Characterizing the Stability Region of IEEE 802.11 Distributed Coordination Function - Part II: A Multi-channel Perspective

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Abstract

In this paper, we characterize the stability region of IEEE 802.11 DCF from a multi-channel perspective. The fundamental conceptual issue accompanying channelization is the notion of channel-switching scheduling policy which introduces a channel-occupancy distribution of each node. We show that channelization is heuristically equivalent to expanding the average backoff window in shaping the stability region. Hence, the convexity of stability region of a multi-channel system under 802.11 DCF is almost ubiquitous. Also, as a result of the comparison among stability regions of different scheduling policies, the existence of throughput-optimal policy is inquired, and some preliminary results on this aspects are also presented.

I. Introduction

Advances in software defined radio in recent years have motivated numerous studies on building agile, channel-aware transceivers that are capable of sensing instantaneous channel quality and making opportunistic channel access and transmission scheduling decisions. By allowing users to dynamically select which channel to use for transmission, these schemes aim to improve the system performance, typically measured by the total (or per user) throughput, the average packet delay and etc, compared to a system with a single channel or more static channel allocations. The main reason behind such improvement lies in temporal, spatial and spectral diversity. That is, the quality of a channel perceived by a user is time-varying, user-dependent, and channel-dependent. This technology development motivates us to continue the study in [1] on stability region of IEEE 802.11 DCF in a single channel setting. The fundamental conceptual issue accompanying channelization is the notion of channel-switching scheduling policy, either centralized or distributed, which introduces a channel-occupancy distribution of each node. Consequently, the attempt rate of a node in a channel is roughly discounted by a factor of its occupancy probability in this channel. In [1], the authors showed that the transmission attempt rate of each node is upper bounded by the reciprocal of the average backoff window size. We may then heuristically argue that this discounting effect from channelization is equivalent to expanding the average backoff window. Hence, following the conclusion in [1] that the stability region under a large backoff window is convex, we may expect that the stability region in a multi-channel system is likely to be convex in most parameterizations. Also, this window expansion effect from the channel occupancy distribution enables us to omit successive attempts even under a small backoff window in the multi-channel case, thus simplifying the analysis without impacting numerical accuracy. Moreover, for a given policy, instead of using the aforementioned metrics we can measure the system performance by its stability region, thus implying the concept of throughput-optimality, and we are then interested in investigating the existence of such a throughput-optimal policy.

We proceed as follows. In Section II, we present our model and state definitions and assumptions. In Section III, we provide a constrained system of equations to quantitatively describe the stability region, followed by the analysis on characteristics of its solutions. These results are then numerically studied and compared to results from simulation in Section IV, and we conclude in Section V.

II. System Model

Consider a multiple access system using the IEEE 802.11 DCF. We assume that

1) the system consists of n nodes (or users interchangeably), indexed by the set $\mathcal{N} = \{1, 2, \ldots, n\}$, each with an infinite buffer; each node uses the same parameterization and has one transceiver;
2) the system is with K channels, indexed by the set $\mathcal{C} = \{1, 2, \ldots, K\}$; all channels are ideal and there is no MAC-level packet discard, i.e., there is no retransmission limit of a packet after collision; all channels are physically symmetric, namely in bandwidth, and the system use the same parameterization for all channels.
3) the queueing process at each node is stationary and ergodic such that Little's law is applicable [2].

We are still unclear about the effects of asymmetry among channels on the system performance, and we leave the asymmetric scenario as our future work. Despite the notion of channel-switching scheduling policy (scheduling policy or policy interchangeably), another rather technical issue introduced by channelization is the heterogeneity of embedded time units among channels. Since the length of a slot in a channel is in nature a random variable that depends on the traffic flows going through, channels are in general strongly asynchronous in the embedded time units. Thus, as nodes transit among channels, we may need to keep changing our reference of embedded time in slot-based analysis. We then define the notions of a slot in different contexts as follows.

