New and Improved Key-Homomorphic Pseudorandom Functions

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Outline

1. Introduction

2. Construction, Parameters and Efficiency

3. Proof of Security (Idea)

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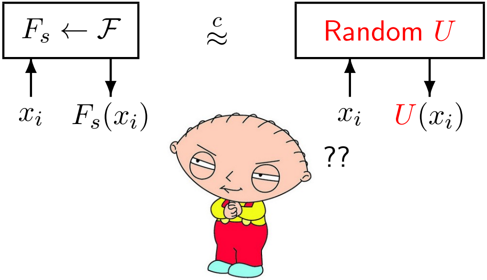
- A family of functions $\mathcal{F} = \{F_s : \{0,1\}^k \to B\}$ such that, given adaptive query access,



- Lots of applications in symmetric key cryptography: encryption, message authentication, friend or foe identification, . . .

(Thanks to Seth MacFarlane for the adversary)

Cooking a (Provably Secure) PRF

1. Goldreich-Goldwasser-Micali [GGM'84]
    - Based on any (doubling) PRG: $F_s(x_1, \ldots, x_k) = G_{x_k}(\cdots(G_{x_1}(s))\cdots)$

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   - Framework: exponentiate to a product of (secret) exponents
   - Security from number-theoretic assumptions (DDH, factoring, ...)
3. Lattice-based direct constructions [BPR'12]
   - Framework: round a product of (secret) matrices/ring elements
   - Security from lattice assumptions (LWE, worst-case lattice problems)

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- Can we obtain similar tradeoffs for KH-PRFs?

Our Results

⋆ New KH-PRFs (from lattices):
  - Polylog $\tilde{O}(1)$ depth (still)
  - Quasi-optimal $\tilde{O}(\lambda)$ key sizes

First sublinear-depth PRFs (KH or otherwise) with $\tilde{O}(\lambda)$ key size!

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Reference	**Key**	**Pub Params**	**Time/Bit**
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Figure : For input length λ with 2^λ security under standard assumptions. Log factors omitted. Ring-based constructions appear in [brackets].

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Full version: `http://eprint.iacr.org/2014/074`

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- Secret key $\mathbf{s} \in \mathbb{Z}_q^n$, pub params $\mathbf{B}_0, \mathbf{B}_1 \in \{0,1\}^{n \times n}$, input $x \in \{0,1\}^k$

$$F_{\mathbf{s}}(x) = \left\lfloor \mathbf{s} \cdot \prod_{i=1}^{k} \mathbf{B}_{x_i} \right\rceil_p$$

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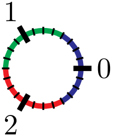

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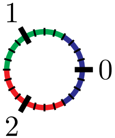

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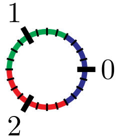

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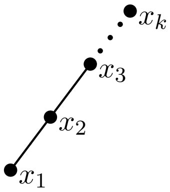

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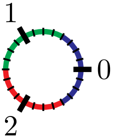

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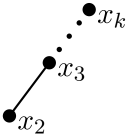

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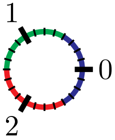

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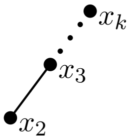

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✗ LWE approx factor grows exponentially in input length k.

Gadget and Bit-Decomposition

- "Gadget" $\mathbb{Z}_q$-matrix $\mathbf{G}$ [MP'12]:

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- A ubiquitous tool in lattice cryptography: FHE [BV'11,GSW'13,AP'14], CCA/IBE/ABE/FHS [MP'12,BGG$^+$'14,GVW'14]

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For matrices $\mathbf{A}_0, \mathbf{A}_1$, full binary tree T and $x \in \{0,1\}^{|T|}$, define $\mathbf{A}_T(x)$:

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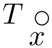

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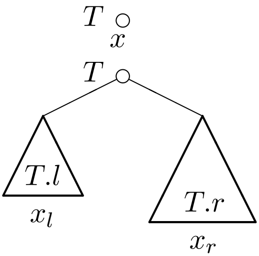

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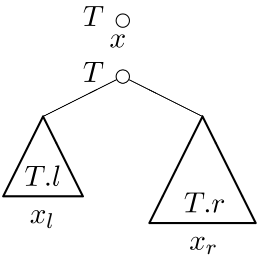

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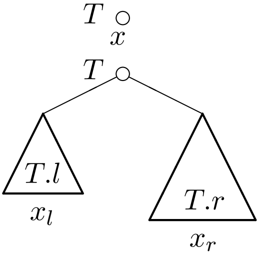

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- Somewhat KH just as in [BLMR'13]. Same applications!

