_p y \right), \quad (4)
\]

that results in \( \lim_{t \to \infty} y(t) = 0 \), where \( (L_y L_f y)^\top \) denotes the Moore–Penrose inverse of the decoupling matrix \( L_y L_f y \).

III. DISTRIBUTED FEEDBACK Controllers

This section develops distributed feedback controllers, based on the decomposition approach, to exponentially stabilize the periodic orbit \( O \) (see Fig. 2(c)). We consider the synthesis problem of local controllers for two subsystems of the hybrid model of locomotion (1). Without loss of generality, we assume that the generalized coordinate vector and control inputs can be decomposed as \( q = \text{col}(q_i, q_j) \) and \( u = \text{col}(u_i, u_j) \), where \( q_i \in \mathbb{Q}_i \) and \( u_i \in \mathcal{U}_i \) denote the local configuration variables and local control inputs for the subsystem \( \Sigma_i \) for \( i \in \{1, 2\} \). The local state variables are further denoted by \( x_i := \text{col}(q_i, \dot{q}_i) \in \mathcal{X}_i \).

1) Decomposition Approach: To develop local controllers, we make the following assumption.

Assumption 2: (State Approximation for Subsystems): We suppose that the subsystem \( \Sigma_i \) for every \( i \in \{1, 2\} \) can approximate the state variables for the other subsystem, i.e., \( \Sigma_j, \ j \neq i \in \{1, 2\} \), by their desired values on the periodic orbit \( O \), that is, \( x_j \approx x_j^*(\tau) \).

We then decompose the equations of motion in (2) as

\[
D_{ii} \ddot{q}_i + D_{ij} \ddot{q}_j + H_i = B_{ii} u_i + B_{ij} u_j
\]

for all \( i \neq j \in \{1, 2\} \). From (5), one can solve for \( \ddot{q}_j \), i.e.,

\[
(D_{ii} - D_{ij} D_{jj}^{-1} D_{ij}) \ddot{q}_i + H_i - D_{ij} D_{jj}^{-1} H_j = (B_{ii} - D_{ij} D_{jj}^{-1} B_{jj}) u_i + (B_{ij} - D_{ij} D_{jj}^{-1} B_{jj}) u_j.
\]
Using Assumption 2, the nonlinear dynamics for subsystem \( i \) can be approximated as follows:

\[
\dot{D}_{ii}(q_i, t) \dot{q}_i + \dot{H}_i(q_i, \dot{q}_i, t) = \dot{B}_i(q_i, t) u_i,
\]

in which \( \dot{D}_{ii} := D_{ii} - D_{ij} D_{jj}^{-1} D_{ji}, \dot{H}_i := H_i - D_{ij} D_{jj}^{-1} H_j - (B_{ij} - D_{ij} D_{jj}^{-1} B_{ji}) u_j, \) and \( \dot{B}_i := B_{ii} - D_{ij} D_{jj}^{-1} B_{ji} \) that are evaluated at \( q_i = q_i^*(\tau), \dot{q}_i = \frac{\partial q_i^*}{\partial \tau}(\tau) \dot{\tau}, \) and \( u_j = u_j^*(\tau) \). Furthermore, the subsystem \( i \) can approximate the nonlinear dynamics of the other subsystem, i.e., the subsystem \( j, \) as

\[
\dot{D}_{jj}(q_j, t) \dot{q}_j + \dot{H}_j(q_j, \dot{q}_j, t) = \dot{B}_j(q_j, t) u_j,
\]

where \( \dot{D}_{jj} := D_{jj} - D_{jj} D_{ji}^{-1} D_{ij}, \dot{H}_j := H_j - D_{jj} D_{ji}^{-1} H_i - (B_{jj} - D_{jj} D_{ji}^{-1} B_{ij}) u_i, \) and \( \dot{B}_j := B_{jj} - D_{jj} D_{ji}^{-1} B_{ij} \) evaluated at \( q_j = q_j^*(\tau), \dot{q}_j = \frac{\partial q_j^*}{\partial \tau}(\tau) \dot{\tau}, \) and \( u_j = u_j^*(\tau) \).