Definition 1: Consider a virtual backoff timer in each channel that counts down according to the 802.11 exponential backoff scheme with an infinite initial value. A *channel-slot (c-slot)* is defined as the time interval between two consecutive decrements. Consider a virtual backoff timer at each node with an infinite initial value, the one that is synchronized with the virtual timer of the channel in which the node resides. A *node-slot (n-slot)* is defined as the time interval between two consecutive decrements of the nodal virtual backoff timer.

Remark 1: There is in fact no inherent difference between notions of the two types of slots; however, the implicit references of timing in embedded slots are explicitly distinguished. This differentiation of the references of time becomes crucial when we define quantities based on the embedded time, the length of which is a random variable and inherently depends on a specific channel, provided that nodes can traverse among different channels. This observation will soon be more concrete in analysis, and without ambiguity in certain context, we will omit the explicit association of a channel (node) index with a slot in the rest of the paper.

We then formally define a channel-switching scheduling policy. We first define some preliminary notation. Define by $\mathbb{1}_i^{(k)}(s)$ the indicator function of the presence of node i in channel k in c-slot s. Define by $\mathcal{I}_i^{s,(k)}$ the space of all possibly available information to node i up to c-slot s in channel k. Assuming perfect record at each node, for any arbitrary realization $I_i^{s,(k)} \in \mathcal{I}_i^{s,(k)}$ and $I_i^{s',(k)} \in \mathcal{I}_i^{s',(k)}$, we have $I_i^{s,(k)} \subset I_i^{s',(k)}$ for all $s < s'$, and $\mathcal{I}_i^{s,(k)}$ is application-dependent. For instance, we may have $\mathbb{1}_i^{(k)}(s) \in I_i^{s,(k)}$, and other information like the current backoff stage and the empirical average throughput up to the c-slot s may also be elements of $I_i^{s,(k)}$. Define by T_k the mapping from a c-slot index in channel k to the real time instant t of the beginning of this slot, and by T_k^{-1} the inverse mapping from a real time instant t to the index of a c-slot in channel k that it is within. We then define a scheduling policy as follows.

Definition 2: 1) Centralized scheduling policy: assuming that $I_i^{s,(k)} \in \mathcal{I}_i^{s,(k)}$ is known by the central controller for all i and k at c-slot[1] s^k with perfect record, $\mathbb{1}_i^{(k)}(s^k+1)$ is then given by

$$\mathbb{1}_i^{(k)}(s^k+1) = g_{i,s^k}^{(k)}(\mathcal{I}^{s^k}),$$

where

$$\mathcal{I}^{s^k} := \big\{ I_i^{s^l,(l)}, \forall i \in \mathcal{N}, \forall l \in \mathcal{C}, \forall s^l \leq T_l^{-1}(T_k(s^k)) \big\}.$$

2) Decentralized scheduling policy: assuming that $I_i^{s,(k)}$ is the private information of node i, and scheduling is determined locally at each node, then

$$\mathbb{1}_i^{(k)}(s+1) = g_{i,s}^{(k)}(I_i^{s,(k)}).$$

In both cases, the sequence $\mathbf{g}^{(k)} := (\mathbf{g}_1^{(k)}, \mathbf{g}_2^{(k)}, \ldots)$ constitutes the corresponding scheduling policy in channel k, where $\mathbf{g}_s^{(k)} = (g_{1,s}^{(k)}, g_{2,s}^{(k)}, \ldots, g_{n,s}^{(k)})$. The collection $\mathbf{g} := \{\mathbf{g}^{(1)}, \mathbf{g}^{(2)}, \ldots, \mathbf{g}^{(K)}\}$ then forms a channel-switching scheduling policy of the system, and the space of all possible policies is denoted by $\mathcal{G}$.

Note that the mixture of centralized and decentralized policies defined above, e.g., locally centralized policies, can be similarly defined. For the rest of this paper, we make the following assumptions about a scheduling policy $\mathbf{g}$.

A1 Under $\mathbf{g}$, the FACS decoupling approximation or simply Bianchi's approximation is still satisfied.

A2 $\mathbf{g}$ is independent of the binary state of queue at any node, where the binary state space is $\mathcal{B} = \{\text{empty, non-empty}\}$.

