Preliminary Results on Correct-by-Construction Control Software Synthesis for Adaptive Cruise Control

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Abstract— **A plethora of driver convenience and safety automation systems are being introduced into production vehicles, such as electronic stability control, adaptive cruise control, lane keeping, and obstacle avoidance. Assuring the seamless and safe integration of each new automation function with existing control functions is a major challenge for vehicle manufacturers. This challenge is compounded by having different suppliers providing software modules for different control functionalities. In this paper, we report on our preliminary steps to address this problem through a fresh perspective combining formal methods, control theory, and correct-by-construction software synthesis. In particular, we begin the process of synthesizing the control software module for adaptive cruise control from formal specifications given in Linear Temporal Logic. In the longer run, we will endow each interacting software module with an *assume-guarantee* specification stating under which environment assumptions the module is guaranteed to meet its specifications. These assume-guarantee specifications will then be used to formally prove correctness of the cyber-physical system obtained when the integrated modules interact with the physical dynamics.**

I. Introduction

Adaptive Cruise Control (ACC) is a driver assistance system designed to provide improved convenience and comfort with respect to conventional cruise control systems. When there is no preceding vehicle in sight, an ACC-equipped vehicle behaves just like one with conventional cruise control, i.e., it will maintain a constant speed set by the driver. When a preceding vehicle is detected and is driving at a speed slower than the preset speed, an ACC-equipped vehicle changes its control objective to maintaining a safe headway (range) or time headway instead. These two modes are commonly known as the speed (cruise) mode, and distance (following) mode, respectively, and the preceding vehicle is commonly called the lead vehicle.

ACC has been available on production vehicles since the mid-1990s. Many of the early ACC systems shut off below a given threshold speed. More recently, full-range (stop-and-go) ACC is available that can bring a vehicle to a full stop and then launch from standstill, and is thus capable of dealing with congested urban traffic. In the last several years, some auto makers (e.g., Volvo and Cadillac) have introduced an automated full-braking function to leverage hardware already installed for ACC, tiptoeing the red-line between comfort/convenience systems and active safety systems. This trend is likely to continue as more automated vehicle control functions are introduced on production vehicles.

In [1], National Highway Traffic Safety Administration (NHTSA) announced its policy on the development of automated vehicles, and defined five levels of vehicle automation: Level 0 (no automation), Level 1 (function-specific automation, such as ACC or electronic stability control), Level 2 (combined function automation, such as ACC plus lane keeping), Level 3 (limited self-driving), and finally Level 4 (full self-driving). All new production cars in the US are already at Level 1, because electronic stability control has been required for all new cars in the US since 2012. A major challenge for control engineers is that as more automation functions are introduced (either required or as options), their seamless integration with existing control functions must be guaranteed. This requirement is important both to ensure vehicle safety and for tort/liability considerations.

This paper is the first step towards the broader objective of synthesizing correct-by-construction control software for automation Level 2 and beyond. By correct-by-construction we mean control software that is guaranteed to meet its formal specifications given certain assumptions on the physical plant and implementation platform. The formal specifications considered in this paper are given in Linear Temporal Logic, a common specification language for software systems. *Here, we begin the task of synthesizing correct-by-construction control software for ACC in the special case that the lead vehicle's speed is constant.* Two synthesis methods are used, one performing set computation directly on the continuous domain, and a second based on finite-state abstractions. The two resulting correct-by-construction controllers are compared by running simulations in Simulink and on a 16 degree of freedom model in CarSim[1]. In subsequent work, we will address stop-and-go ACC with variable lead-vehicle speed, correct-by-construction control software for lane keeping, and correct-by-construction control software when ACC and lane keeping are simultaneously active.

II. Adaptive Cruise Control

For ACC systems, the most important function is to maintain a safe and comfortable range from the preceding vehicle. Whether a range is safe or not is based on metrics

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[1]CarSim is a vehicle simulation package that is widely used in industry. It is a registered trademark of Mechanical Simulation Corporation, Ann Arbor, MI.

such as time-to-collision or deceleration needed to avoid a crash. Whether a range is comfortable or not is subjective, and may consider additional factors such as value and rate of change of range, time-headway, and jerk.

A. Past Work

Adaptive cruise control started as an extension of conventional cruise control (CCC), which was how it was described in relevant ISO [2] and SAE [3] standards. In these early design concepts, ACC was a phase or mode of the overall control system, and used existing CCC hardware architecture: in the ACC mode, the speed command to the CCC servo-loop was adjusted to achieve the desired range control objective (e.g., [4]). This nested control architecture resulted in slower response (in comparison to when throttle and brake are controlled directly), which was observed in prototype ACC vehicles in field tests [5]. Comparison between the velocity-command approach and acceleration command approach was analyzed in [6].

The potential of ACC-equipped vehicles for improving traffic flow and safety has been studied extensively since the 1990s [7][8]. In addition to traffic flow and safety, string stability [9][10][11], congestion [12], fuel economy [13] and integration with crash avoidance [14] have also been studied. An extensive survey on ACC designs, including the underlying control concepts, is given in [15]. In recent years, MPC has been widely used in ACC design [16].