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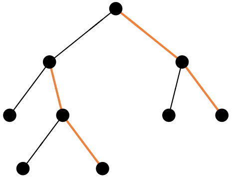

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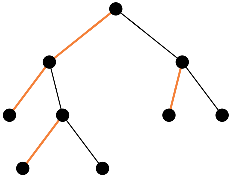

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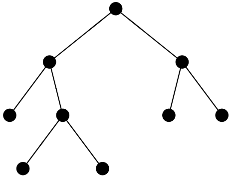

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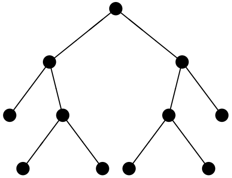

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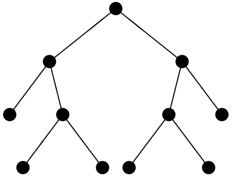

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Instantiations

$e(T)$	$s(T)$	Key	Params
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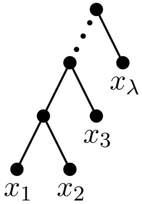
"Left Spine"



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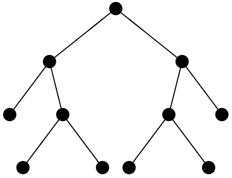

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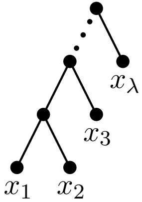
[BLMR'13]

Construction!



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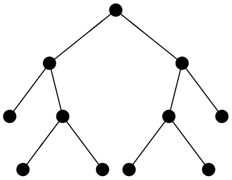

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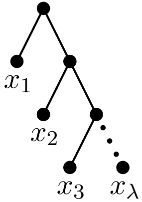

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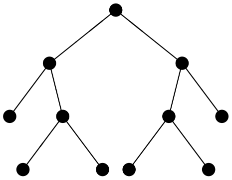


$s = 2,\ e = 2$

Instantiations

$e(T)$	$s(T)$	Key	Params
$\lambda - 1$	1	λ^3	λ^6
1	$\lambda - 1$	λ	λ^2
$\approx \log_4(\lambda)$	$\approx \log_4(\lambda)$	λ	λ^2

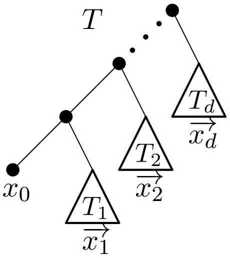
Outline

1. Introduction

2. Construction, Parameters and Efficiency

3. Proof of Security (Idea)

Proof Idea



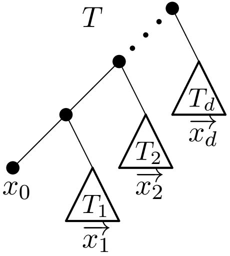
$$F_{\mathbf{s}}(x) = \left\lfloor \mathbf{s} \cdot \mathbf{A}_{x_0} \cdot \mathbf{G}^{-1}(\mathbf{A}_{T_1}(\overrightarrow{x_1})) \cdots \right\rceil_p$$

$$\stackrel{s}{\approx} \left\lfloor \underbrace{(\mathbf{s} \cdot \mathbf{A}_{x_0} + \mathbf{e}_{x_0})}_{\mathbf{u}_{x_0}} \cdot \mathbf{G}^{-1}(\mathbf{A}_{T_1}(\overrightarrow{x_1})) \cdots \right\rceil_p$$

$$\stackrel{c}{\approx} \left\lfloor \mathbf{u}_{x_0} \cdot \mathbf{G}^{-1}(\mathbf{A}_{T_1}(\overrightarrow{x_1})) \cdots \right\rceil_p$$

Proof Idea

✓ New Idea: $\mathbf{u} = \mathbf{s} \cdot \mathbf{G} + \mathbf{v}$ for uniform, *independent* $\mathbf{s}$ and $\mathbf{v} \in \mathcal{P}(\mathbf{G})$.