### 2. Output Modification and Local Controllers: The virtual constraints (3) can be decomposed as \( y = \text{col}(y_1, y_2) \) for some local outputs \( y_1 \) and \( y_2 \) with the property \( \dim(y_1) \leq \dim(u_i) \), \( i \in \{1, 2\} \). In particular, we assume that

\[
y_i = C_{ii} (q_i - q_i^*(\tau)) + C_{ij} (q_j - q_j^*(\tau))
\]

for all \( i \neq j \in \{1, 2\} \). Using Assumption 2, the local output \( y_i \) can be reduced to \( y_i = C_{ii} (q_i - q_i^*(\tau)) \). Our previous work [22] showed that such local virtual constraints may not stabilize the orbit \( O \) for the hybrid model of locomotion. To tackle this challenge, we make the following assumption.

**Assumption 3: (Measurable Global Variables):** We suppose that there are some smooth functions \( \psi_i(x) = \psi_i(x_1, x_2) \), referred to as measurable global variables, that 1) depend on the global states \( x_1 \) and \( x_2 \), and 2) are measurable for the subsystem \( \Sigma_i \) along the trajectories of the hybrid model via some sensors.

We next consider some measurable holonomic global variables for the subsystem \( i \) represented by \( \psi_i(q) \). We further suppose that both \( \psi \) and \( \psi_1 \) are measurable for the subsystem \( \Sigma_i \). Our motivation for this assumption comes from the fact that robotic prosthetic legs can be equipped with some inertial measurements units (IMUs) attached to the human thigh to measure the absolute position and velocity variables that depend on the global states \( (q, \dot{q}) \). We then modify the local output for the subsystem \( \Sigma_i \) as follows:

\[
y_i = C_{ii} (q_i - q_i^*(\tau)) + E_i (\psi_i(q) - \psi_i^*(\tau)),
\]

where \( \psi_i^*(\tau) \) denotes the desired evolution of the measurable global variables \( \psi_i \) on \( O \) in terms of the phasing variable \( \tau \). The local output (10) is further parameterized by an output matrix \( E_i \) to be determined in Section IV. Differentiating the output (10) results in

\[
\ddot{y}_i = E_i \left( \ddot{q}_i - \frac{\partial^2 q_i^*(\tau)}{\partial \tau^2} (\dot{\tau})^2 \right) + E_i \left( \frac{\partial \psi_i}{\partial q} \dot{\psi}_i - \frac{\partial \psi_i}{\partial q} q - \frac{\partial^2 \psi_i}{\partial \tau^2} (\dot{\tau})^2 \right),
\]

in which the acceleration terms, i.e., \( \ddot{q}_i \) and \( \ddot{\psi}_i = \text{col}(\ddot{q}_1, \ddot{q}_2) \), can be approximated using the nonlinear dynamics (7) and (8). Furthermore, the nonlinear terms \( \frac{\partial \psi_i}{\partial q} \dot{\psi}_i \) and \( \frac{\partial \psi_i}{\partial \tau} (\dot{\psi}_i) \dot{\tau} \) can be computed utilizing Assumption 2. This would result in an input-affine output dynamics as follows:

\[
\dot{y}_i = A_i (\tau, x_i) u_i + b_i (\tau, x_i) = -k_d \dot{y}_i - k_p y_i.
\]

From the output dynamics (12), one can solve for the minimum 2-norm local control solution, that is,

\[
u_i = \Gamma_i (\tau, x_i, \psi_i, \dot{\psi}_i) := -A_i^T (b_i + k_d \dot{y}_i + k_p y_i).
\]

### IV. Exponential Stability Analysis

This section investigates the exponential stabilization problem of the periodic orbit \( O \) under the proposed HZD-based distributed feedback controllers (13). We study the Jacobian matrix of the Poincaré map with distributed controllers and then set up an optimization-based approach to tune the parameters of the local controllers.