Although driver assistance and safety modules such as ACC have been investigated for many years, the emphasis on the correctness of a module's software implementation is much more recent. Safety verification of evasive maneuvers for autonomous vehicles is addressed in [17]. By computing the reachable set of each vehicle under different types of uncertainties and disturbances, safety is ensured whenever the reachable sets for different vehicles are disjoint. The computation of reachability sets for hybrid systems is known to be expensive and in [18] verification is done through counterexample-guided search. Rather than working with a detailed model of the system to be verified, an abstraction is used. Verifying the abstraction leads to counterexamples that may be spurious, i.e., they may not be true behaviors of the real system. However, since reachability analysis on the abstraction is cheaper, a two-step approach is taken: 1) the abstract model is used to obtain counterexamples; 2) the counterexamples are proved or disproved on a detailed model of the system to be verified. The previous approaches to verification fall in the class of model checking, given a specification one checks (by reachability computation or counterexample-guided search) if the specification is met. A different approach, considered in [19], is theorem proving. Here, one writes the assumptions about the system and its environment in a convenient logic and proves a theorem stating that the desired specification follows from the assumptions. A related approach is the use of satisfiability (SAT) and satisfiability modulo theory (SMT) solvers instead of a customized logic as was done in [20]. Unlike all the aforementioned approaches, we do not verify an existing software module. Instead, we synthesize a software module that is guaranteed to satisfy the specification by construction, hence the term correct-by-construction.

B. Simplified Model For Constant Lead Vehicle Speed

The vehicle is modeled as a (lumped) point mass m moving along a straight line. The net action of braking and engine torque applied to the wheels is lumped as a net force F_w acting on the mass of the vehicle, while the combined aerodynamic and rolling resistance is gathered into a net force F_r,

$$m\dot{v} = F_w - F_r. \tag{1}$$

In the above equation, F_w is viewed as the control input and is assumed to be bounded by

$$-0.3mg \leq F_w \leq 0.2mg, \tag{2}$$

where g is the gravitational constant. Such a bound is consistent with non-emergency braking and acceleration, and thus with the "driver convenience" notion of ACC. The term F_r is represented by $F_r = f_0 + f_1 v + f_2 v^2$ [7]. We limit the admissible velocities to a bounded set $\mathcal{V} = [v_{min}, v_{max}]$ with $v_{min} \geq 0$.

To include a lead vehicle in the system description, we use a hybrid system model with two discrete modes M_1 and M_2, called `no lead car` and `lead car` mode, respectively. The `lead car` mode M_2 has an additional continuous state h which measures the headway to the lead car. The continuous dynamics of M_1 are those of (1) while the continuous dynamics in mode M_2 contain an additional equation describing the dynamics of the headway:

$$\begin{aligned} m\dot{v} &= F_w - f_0 - f_1 v - f_2 v^2 \\ \dot{h} &= v_L - v \end{aligned} \tag{3}$$

The two modes have different state spaces, mode M_1 has state space $\mathcal{V}$ while mode M_2 has state space $\mathcal{V} \times \mathcal{H}$, where $\mathcal{H} = [0, h_{max}]$ for an upper limit h_{max} on the radar range.

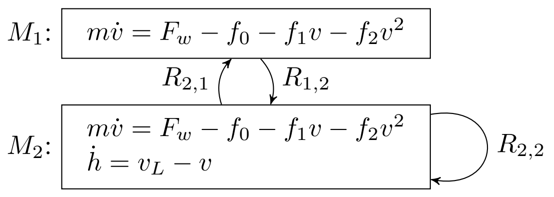
In practice, the system is in M_2 if there is a car within the radar range, and in M_1 otherwise. Switching between the two states is governed by lane changes of lead cars, which are modeled using reset maps $R_{1,2} : \mathcal{V} \to 2^{\mathcal{V}\times\mathcal{H}}$, $R_{2,1} : \mathcal{V} \times \mathcal{H} \to \mathcal{V}$ and $R_{2,2} : \mathcal{V} \times \mathcal{H} \to 2^{\mathcal{V}\times\mathcal{H}}$.

$$\begin{aligned} R_{1,2}(v) = \{(v, \bar{h}) : \bar{h} \in \mathcal{H}\}, \quad R_{2,1}(v,h) = \{v\}, \\ R_{2,2}(v,h) = \{(v, \bar{h}) : \bar{h} \in \mathcal{H}\}. \end{aligned} \tag{4}$$

Here $R_{1,2}$ models a transition from the `no lead car` mode M_1 to the `lead car` mode M_2, where the headway is initialized to some $\bar{h} \in \mathcal{H}$. Similarly, $R_{2,2}$ models situations where the radar reading suddenly changes as a result of lane changes undertaken by cars in front.

The switching is assumed to be non-deterministic, except for the case when h reaches h_{max} in mode M_2, in which case a forced transition to mode M_1 occurs. The complete hybrid model is shown schematically in Fig. 1. The vehicle parameters[2] we assume have been taken from the "S-Class

[2]In SI units, $m = 1370$ kg, $f_0 = 3.8 \times 10^{-3} \times mg$ N, $f_1 = 2.6 \times 10^{-5} \times mg$ Ns/m, $f_2 = 0.4161$ Ns^2/m^2.



M_1:
$m\dot{v} = F_w - f_0 - f_1 v - f_2 v^2$
$R_{2,1}$
$R_{1,2}$
M_2:
$m\dot{v} = F_w - f_0 - f_1 v - f_2 v^2$
$\dot{h} = v_L - v$
$R_{2,2}$

Fig. 1: Hybrid system model.

Sedan" model in CarSim. We also restrict the domain to v_{min} = 0 m/s, v_{max} = 35 m/s, h_{min} = 0 m, h_{max} = 300 m, where the latter models the radar range.

For simplicity, we make the following assumptions:

(A.1) The velocity v_L of the lead car is known and constant, i.e., $\dot{v}_L = 0$. In the following we assume $v_L = 12$ m/s.