A3 $\mathbf{g}$ is *persistent* in a channel for the entire service process of a packet, that is, a channel-switching decision is only made before or after the service process of a packet.

Remark 2: **A3** also means that a channel switching is only scheduled at the edge of a c-slot. However, the edges of c-slots in the two channels that a node switches between may not be aligned then, and we then assume the synchronization of the nodal backoff time with the channel timer is done with the beginning of the next c-slot in the new channel. Hence, there may be a fragment of time after switching that the nodal timer is "indefinite" about its channel allocation, as illustrated in Appendix A.

As aforementioned, $\mathbf{g}$ introduces a channel occupancy distribution for each node and we characterize it as follows. Define by $\mathcal{Q}_i = \{q_i^{(k)}, k \in \mathcal{C}\}$ the *equilibrium* channel occupancy distribution *in n-slots* of node i. Denote by t^- the beginning of an arbitrary n-slot, and then $q_i^{(k)}$ is given by

$$q_i^{(k)} = P\big\{\text{node } i \text{ is in channel } k \text{ at } t^-\big\}.$$

[1]Here we use the superscript to avoid ambiguity.

Define by $\widehat{\mathcal{Q}}_i = \{\hat{q}_i^{(k)}, k \in \mathcal{C}\}$ the *equilibrium* channel occupancy profile *in c-slots* of node i. Similarly, denote by $\hat{t}^-$ the beginning of an arbitrary c-slot in channel k, and $\hat{q}_i^{(k)}$ is then given by

$$\hat{q}_i^{(k)} = P\big\{\text{node } i \text{ is in channel } k \text{ at } \hat{t}^-\big\}.$$

Note that $\sum_{k\in\mathcal{C}} \hat{q}_i^{(k)}$ may not be one, and $\widehat{\mathcal{Q}}_i$ is thus not a distribution in any probabilistic sense. However, $\widehat{\mathcal{Q}}_i$ is associated with $\mathcal{Q}_i$ by

$$\hat{q}_i^{(k)} \approx \frac{q_i^{(k)}}{\sum_{\substack{l\neq k\\ l\in\mathcal{C}}} \left(\frac{q_i^{(l)}\mathbb{E}[slot_{+i}^{(l)}]}{\mathbb{E}[slot_{-i}^{(k)}]}\right) + q_i^{(k)}}$$

for all k, where $\mathbb{E}[slot_{+i}^{(k)}]$ ($\mathbb{E}[slot_{-i}^{(k)}]$) are the conditional average lengths of a c-slot in channel k in seconds, given the presence (resp. absence) of node i therein. We show in Appendix A that this approximation becomes equality if we assume the edges of slots in two channels are aligned when there is a node switching between them. Define by $\widetilde{\mathcal{Q}}_i = \{\tilde{q}_i^{(k)}, k \in \mathcal{C}\}$ the *equilibrium* packet assignment distribution of node i, where

$$\tilde{q}_i^{(k)} = P\big\{\text{an arbitrary packet of node } i \text{ is served in channel } k\big\}.$$

$\widetilde{\mathcal{Q}}_i$ and $\mathcal{Q}_i$ are associated by a well-defined correspondence which is specified at the end of this section after defining other required quantities.

Let the data arrival rate at node i be λ_i bits per second, where $i \in \mathcal{N}$, and let $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n)$. We then define the stability region of system as follows.

Definition 3: The *stability region* of system $\Lambda^{\mathbf{g}}$ given a scheduling policy $\mathbf{g}$ is the set

$$\Lambda^{\mathbf{g}} := \left\{ \boldsymbol{\lambda} \in \mathbb{R}_+^n \;\middle|\; \begin{array}{c}\text{the queue lengths at all nodes are}\\ \text{bounded when the data arrival rates}\\ \text{are } \boldsymbol{\lambda} \text{ under 802.11 DCF with } \mathbf{g}\end{array} \right\}.$$