$$
\begin{aligned}
F_{\mathbf{s}}(x) &= \left\lfloor \mathbf{s} \cdot \mathbf{A}_{x_0} \cdot \mathbf{G}^{-1}(\mathbf{A}_{T_1}(\overrightarrow{x_1})) \cdots \right\rceil_p \\
&\overset{s}{\approx} \left\lfloor \underbrace{(\mathbf{s} \cdot \mathbf{A}_{x_0} + \mathbf{e}_{x_0})}_{\mathbf{u}_{x_0}} \cdot \mathbf{G}^{-1}(\mathbf{A}_{T_1}(\overrightarrow{x_1})) \cdots \right\rceil_p \\
&\overset{c}{\approx} \left\lfloor \mathbf{u}_{x_0} \cdot \mathbf{G}^{-1}(\mathbf{A}_{T_1}(\overrightarrow{x_1})) \cdots \right\rceil_p
\end{aligned}
$$

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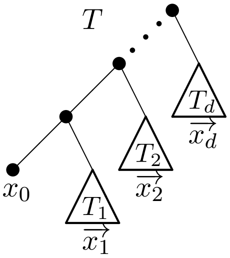

$$F_{\mathbf{s}}(x) = \left\lfloor \mathbf{s} \cdot \mathbf{A}_{x_0} \cdot \mathbf{G}^{-1}(\mathbf{A}_{T_1}(\overrightarrow{x_1})) \cdots \right\rceil_p$$

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$$\overset{c}{\approx} \left\lfloor \mathbf{u}_{x_0} \cdot \mathbf{G}^{-1}(\mathbf{A}_{T_1}(\overrightarrow{x_1})) \cdots \right\rceil_p$$

$$= \left\lfloor \mathbf{s}_{x_0} \cdot \mathbf{A}_{T_1}(\overrightarrow{x_1}) \cdot \mathbf{G}^{-1}(\mathbf{A}_{T_2}(\overrightarrow{x_2})) \cdots + \mathbf{v}_{x_0} \cdot \mathbf{G}^{-1}(\mathbf{A}_{T_1}(\overrightarrow{x_1})) \cdots \right\rceil_p$$

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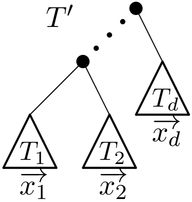

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$$= \left\lfloor \mathbf{s}_{x_0} \cdot \mathbf{A}_{T'}(\overrightarrow{x_1} \| \cdots \| \overrightarrow{x_d}) + \mathbf{v}_{x_0} \cdot \mathbf{G}^{-1}(\mathbf{A}_{T_1}(\overrightarrow{x_1})) \cdots \right\rceil_p$$

Proof Idea

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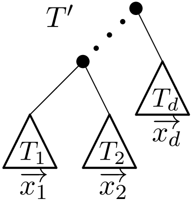

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$$= \left\lfloor \mathbf{s}_{x_0} \cdot \mathbf{A}_{T'}(\overrightarrow{x_1} \| \cdots \| \overrightarrow{x_d}) + \mathbf{v}_{x_0} \cdot \mathbf{G}^{-1}(\mathbf{A}_{T_1}(\overrightarrow{x_1})) \cdots \right\rceil_p$$

$$\cdots \stackrel{c}{\approx} \left\lfloor \mathbf{s}_x + \mathbf{v}_{x_0} \mathbf{G}^{-1}(\mathbf{A}_{T_1}(\overrightarrow{x_1})) \cdots + \text{other } \mathbf{v} \text{ terms} \right\rceil_p \stackrel{s}{\approx} U(x). \qquad \blacksquare$$

Conclusions

Our main contributions

- New KH-PRFs from lattices: quasi-optimal key sizes, polylog depth
- New proof technique

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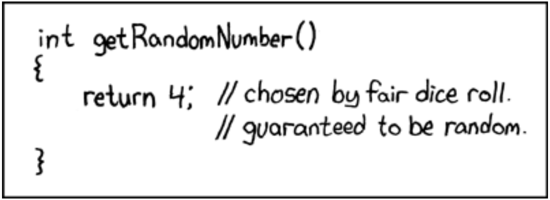
The Last Word [Mun'07]



(Image source: http://xkcd.com/221/)