(A.2) There is at most one lead car within the radar range at all times, i.e., $R_{2,2}(v,h) = \emptyset$ for all $(v,h) \in \mathcal{V} \times \mathcal{H}$.

C. ACC Formal Specification

In this section we formalize the adaptive cruise control requirements using Linear Temporal Logic (LTL). Introducing the *time headway*, defined as $\tau = h/v$, we summarize the requirements defined by the International Organization of Standardization, see [2, Chapter 6], as follows.

1) ACC operates in two modes: the `no lead car` mode and the `lead car` mode;
2) in `no lead car` mode, a preset desired speed v_{des} eventually needs to be reached and maintained;
3) in `lead car` mode, a desired lower bound on safe time headway τ_{des} to the lead vehicle and an upper bound on a desired velocity v_{des} eventually needs to be reached and maintained; and the time headway needs to satisfy $\tau \geq 1$ at all times,
4) independently of the mode, the input constraint (2) needs to be satisfied at all times.

Note that in requirement 3) different choices of time or range headway are possible, see e.g. [21].

We proceed with the translation of requirements 1)–4) into LTL. The basic building blocks of an LTL specification are the so-called *atomic propositions*. The set of atomic propositions represents the quantities necessary to express the desired behavior. For example, to be able to refer to the two modes of ACC in the specification, we introduce the atomic proposition M_1 that is satisfied when the vehicle is in `no lead car` mode and the atomic proposition M_2 that is satisfied when the vehicle is in `lead car` mode. We also introduce the atomic propositions G_1, G_2, S_1, and S_2. We use G_1 and G_2 to express requirements 2) and 3), i.e., a desired velocity v_{des} and a desired lower bound on time headway τ_{des} as well as an upper bound on desired velocity v_{des}, should be attained (and maintained) if the vehicle satisfies M_1 and M_2, respectively. We identify G_1 and G_2 with subsets of the state space of (3) in the sense that G_1 and G_2 are satisfied whenever the state of the hybrid system belongs to the following sets:

$$G_1 = \left\{ v : v \in [v^-, v^+] \right\}, \tag{5}$$

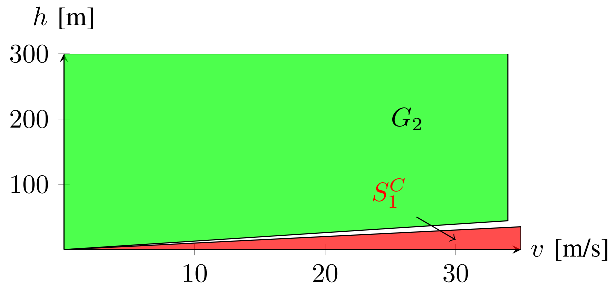


Fig. 2: State space regions for mode M_2.

$$G_2 = \left\{ (v,h) : h/v \geq \tau_{des}, v \leq v^+, h \in \mathcal{H} \right\}. \tag{6}$$

Here v^- and v^+, define the interval of tolerated deviation around the desired speed v_{des}. The set G_2 is depicted in Fig. 2. The sets S_1 and S_2 are used to express the constraints that need be satisfied at all times, i.e., the distance constraint and input constraint. These sets are given by

$$S_1 = \{ (v,h) : h/v \geq 1, v \in \mathcal{V}, h \in \mathcal{H} \}, \tag{7}$$

$$S_2 = \{ F_w : F_w \in [-0.3mg, \, 0.2mg] \}. \tag{8}$$

In summary, the set of atomic propositions is given by $\{M_1, M_2, G_1, G_2, S_1, S_2\}$.

The specifications considered in this paper can be expressed using the atomic propositions, the propositional logic conjunction "$\wedge$" and negation "$\neg$", and the temporal operators always "$\Box$". Disjunction "$\vee$" and implication "$\Rightarrow$" can be constructed from conjunction and negation. Eventually "$\Diamond$" can be constructed from negation and always. We interpret LTL formulas over infinite sequences (μ, ξ, ν) where the signal $\mu : \mathbb{N} \to \{M_1, M_2\}$ specifies the current mode of the system and $\xi : \mathbb{N} \to \mathbb{R}^2$ is a sample-and-hold trajectory of (3) given the input signal $\nu : \mathbb{N} \to \mathbb{R}$ generated by the ACC. We refer to the triple of such sequences (μ, ξ, ν) as a *behavior of the closed-loop system*, i.e., (3) controlled by the ACC.

A behavior (μ, ξ, ν) is said to *satisfy* $\Box p$ or $\Diamond p$ if the atomic proposition p holds true at every time step or at some future time step(s), respectively. A closed-loop system satisfies an LTL formula φ if every system behavior (μ, ξ, ν) satisfies φ. For reasons of space, we do not provide the syntax and semantics of LTL and refer the interested reader to [22]. Instead, we continue with two examples. Consider the simple formulas $\varphi_1 = \Box(M_1 \wedge G_1 \wedge S_2)$ and $\varphi_2 = \Diamond(M_2 \wedge G_2)$. A behavior satisfies φ_1 if $\mu_t = M_1$ and $(\xi_t, \nu_t) \in G_1 \times S_2$ holds for *all* $t \in \mathbb{N}$. A behavior satisfies φ_2 if $\mu_t = M_2$ and $\xi_t \in G_2$ holds for *some* time $t \in \mathbb{N}$.