For any given $\boldsymbol{\lambda}$, whether $\boldsymbol{\lambda} \in \Lambda$ is determined by the utilization factor of each node, denoted by ρ_i for node i,or equivalently the probability that the queue at node i is non-empty at an arbitrary real time instant. Let $\hat{\rho}_i^{(k)}$ be the probability that the queue at node i is non-empty at the beginning of an arbitrary c-slot in channel k, denoted by t^-. $\hat{\rho}_i^{(k)}$ is then given by

$$\hat{\rho}_i^{(k)} = P\big\{\text{the queue at node } i \text{ is non-empty at } t^-\big\},$$

Similar to the single channel case, we have $\hat{\rho}_i^{(k)} \leq \rho_i$ where equality holds if and only if $\rho_i = 1$ or $\rho_i = 0$, i.e., node i is either saturated or. In parallel, define

$$\hat{\hat{\rho}}_i^{(k)} = \frac{\rho_i \mathbb{E}[slot_{i,\overline{Q}}^{(k)}]}{\rho_i \mathbb{E}[slot_{i,\overline{Q}}^{(k)}] + (1-\rho_i)\mathbb{E}[slot_{i,Q}^{(k)}]},$$

where $\mathbb{E}[slot_{i,Q}^{(k)}]$ ($\mathbb{E}[slot_{i,\overline{Q}}^{(k)}]$) is the conditional average length of an arbitrary c-slot in channel k, given that the queue at node i is non-empty (resp. empty) at the beginning of slot. Using the similar argument in [1], we have

$$\hat{\rho}_i^{(k)} \approx \hat{\hat{\rho}}_i^{(k)}.$$

Let $\tau_i^{(k)}$ be the probability that node i initiates a transmission attempt in an arbitrary c-slot in channel k. Then, we have the following lemma.

Lemma 1: $\tau_i^{(k)}$ is given by $\tau_i^{(k)} = \hat{q}_i^{(k)}\hat{\rho}_i^{(k)}/\overline{W}_i^{(k)}$, where $\overline{W}_i^{(k)}$ is the average backoff length of node i in channel k, and the term *backoff length* means the selected timer value plus 1.

Proof:

$$\begin{aligned} Tx &:= \big\{\text{node } i \text{ initiates a first-attempt in a c-slot in channel } k\big\};\\ Q(\overline{Q}) &:= \big\{\text{the queue at node } i \text{ is non-empty (empty) at the beginning of a c-slot in channel } k\big\};\\ Ch(k)(\overline{Ch(k)}) &:= \{\text{node } i \text{ is present (absent) in channel } k \text{ in a c-slot}\}. \end{aligned}$$

We then have

$$\begin{aligned} \tau_i^{(k)} &= P(Tx|\overline{Q}) \cdot P(\overline{Q}) + P(Tx|Q) \cdot P(Q)\\ &= P(Tx|\overline{Q}) \cdot P(\overline{Q}) + P(Tx|Q, Ch(k)) \cdot P(Ch(k)|Q) \cdot P(Q) + P(Tx|Q, \overline{Ch(k)}) \cdot P(\overline{Ch(k)}|Q) \cdot P(Q). \end{aligned}$$

Note that $P(Tx|Q, Ch(k)) = \frac{1}{\overline{W}_i^{(k)}}$ and $P(Ch(k)|Q) = \hat{q}_i^{(k)}$, we obtain[2]

$$\begin{aligned}\tau_i^{(k)} &= 0 \cdot (1-\hat{\rho}_i^{(k)}) + \frac{1}{\overline{W}_i^{(k)}} \cdot \hat{q}_i^{(k)} \cdot \hat{\rho}_i^{(k)} + 0 \cdot (1-\hat{q}_i^{(k)}) \cdot \hat{\rho}_i^{(k)} \\ &= \frac{\hat{q}_i^{(k)} \hat{\rho}_i^{(k)}}{\overline{W}_i^{(k)}}.\end{aligned}$$

■

Remark 3: 1) With **A3**, $\overline{W}_i^{(k)}$ is given by

$$\overline{W}_i^{(k)} = \frac{1}{2}\left[W\left((1-p_i^{(k)})\sum_{j=0}^{m-1}(2p_i^{(k)})^j + (2p_i^{(k)})^m\right) + 1\right],$$

where $p_i^{(k)}$ is the probability of collision given a transmission attempt and W is the initial size of backoff window.