**A. Poincaré Sections Analysis for Closed-Loop Systems**

The evolution of the closed-loop mechanical system with centralized and distributed feedback controllers can be expressed by the following hybrid models

\[
\Sigma_c^i := \begin{cases}
\dot{x} = f_c^i (\tau, x), & x \in X, \\
\tau^+ = \Delta (x^-), & x^- \in X \cap S
\end{cases}
\]

and

\[
\Sigma_c^d := \begin{cases}
\dot{x} = f_c^d (\tau, x), & x \in X, \\
\tau^+ = \Delta (x^-), & x^- \in X \cap S,
\end{cases}
\]

where \( \dot{\Gamma}(\tau, x) := \text{col}(\Gamma_1(\tau, x_1, \psi_1, \dot{\psi}_1), \Gamma_2(\tau, x_2, \psi_2, \dot{\psi}_2)) \) represents the distributed controllers, and \( f_c^d (\tau, x) := f(x) + g(x) \Gamma(\tau, x) \) and \( f_c^d (\tau, x) := f(x) + g(x) \Gamma(\tau, x) \) denote the closed-loop vector fields with centralized and distributed feedback laws, respectively. For future purposes, we define
the augmented states as \( x_a := \text{col}(x, \tau) \in \mathcal{X}_a := \mathcal{X} \times \mathbb{R}_+ \), augmented vector fields as \( f^{cl}_a(x_a) := \text{col}(f^{cl} (\tau, x), 1) \), and \( \Delta^{cl}(x_a) := \text{col}(\Delta(x), 0) \), and \( S_a := \mathcal{X} \times \mathbb{R}_+ \). We then denote the Poincaré maps for the closed-loop hybrid systems (14) and (15) by \( P_a \colon S_a \to S_a \) and \( P_a' \colon S_a \to S_a \), respectively.

**Lemma 1:** (Fixed Points) Under Assumptions 1-3, \( x^* := \text{col}(x^*_a, 1) \) is a fixed point for the Poincaré maps, i.e., \( P_a(x^*_a) = P_a'(x^*_a) = x^*_a \), where \( x^*_a := \text{col}(x^*_a(1) = \mathcal{O} \cap S) \).

The proof is immediate according to the construction procedure. From Lemma 1, \( O_a := \{ (x, \tau) \mid x = x^*(\tau), 0 \leq \tau < 1 \} \) is a periodic orbit for the closed-loop hybrid systems (14) and (15). Let us denote the time evolution of the augmented state trajectory along \( O_a \) (i.e., the closure of \( O_a \)) by \( x^*_a(t) \) for \( 0 \leq t \leq T^* \). We then define the Jacobian matrices of the augmented vector fields along \( x^*_a(t) \) by \( A(t) \) and \( \hat{A}(t) \), that is, \( A(t) := \frac{\partial f^{cl}_a(x^*_a(t))}{\partial x_a} \) and \( \hat{A}(t) := \frac{\partial f^{cl}_a(x^*_a(t))}{\partial x_a} \) for \( t \in [0, T^*] \). We now make the following assumption.

**Assumption 4:** There are \( L > 0 \) and \( \mu > 0 \) such that for all \( t \in [0, T^*] \), \( \|A(t)\|_2 \leq L \) and \( \|A(t) - \hat{A}(t)\|_2 \leq \mu \).

**Lemma 2:** (Variational Equations): Under Assumptions 1-4, consider \( \tilde{\Phi}(t) = A(t)\Phi(t) \) and \( \tilde{\Phi}(t) = \hat{A}(t)\Phi(t) \) for \( t \in [0, T^*] \) with \( \Phi(0) = \tilde{\Phi}(0) = I \). Let us define \( \varphi(t) := \text{vec}(\tilde{\Phi}(t)) \) and \( \varphi(t) := \text{vec}(\hat{\Phi}(t)) \), where “vec” denotes the vectorization operator. Then, there is \( \delta > 0 \) such that for all \( t \in [0, T^*] \), \( \|\varphi(t) - \hat{\varphi}(t)\|_2 \leq \frac{\mu}{L} (\exp(LT^*) - 1) \).

**Proof:** From properties of the vectorization operator, \( \tilde{\varphi} = \text{vec}(A(t)\Phi(t)) = (I \otimes A(t))\text{vec}(\Phi(t)) = (I \otimes A(t))\varphi \), and similarly \( \tilde{\varphi} = (I \otimes A(t))\hat{\varphi} \). We denote the vectorization operator by \( \text{vec}(\cdot) \), boundedness of the solutions, i.e., \( \exists \delta > 0 \) such that \( \|\varphi(t)\|_2 \leq \delta \) and \( \|\hat{\varphi}(t)\|_2 \leq \delta \) for \( t \in [0, T^*] \), and applying [27, Theorem 3.4, pp. 96] to the ODEs complete the proof.

We are now in a position to present the following theorem.