The ACC specification can be described by the LTL formula ψ:

$$\Box\big((M_1 \vee S_1) \wedge S_2\big) \wedge \Box\Big(\bigwedge_{i=1}^{2} \Box M_i \implies \Diamond\Box G_i \Big). \tag{9}$$

The first term $\Box\big((M_1 \vee S_1) \wedge S_2\big)$ ensures that the input constraint and the distance to the lead vehicle constraint are always met. Note that the distance constraint only needs to be satisfied in mode M_2, hence the disjunction $M_1 \vee S_1$. We specify with the second term $\Box\big(\wedge_{i=1}^{2}\Box M_i \implies \Diamond\Box G_i\big)$ that the desired velocity/time headway G_i should eventually be reached and kept invariant, given that the system is in mode M_i.

Notice that the specification gives no guarantees of reaching the desired time headway and velocity, when the system switches between M_1 and M_2 infinitely often. This is in compliance with the ISO standards [2], which does not specify what the system should guarantee in such a scenario. Moreover, when the controller for mode M_i is constructed, as long as the system is in M_i, it guarantees progress towards G_i. This property is desirable and it holds by construction because the controller cannot know if a mode change is going to occur in the future or not.

D. Problem statement

The goal of this paper is to synthesize a controller for the hybrid system in Fig. 1 with reset maps given by (4) so that every behavior of the closed-loop system satisfies the specification ψ given in (9). However, if the lane-change behavior of the lead car is not properly constrained, e.g. if a car with very low speed may cut right in front of an ACC-equipped car traveling at higher speed, no controller can prevent collisions, hence ψ cannot be satisfied.

Motivated by this fact, the formal controller synthesis problem can now be stated as follows:

Problem 1: Assume (A.1) and (A.2) hold. Synthesize a controller and a control domain $\mathcal{D} \subseteq \mathcal{V} \times \mathcal{H}$ so that every behavior (μ, ξ, ν) of the closed-loop system satisfies ψ if the values of $R_{1,2}(v)$ are restricted to $\mathcal{D}$ for all $(v,h) \in \mathcal{V} \times \mathcal{H}$.

In what follows, we present two approaches to this problem. Both approaches seek to synthesize controllers with control domains that are as large as possible. In general, finding a maximal $\mathcal{D}$ can be hard [23]. The presented approaches are approximate in the sense that although the synthesized controllers are provably-correct, the synthesized control domains are not necessarily maximal.

III. Solution by Computations on the Continuous State Space

In this section a solution strategy based on set computations on the continuous domain $\mathcal{V} \times \mathcal{H}$ is presented. We proceed by separately synthesizing controllers for the two modes M_1 and M_2, such that, respectively, the specifications

$$\Diamond\Box G_1 \wedge \Box S_2 \tag{10}$$

$$\Diamond\Box G_2 \wedge \Box(S_1 \wedge S_2) \tag{11}$$

are satisfied when there is no mode switching. Then, we show how these two controllers can be implemented together to satisfy the original specification in (9) under mode switching.

A. Linearized Model

The solution strategy presented in this section relies on computation of reachable sets. Such computations are hard to perform for general nonlinear systems, we therefore linearize the polynomial system (3) around a nominal velocity $\bar{v}$ to obtain

$$\begin{aligned} \dot{v} &= \frac{1}{m}(\bar{F}_w - \bar{f}_0 - \bar{f}_1 v), \\ \dot{h} &= v_L - v, \end{aligned} \tag{12}$$

for $\bar{f}_0 = f_0 - f_2\bar{v}^2$ and $\bar{f}_1 = f_1 + 2f_2\bar{v}$. Assuming that a control input $\bar{F}_w$ has been computed for the linearized system (12), the modified control $F_w = \bar{F}_w + f_2(v-\bar{v})^2$ can be used to make the nonlinear system behave in accordance with the linear system. Effectively, this linearization procedure moves the nonlinearity from the system dynamics to appropriate bounds on the input. To ensure that F_w stays inside the required range, we conservatively restrict $\bar{F}_w$ to the interval

$$-0.3mg \leq \bar{F}_w \leq 0.2mg - \gamma, \tag{13}$$

where $\gamma = \max_{v \in [v_{min}, v_{max}]} f_2(v-\bar{v})^2$. The correction term γ is small for typical parameter values and the level of conservativeness is therefore not very severe. By construction, the following now holds.

Proposition 1: For any $(v_0,h_0),(v_1,h_1) \in \mathcal{V}\times\mathcal{H}$, if there exists an input $\bar{F}_w : [0,T] \to [-0.3mg, 0.2mg-\gamma]$ that steers the state of (12) from (v_0,h_0) to (v_1,h_1) in time T, then there exists an input $F_w : [0,T] \to [-0.3mg, 0.2mg]$ that steers the state of (3) from (v_0,h_0) to (v_1,h_1) in time T.

It is therefore sufficient to study reachability for the linearized system, which is described in the subsequent section.

B. Backwards-Time Reachability for Linear Systems

A *polyhedral set*, or *polyhedron*, is a set defined by linear inequalities. In this section we outline how the following problem can be solved numerically:

Problem: Given a final polyhedral set $X_1 = \{x \in \mathbb{R}^n : L_x x \leq l_x\}$, a linear time-invariant system

$$\Xi : \quad \dot{x} = Ax + Bu + K, \tag{14}$$

a time step ΔT, and (possibly state-dependent) linear constraints on the input represented as $H_x x + H_u u \leq h_{xu}$, find an initial set $Pre_\Xi(X_1) \subset \mathbb{R}^n$ such that for all $x_0 \in Pre_\Xi(X_1)$, there exists a constant control u_0 such that

1) the control constraint $H_x x_0 + H_u u_0 \leq h_{xu}$ is satisfied,
2) the final set X_1 is reached at time ΔT by applying u_0, i.e. $A_d x_0 + B_d u_0 + K_d \in X_1$, where $A_d = e^{A\Delta T}$, $B_d = \int_0^{\Delta T} e^{As}B\mathrm{d}s$ and $K_d = \int_0^{\Delta T} e^{As}K\mathrm{d}s$ define the time-discretized version Ξ_d of Ξ. ■

The constraint 1) above can be expressed as $L_x(A_d x_0 + B_d u_0 + K_d) \leq l_x$ and combined with 2) the joint constraints on initial state and input can be written as

$$\begin{bmatrix} L_x A_d & L_x B_d \\ H_x & H_u \end{bmatrix} \begin{bmatrix} x_0 \\ u_0 \end{bmatrix} \leq \begin{bmatrix} l_x - L_x K_d \\ h_{xu} \end{bmatrix}. \tag{15}$$

Let L be the left-hand side matrix and l the right-hand side vector in (15), which together define a polyhedron $P = \{(x,u) \in \mathbb{R}^{n\times m} : L\begin{bmatrix} x^T & u^T \end{bmatrix}^T \leq l\}$ in $x-u$-space containing all possible combinations of initial states and inputs that reach X_1. To separate out the set of all initial states, we project P onto its x-coordinates. If Π_x is the x-projection operator, then $Pre_{\Xi}(X_1) = \Pi_x(P)$. The projection of a polyhedron can be computed with for example the Multi-Parametric Toolbox (MPT) [24].

C. Finding and Steering to a Controlled-Invariant Set

In this section we outline how to solve a problem of type

$$\Diamond\Box G \wedge \Box S \tag{16}$$

by performing reachability calculations.

An equivalent condition to $\Diamond\Box G$ for deterministic systems is $\Diamond G \wedge \Box(G \rightarrow \Box G)$, i.e. ensure that G is eventually reached and that the system never leaves G after reaching it for the first time. To find a strategy for staying in G we search for a *controlled-invariant set* C contained in G, where controlled-invariance is defined as follows.

Definition 1: A set C is *controlled-invariant* for the discrete-time system Ξ_d if for any $x_0 \in C$, there exists an input u such that (x_0, u) satisfies the input constraint and such that $A_d x_0 + B_d u + K_d \in C$.

In the following, we propose two methods for finding a controlled-invariant set C contained inside a given set G. Both methods rely on finding fixed points for the operator $K_G(C) = Pre_{\Xi}(C) \cap G$.

The *outside-in* Algorithm 1 starts with the whole set G and gradually shrinks it until only the controlled-invariant part remains. This algorithm converges to the maximal controlled-invariant set contained in G [25], [26]. However, it may not terminate and manual termination gives an over approximation of the maximal controlled-invariant set in G. On the other hand, if a 'small' controlled-invariant set C^* can be found inside G, the *inside-out* Algorithm 2 [27] grows C^* as much as possible inside of G. The initial 'small' controlled-invariant set C^* can for example be some natural equilibrium point of the system. The advantage of Algorithm 2 is that the iteratively updated set $\tilde{C}$ is controlled-invariant in every iteration. It can therefore be terminated early and still produce a non-maximal controlled-invariant set inside of G.

Algorithm 1 Outside-in controlled-invariant set computation.

```
function CONINVOI($\Xi, G$)
    $\tilde{C} := G, \; C := \emptyset$
    while $\tilde{C} \neq C$ do
        $C := \tilde{C}, \; \tilde{C} := Pre_{\Xi}(C) \cap G$
    return $C$
```

Finally, when a controlled-invariant set $C \subseteq G$ has been found, we find a list of sets $C_2, \ldots, C_I$, such that C_i is reachable from C_{i+1}, using Algorithm 3. By ensuring that all sets C_i are contained in the safe set S and denoting $C_1 = C$, the control strategy defined as "when in C_i go to $C_{\max(1,i-1)}$ in finite time" will satisfy (16). This control strategy can be implemented using any method capable of generating input that takes the plant to a set defined by linear inequalities. A natural choice is to use Model-Predictive Control (MPC) with a quadratic cost criterion for the discrete-time system Ξ_d, which will result in a piecewise-constant control signal for the continuous system. The domain $\mathcal{D}$ of this controller is $\cup_{i=1}^{I} C_i$.

Algorithm 2 Inside-out controlled-invariant set computation.

```
function CONINVIO($\Xi, G, C^*$)
    $\tilde{C} := C^*, \; C := \emptyset$
    while $\tilde{C} \neq C$ do
        $C := \tilde{C}, \; \tilde{C} := Pre_{\Xi}(C) \cap G$
    return $C$
```

Algorithm 3 Find a chain of sets in S that reaches C_1.

```
function CONTROLCHAIN($\Xi, C_1, S, I$)
    for i=2:I do
        $C_i = Pre_{\Xi}(C_{i-1}) \cap S$
    return $C_2, \ldots C_I$
```

D. Application to ACC problem

In this section we construct controllers K_1 and K_2 for the two modes M_1 and M_2 such that the specifications (10) and (11) are satisfied, for parameters $\tau_{des} = 1.3$ s, $v^- = 28$ m/s, $v^+ = 34$ m/s. The sets of initial conditions for which these controllers satisfy their respective specifications, denoted $\mathcal{D}_1 \subset \mathcal{V}$ and $\mathcal{D}_2 \subset \mathcal{V} \times \mathcal{H}$, will define the necessary restrictions on the reset maps of the hybrid system. We use a time discretization $\Delta T = 0.5$ s.