**Theorem 1:** (Upper Bound on the Norm of the Difference of the Jacobian Matrices): Under Assumptions 1-4, there is a constant \( \beta > 0 \) such that

\[
\left\| \frac{\partial P_a}{\partial x_a}(x^*_a) - \frac{\partial P_a'}{\partial x_a}(x^*_a) \right\|_2 \leq \frac{\mu \beta}{L} (\exp(LT^*) - 1).
\]

**Proof:** From [28, Theorem 1] and [29, Appendix D], the Jacobian linearization of the Poincaré return map can be given by \( \frac{\partial P_a}{\partial x_a}(x^*_a) = \Pi \Phi(T^*) \frac{\partial \Phi}{\partial x_a}(x^*_a) \) and \( \frac{\partial P_a'}{\partial x_a}(x^*_a) = \Pi \Phi(T^*) \frac{\partial \Phi}{\partial x_a}(x^*_a) \), where \( \Pi \) is the saltation matrix, defined by \( \Pi := I - 1/(\Delta^{cl}(x^*_a) f^{cl}_a(x^*_a) f^{cl}_a(x^*_a) \frac{\partial \Phi}{\partial x_a}(x^*_a)) \). Here, we assume that the augmented switching manifold \( S_a \) can be expressed as the zero-level set of the switching function \( s_a(x_a) \). From Lemma 1, \( f^{cl}_a(x^*_a) = f^{cl}_a(x^*_a) \), and hence, \( S_a \) is indeed the saltation matrix for both Jacobian matrices. From the properties of the vectorization operator, there is a scalar \( \gamma > 0 \) such that \( \|\Phi(T^*) - \hat{\Phi}(T^*)\| \leq \gamma (\varphi(T^*) - \hat{\varphi}(T^*)) \) in combination with Lemma 2 and norm properties results in

\[
\|\frac{\partial P_a}{\partial x_a}(x^*_a) - \frac{\partial P_a'}{\partial x_a}(x^*_a)\|_2 \leq \left\| \Pi \right\| (\frac{\partial \Phi}{\partial x_a}(x^*_a) - \frac{\partial \Phi}{\partial x_a}(x^*_a)) \leq \left\| \Pi \right\| \frac{\mu \beta}{L} (\exp(LT^*) - 1). \]

Finally, defining \( \beta := \delta \gamma \left\| \Pi \right\| \frac{\partial \Phi}{\partial x_a}(x^*_a) \), completes the proof.

**B. Distributed Controller Synthesis**

In order to synthesize distributed controllers, we look for some output matrices \( E_i, i \in \{1, 2\} \) in (10) to exponentially stabilize \( O \) via proposed local controllers. For this purpose, let us denote the columns of \( E_i \) by \( \xi_i \), that is, \( \xi_i := \text{vec}(E_i) \). The local controllers \( T_i \) in (13) can be then represented by parameterized feedback laws \( T_i(\tau, x, \psi_i, \psi_i, \xi_i) \). We further define the control parameters as \( \xi := \text{col}(\xi_1, \xi_2) \). This allows us to parameterize the Jacobian matrix of the Poincaré map by \( \xi \) which can be denoted by \( \frac{\partial P_a}{\partial x_a}(x^*_a, \xi) \). The exponential stabilization problem then consists of finding the controller parameters \( \xi \) such that the eigenvalues of \( \frac{\partial P_a}{\partial x_a}(x^*_a, \xi) \) strictly lie inside the unit circle. We then set up a nonlinear program (NLP) to look for stabilizing parameters \( \xi \) as follows:

\[
\min_{\xi} \frac{1}{2} \left\| \xi \right\|^2_2 \quad \text{s.t.} \quad \text{eig} \left( \frac{\partial P_a}{\partial x_a}(x^*_a, \xi) \right) < 1. \quad (16)
\]

Here, we search for the minimum 2-norm controller parameters to make modified outputs (10) close to the decomposed ones (9) while imposing the exponential stability constraint. Our previous work in [22] synthesized distributed controllers by looking for both output matrices \( C_{ii} \) and \( E_i \) in (10) which may have a significant number of decision variables and hence, it may result in a large-scale and computationally intensive NLP. The proposed approach, however, significantly reduces the number of parameters to be optimized for the synthesis of distributed controllers. More specifically, we do not search for \( C_{ii} \) and only focus on \( E_i \).