In mode M_1 the requirement on K_1 is $\Diamond\Box G_1$, which can be interpreted as conventional (non-adaptive) cruise control. It is possible to use the method presented in Section III-C to synthesize K_1. However, for simplicity, we choose to use MPC to design a K_1 which satisfies (10) for any trajectory starting in $\mathcal{D}_1 = \mathcal{V}$.

Turning to controller design for mode M_2, we now apply the solution strategy in Section III-C to find a controller K_2 that satisfies (11). If Ξ denotes the linearized dynamics (12), we proceed as follows; 1) call CONINVOI(Ξ, G_2) or CONINVIO(Ξ, G_2, C^*) to find a controlled-invariant set C_1 contained in G_2, 2) call CONTROLCHAIN(Ξ, C_1, S_1, I) to find a chain of sets $C_2, \ldots C_I$ in S_1 such that C_i is reachable from $C_{\max(1,i+1)}$.

For this example on a 3.4 Ghz iMac, Algorithms 1 and 2 finish in 15 and 17 milliseconds, respectively, and return an over- and under-approximation of the same controlled-invariant set $C_1 \subset G_2$. The call to Algorithm 3 takes 20 milliseconds on the same computer and yields two additional sets C_2 and C_3 such that $C_{\max(1,i-1)}$ is reachable from C_i. These sets are depicted in Fig. 3.

Let K_2 be a controller that implements the control strategy 'when in C_i, go to $C_{\max(1,i-1)}$'. By construction, K_2 then satisfies (11) for initial conditions in $\mathcal{D}_2 = \cup_i C_i$.

Proposition 2: A controller that uses the controller K_1 when in mode M_1 and the controller K_2 when in mode M_2 solves Problem 1 with control domain $\mathcal{D} = \mathcal{D}_2$.

Proof: Let $\Gamma^{(v,h)}(\mathcal{D}_1)$ be the lifting of $\mathcal{D}_1$ to $\mathcal{V} \times \mathcal{H}$, that is, $\Gamma^{(v,h)}(\mathcal{D}_1) \doteq \mathcal{V} \times \mathcal{D}_1$. The result follows from the facts that $R_{2,1}(\mathcal{D}_2) \subseteq \mathcal{D}_1$ and $\mathcal{D}_2 \subseteq \Gamma^{(v,h)}(\mathcal{D}_1) \cap \mathcal{D}_2$. ■

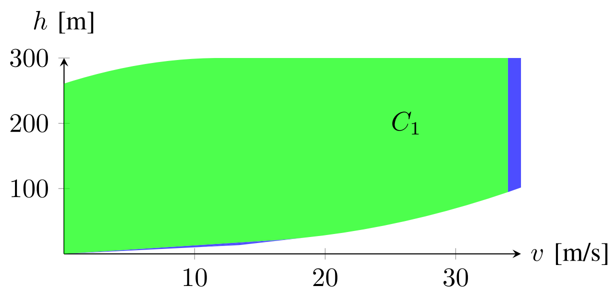


Fig. 3: The overlapping sets $C_1 \subset C_2 \subset C_3$. The control-invariant set C_1 is pictured in green and is entirely visible. C_2 and C_3, which are both subsets of G, are colored in blue and constitute the region from where C_1 is reachable.

To discriminate between the control signals that correctly implement 'when in C_i, go to $C_{\max(1,i-1)}$', we use MPC with optimization weights that punish deviations from the desired speed or the desired time headway, depending on the current state of the system. This allows us to tune the controller in order to enhance the driving experience while still honoring the formal safety specifications.

IV. Solution via PESSOA

In this section we describe how we compute a controller that enforces the desired behavior (9) on the system (3) using the MATLAB toolbox PESSOA [28]. The computation is based on a discrete abstraction of the system (3).

The abstraction is computed by a discretization of the state space, input space and time. The result is a finite state transition system $\Sigma = (Q, U, \delta)$, where Q is the set of states, U is the set of inputs and $\delta : Q \times U \to 2^Q$ is the transition relation. Note that Q forms a grid on the state space. Therefore the sets G_1, G_2 and S_1, S_2 can easily be mapped to Q by computing the grid points that fall inside the regions. We denote the corresponding sets in Q as $G_{1,a}, G_{2,a}$ and $S_{1,a}, S_{2,a}$. To be able to satisfy the input constraints we set $U = S_{2,a}$.

A. Controller Synthesis

We use $[\![\psi]\!]$ to denote the states for which there exists a control strategy that enforces ψ. From the set $[\![\psi]\!]$ we can easily construct a strategy. For reasons of space, however, we will only discuss the computation of $[\![\psi]\!]$ at an intuitive level.

All the controller synthesis algorithms in PESSOA are based on fixed-point solutions to certain types of games on finite graphs. Termination of these algorithms is guaranteed by the finiteness of the set of states Q. The interested reader is referred to [29] for further details on the algorithmic solution of such games.

To find $[\![\psi]\!]$, we intoduce Algorithm 4. Here, PESSOA first handles the safety part of the specification, i.e. $\Box S_{1,a}$, by solving a safety game via Algorithm 5. The result is a set-valued controller that determines the largest set of inputs that can be applied to each state while enforcing the safety specification. We then restrict the inputs of the abstraction to the inputs given by this controller and obtain a new system that will be used to compute the remaining specification via Algorithm 4. Since we assume that the system can change

Algorithm 4 Computation of $[\![\psi]\!]$.

```
function SYNTHESIZE($\Sigma$, $G_{1,a}$, $G_{2,a}$)
    $\tilde{X} := Q, \quad X := \emptyset$
    while $\tilde{X} \neq X$ do
        $X := \tilde{X}, \quad \tilde{X} := \bigcap_{i=1}^{2} [\![\Diamond\Box G_{i,a} \wedge \Box X]\!]$
    return $X$
```

from any mode to any other mode at any time, the resulting controller should be able to enforce $\Diamond\Box G_{i,a}$ for $i = \{1, 2\}$ at all times.

The first iteration of the algorithm computes $\bigcap_{i=1}^{2} [\![\Diamond\Box G_{i,a}]\!]$. These are the states from which there exist controllers that enforce $\Diamond\Box G_1$ and $\Diamond\Box G_2$. However, since the modes can change nondeterministically, this is not enough. These controllers also have to make sure that, the system always remains in the set $\bigcap_{i=1}^{2} [\![\Diamond\Box G_{i,a}]\!]$. Hence, in the next iteration we add this safety constraint and compute $\bigcap_{i=1}^{2} [\![\Diamond\Box G_{i,a} \wedge \Box \bigcap_{i=1}^{2} [\![\Diamond\Box G_{i,a}]\!]]\!]$. We continue the same procedure until a fixed-point is reached.

Note that at each iteration of Algorithm 4, we compute a controller which enforces $\Diamond\Box G_{i,a} \wedge \Box K_{i-1}$, where K_{i-1} is the set of states computed in iteration $i-1$. This is a "reach and stay while stay" specification which is supported in PESSOA. Again first, PESSOA handles the safety part of the specification and then it computes the domain of the controller for the "reach and stay" part via Algorithm 6. In these algorithms, we denote the predecessor operator by, $Pre_\Sigma : 2^Q \to 2^Q$, where $Pre_\Sigma(Q') = \{q \in Q \,|\, \exists u \in U : \delta(q, u) \in Q'\}$.

Algorithm 5 Computation of $[\![\Box K]\!]$.

```
function STAY($\Sigma$, $K$)
    $\tilde{X} := Q, \quad X := \emptyset$
    while $\tilde{X} \neq X$ do
        $X := \tilde{X}, \quad \tilde{X} := Pre_\Sigma(X) \cap K$
    return $X$
```

Algorithm 6 Computation of $[\![\Diamond\Box K]\!]$.

```
function REACHSTAY($\Sigma$, $K$)
    $\tilde{X} := Q, \quad X := \emptyset, \quad \tilde{Y} := \emptyset, \quad Y := Q$
    while $\tilde{Y} \neq Y$ do
        $Y := \tilde{Y}$
        while $\tilde{X} \neq X$ do
            $X := \tilde{X}, \quad \tilde{X} := (Pre_\Sigma(X) \cap K) \cup Pre_\Sigma(Y)$
        $\tilde{Y} := \tilde{X}, \quad \tilde{X} := Q$
    return $Y$
```

B. Solving the ACC problem

When using PESSOA to synthesize a controller, the first step is the computation of a discrete abstraction Σ of (3).

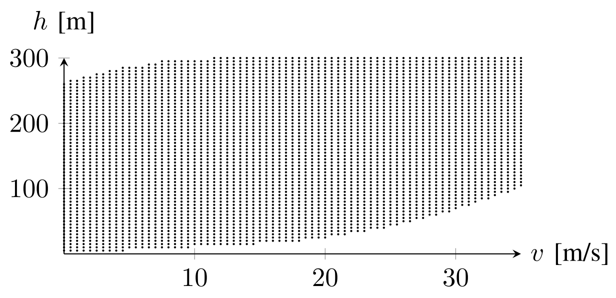


Fig. 4: The domain of the controller enforcing (9) on the system (3), i.e. $[\![\psi]\!]$. Each point represents a discrete state. 90% of the points in the h-coordinate have been removed for visibility.

The computation is based on the algorithms described in [30] and supported by PESSOA. For our case, we chose the state space and input space discretization parameters to be $\eta = 0.5$ and $\mu = 0.2$, respectively. Moreover, we use $\Delta T = 0.5$ for the sampling time and the same problem parameter values as in Section III-D. Afterwards, we run the algorithms described in the previous subsection and obtain a controller that enforces the formula ψ given in (9) on the system (3). It took 30.65 seconds to synthesize the controller on a 3.4 GHz iMac. Fig. 4 illustrates the domain of the controller synthesized in PESSOA.

The controller synthesized by PESSOA is given by a large look-up table which is stored on the hard disk in terms of a binary decision diagram (BDD). Based on the current mode and state of the system, the BDD provides all inputs that are valid so that the closed-loop system satisfies the specification. It is straightforward to implement the controller on an embedded device, since it basically reduces to querying the BDD at each sampling time. The time to query the BDD is at least two magnitudes smaller than the sampling time and therefore does not constitute a problem.

V. Evaluation and Comparison in CarSim

The two controllers designed in Sections III and IV have been tested in a Simulink environment using the polynomial model (3). Fig. 5a shows simulation results in the following scenario; at $t = 0$ the speed of the ACC-equipped car is 5 m/s and there is a lead car 250 m away which leaves the lane at $t = 50$ s. A new car cuts 100 meters in front at $t = 100$ s and leaves the lane at $t = 150$ s. As can be seen, the formal requirements on input, velocity and time headway are satisfied for the two modes.

The controller from Section III has also been tested in CarSim, as are shown in Fig. 5b. A video is also available at `http://web.eecs.umich.edu/cpswiki/public_media/`.

VI. Discussion on Assumptions

We made some simplifying assumptions with regard to the lead car speed (A.1) and the number of cars within the radar range (A.2). Next, we discuss implications of these assumptions and how they can be relaxed.