**V. Simulation Results**

The objective of this section is to numerically verify the theoretical results for human-robot locomotion. We consider a lower-extremity amputee as the human subsystem and assume that a 3D tree-like structure with square-profile foot represents the human body (see Fig. 2(a), 2(c)). The robot subsystem on the other hand comprises a knee-ankle powered prosthesis where the foot is modeled with a square profile (see Fig. 1). While we model the powered prosthesis after [30], we extend it with an additional “passive” DOF, attributed to the compliance in the ankle roll for simulation purposes. Under this consideration, we assume the entire structure to consist of 19 DOFs. The first 6 DOFs are attributed to the absolute position and orientation of the human body. The remaining 13 DOFs are the internal shaping variables that are distributed between both the legs as follows: 3 DOFs are associated with the hip to model the roll, pitch, yaw angles, one DOF captures the pitch joint at the knee, further 2 DOFs are allocated to the ankle pitch and roll, and one “passive” DOF is employed to model the point of attachment of the prosthesis to the residual human leg. This completes the state vector \( x := \text{col}(q, \dot{q}) \in \mathcal{X} \subset \mathbb{R}^{38} \) and the control vector \( u \in \mathcal{U} \subset \mathbb{R}^{11} \). We adopt the kinematic and dynamic parameters of [31] and [30] for the human and robot components of the system, respectively.
In this paper, we consider a 4-domain walking cycle to include flat-footed phases (domains \( \nu_1 \) and \( \nu_3 \)) and push-off phases (domains \( \nu_2 \) and \( \nu_4 \)) that mimic natural human walking (see Fig. 2(b)). With this directed graph, we formulate an optimization problem via FROST [32] to solve for two different desired periodic gaits, \( \mathcal{O} \) with a speed of 0.2 (m/s) and 0.3 (m/s). To demonstrate the power of the proposed approach in synthesizing local controllers, we first design centralized controllers as in (4) such that the 0.2 (m/s) and 0.3 (m/s) gaits becomes stable and unstable, respectively. We then systematically proceed with the proposed decomposition approach by separating the human-robot system into two discrete subsystems at the point of severance, located just above the left knee. The human subsystem \( \Sigma_1 \) retains the base DOFs in addition to its 9 DOFs which results in \( \dim(x_1) = 30 \) and \( \dim(\nu_1) = 9 \). Similarly, the robotic subsystem \( \Sigma_2 \) has 4 DOFs with two passive joints, including the ankle roll and point of attachment of two subsystems, i.e., \( \dim(x_2) = 8 \) and \( \dim(\nu_2) = 2 \). In addition, we utilize a set of global position and velocity variables, \( \psi_2(q) \) and \( \dot{\psi}_2(q, \dot{q}) \), that are obtained from a “single” IMU located on the residual human thigh. The human subsystem has no avail of the global IMU variables, indicating that \( \nu_1 = \emptyset \) as it already has access to its own absolute position and orientation. We remark that in this formulation, the local controller for the robotic subsystem in (10) has access to at most 14 state variables. This is a direct result of its 8 internal states and the availability of at most 6 global variables from the IMU in the form of \( \psi \in \{r, p, y\} \) and \( \dot{\psi} = \{\dot{r}, \dot{p}, \dot{y}\} \), in which \( r, p, y \) denote the roll, pitch, and yaw angles, respectively. Likewise, local controller for the human subsystem in (10) has access to at most 30 state variables.