A cruise controller that has practical value must be able to handle a lead car that changes speed. Relaxing the assumption (A.1) introduces environmental uncertainty into the system and increases the dimension of the state space from two to three. The theory of approximate finite-state abstractions, upon which PESSOA is based, already supports adversarial environments such as a lead car with changing velocity. Similarly, for the invariant-set based approach, the notions of a controlled-invariant set and reachability can be replaced with those of a *robustly controlled-invariant set* and "robust" reachability for systems with uncertainty. The increase in the dimension of the model, from two to three states, will render the synthesis of a controller in PESSOA or based on invariant-sets more challenging from a computational point of view.

On the other hand, the assumption (A.2) is rather technical and it is adopted to avoid the pathological cases where lead cars cut into the lane infinitely often in a way to prevent maintaining the desired time headway while always in mode M_2. The synthesized controllers are still correct when this assumption is relaxed to either (i) $R_{2,2}(v,h) = \mathcal{D}$ for all $(v,h) \in \mathcal{V} \times \mathcal{H}$ and there are finitely many resets, or (ii) $R_{2,2}(v,h) = G_2$ for all $(v,h) \in \mathcal{V} \times \mathcal{H}$.

VII. Conclusions

In this paper, we formalized the ACC problem using a hybrid dynamical system model with two modes and an LTL specification. Then, we presented two solution approaches to synthesize correct-by-construction control software for ACC. Both approaches rely on a fixed-point-based characterization of the LTL specification, one computing such fixed-points directly on the continuous state-space, the other on a finite-state abstraction of the nonlinear dynamics. Each approach has certain advantages and disadvantages: (i) Termination of fixed-point-based algorithms on continuous state-spaces is not guaranteed whereas termination is always guaranteed when working with finite-state abstractions. (ii) Approximate finite-state abstractions can be computed directly for nonlinear dynamics, whereas the current implementation of the invariant set based approach requires linearization of the dynamics and error bounds between the actual model and the linearized model. (iii) It is currently not possible to handle quantitative objectives in PESSOA, whereas the invariant-set based approach readily gives the designer the flexibility to do optimizations to shape the transient behavior or to encode "soft" constraints within MPC.

It can happen that an ACC system cannot maintain safety within specified limits on braking (e.g., (2)), such as in the event of a car cutting dangerously close into a lane. In this case, the ACC system is expected to alert the driver through automated emergency braking and/or other feedback cues, and hand control of the vehicle back to the driver. An advantage of the presented formal methods is that the safe set (i.e., control domain $\mathcal{D}$) is explicitly computed and thus conditions for passing control to an emergency braking module are clearly defined.

Acknowledgement

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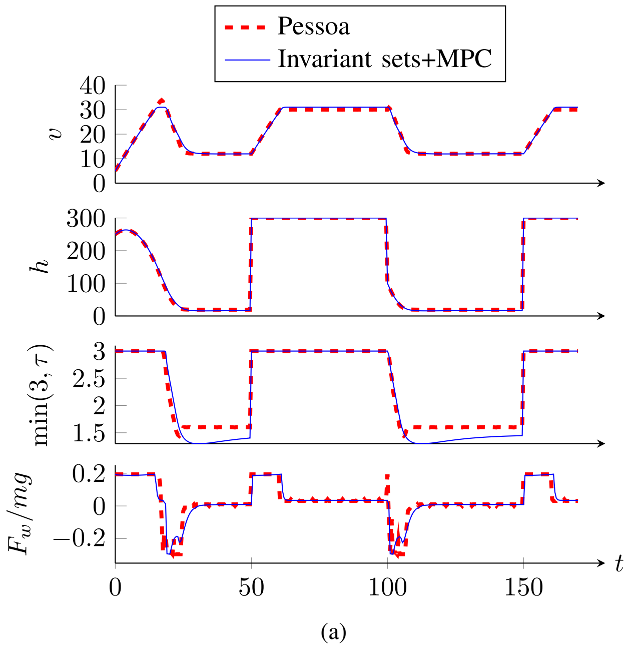

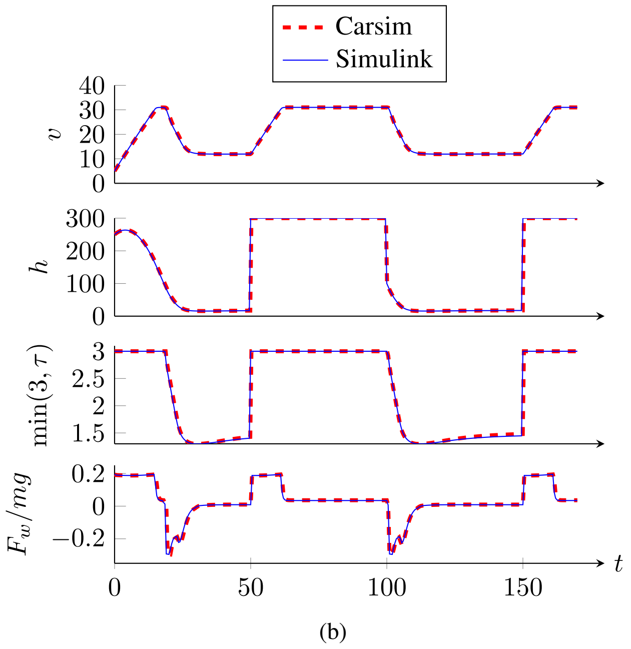

Fig. 5: Simulation results for (a) both controllers in Simulink and (b) the controller from Section III in Simulink and CarSim. The plots show, from top to bottom: speed, headway, time headway, and normalized applied wheel force.

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