The entire mechanical system has 8 DOFs during the push-off phases and 13 DOFs during the flat-footed phases. In order to have a nontrivial zero dynamics, we consider 7-dimensional and 11-dimensional outputs (3) in push-off and flat-footed phases, respectively. We then decompose \( y \) into \((y_1, y_2)\) such that \( y_1 \) includes the DOFs assigned to the human subsystem and \( y_2 \) includes the ones for the robotic subsystem. We remark that for the push-off domains \((\nu_1 \) and \( \nu_3 \)), \( \dim(y_1) = 6 \) and \( \dim(y_2) = 1 \), and the corresponding decoupling matrices in (13) become \( A_1 \in \mathbb{R}^{6 \times 9} \) and \( A_2 \in \mathbb{R}^{1 \times 2} \). In addition for the flat-footed domains \((\nu_2 \) and \( \nu_4 \)), \( \dim(y_1) = 9 \), \( \dim(y_2) = 2 \), \( A_1 \in \mathbb{R}^{9 \times 9} \), and \( A_2 \in \mathbb{R}^{2 \times 2} \). Next, we parameterize the local outputs for the robotic subsystem but not the human subsystem as (10) by output matrices \( E_2 \) over three domains. Here, we assume a local output regulating controller for the human subsystem as [22], [33]. In our parameterization, the Poincaré map is only parameterized by four parameters related to the robotic subsystem, i.e., \( \xi \in \mathbb{R}^4 \), whereas the synthesis problem in [22, Sec. VII.B] included 78 parameters for a model with less DOFs (i.e., 94.8% reduction in controller parameters). The local parameters for the robotic leg are then effectively optimized by the NLP (16) for exponential stability. Here the Poincaré section is taken at the beginning of domain \( \nu_1 \) and the Poincaré map is \( 15 = 2 \times 8 - 1 \) dimensional as the mechanical system has 8 DOFs in domain \( \nu_1 \). In addition, we make use of a total of four parameters to parameterize the local outputs over three domains. The four optimal values of \( E_2 \) are \{0.0571, 0.1017, −0.4186, 2.0232\} that correspond to the IMU pitch, roll, pitch, and yaw, respectively (see [34]). Figure 3(a)-(b) illustrates the phase portraits for the human pitch and yaw angles, respectively, over 100 consecutive steps for the 0.2 (m/s) gait. Convergence to the periodic orbit is clear. To show that the distributed architecture does not rely on the existence of a stabilizing centralized controller, we repeat the same procedure for the 0.3 (m/s) gait with an unstable centralized controller. Figures 3(c)-(d) demonstrate the distributed controller’s ability to stabilize the gait despite having an originally unstable centralized controller.

Robustness Analysis: To illustrate robustness against external disturbances, we induce persistent forces during the prosthetic leg’s swing phase \((\nu_2)\) for every step taken. First, a persistent force of 125 (N) is exerted along the x-axis. In another simulation, a persistent force of 200 (N) is applied in the direction of negative y-axis. Figures 4(a)-(b) demonstrate convergence to a stable orbit indicating the controller’s positive response to the provided disturbances. Furthermore, to show the controller’s robustness to uncertainties in dynamic parameters, we vary the mass of the torso while still employing the distributed controller synthesized with the original torso mass of 40.82 (kg). Two simulations, one with torso mass of 32 (kg) and the other with torso mass of 45 (kg), demonstrate stability by converging to periodic orbits as illustrated in Figs. 4(c)-(d).

To draw meaningful differences between the existing centralized and the proposed distributed controllers, we induce white Gaussian noise with a signal-to-noise-ratio (SNR) of 42 (db) into the velocity components of the actuated prosthetic DOFs. Figures 5(a)-(b) show torques captured over one step in the human subsystem for the centralized and distributed control frameworks, respectively. It can be seen from Fig. 5(a) that the induced sensor noise in the prosthetic joints propagates and manifests itself in the joint torques of the human subsystem under the centralized architecture. However, joint torques in the human subsystem are unaffected under the distributed framework (see Fig. 5(b)). Additionally, we note that the amplitude of noise is significantly higher in the centralized control framework.
simulations together with the optimal modified outputs can be found online [34].

VI. CONCLUSIONS

This paper established a formal foundation, based on decomposition, HZD, and optimization, to synthesize distributed controllers for exponential stabilization of cooperative human-robot locomotion. The proposed approach assumes a set of HZD-based centralized controllers and then decomposes the dynamics and virtual constraints while parameterizing them with some adjustable local parameters. This decomposition method significantly reduces the parametric space resulting in a small-scale optimization problem compared to previous approaches for synthesizing local controllers. Furthermore, the Poincaré map for the closed-loop hybrid models of locomotion with distributed controllers is investigated and compared to that with centralized ones. We numerically validated the proposed distributed scheme for stable, robust, and multi-domain human-robot locomotion with a powered prosthetic leg and 19 DOFs. For future research, we will investigate the design of robust distributed controllers for locomotion on rough terrains. We will also experimentally evaluate the approach on powered prosthetic legs attached to lower limb amputees.

REFERENCES

[34] [Online]. Available: https://youtu.be/MiGhjggJhpI