2) With the notion of $\tau_i^{(k)}$, the family of quantities $\mathbb{E}[slot_{\{\cdot\}}^{(k)}]$, the conditional average length of a c-slot in channel k given the event $\{\cdot\}$, is well defined, and the results are reported Appendix B.

Define $p_i^{(k)}$ the probability of collision given a transmission from node i in channel k. Then, we have

$$q_i^{(k)} \approx \frac{\tilde{q}_i^{(k)} \frac{\hat{q}_i^{(k)}}{\tau_i^{(k)}(1-p_i^{(k)})}}{\sum_{l \in \mathcal{C}} \tilde{q}_i^{(l)} \frac{\hat{q}_i^{(l)}}{\tau_i^{(l)}(1-p_i^{(l)})}},$$

where the approximation becomes equality under the same condition as $\hat{q}_i^{(k)}$, and is justified in the appendix as well.

III. Multi-channel Analysis

A. The stability region equation $\Sigma^{\mathbf{g}}$

Given any scheduling policy $\mathbf{g}$, Let $\Lambda^{\mathbf{g}}$ be the corresponding stability region.

Theorem 1: $\boldsymbol{\lambda} \in \Lambda^{\mathbf{g}}$ if and only if there exists at least one solution $\boldsymbol{\tau} = (\boldsymbol{\tau}^{(k)}, k \in \mathcal{C})$ where $\boldsymbol{\tau}^{(k)} = (\tau_i^{(k)}, i \in \mathcal{N})$ to the following constrained system of equations $(\Sigma^{\mathbf{g}}, \mathrm{C}, \boldsymbol{\lambda})$,

$$\Sigma^{\mathbf{g}} : \begin{cases} \tau_i^{(k)} = \dfrac{\hat{q}_i^{(k)} \hat{\rho}_i^{(k)}}{\overline{W}_i^{(k)}} & \text{(a)} \\ p_i^{(k)} = 1 - \prod_{j \neq i}(1-\tau_j^{(k)}) & \text{(b)} \\ \rho_i = \min\left\{\dfrac{\lambda_i}{P}\sum_{k \in \mathcal{C}}\left[\tilde{q}_i^{(k)}\left(\dfrac{\overline{W}_i^{(k)}-1}{1-p_i^{(k)}}\mathbb{E}[slot_{i,Q,\overline{Tx}}^{(k)}] + T_c^{(k)}\dfrac{p_i^{(k)}}{1-p_i^{(k)}} + T_s^{(k)}\right)\right], 1\right\} & \text{(c)} \end{cases}$$

subject to

$$\mathrm{C} : \begin{cases} 0 \leq \tau_i^{(k)} \leq 1 & \text{(i)} \\ 0 \leq \rho_i < 1 & \text{(ii)} \end{cases}$$

where $i \in \mathcal{N}$ and $k \in \mathcal{C}$; P is the packet payload size; $\mathbb{E}[slot_{i,Q,\overline{Tx}}^{(k)}]$ is the conditional average length of a c-slot in channel k given that the queue at node i is non-empty but i does not transmit in this slot.

Proof: The proof is an immediate extension of the proof of Theorem 1 in [1], given the assumptions made about $\mathbf{g}$. ■

Remark 4: 1) In the rest of the paper, we assume that $\mathcal{Q}_i^{\mathbf{g}}$ are constants that are predeterminable for all i. In addition, the explicit upper bound on ρ_i in $\Sigma^{\mathbf{g}}$(c) can be omitted due to C(ii).

2) With $K = 1$ and $q_i^{(1)} = 1$ for all $i \in \mathcal{N}$, $\Sigma^{\mathbf{g}}$ degrades to Σ in the single channel case [1], and $\Sigma^{\mathbf{g}}$ then forms the unified framework for the rest of our analysis.

[2] As noted in [1], the first equality is technically an approximation.

B. Characteristics of solutions to $\Sigma^{\mathbf{g}}$

Like in the single channel case, we use Brouwer's fixed point theorem to prove the existence of solutions to $\Sigma^{\mathbf{g}}$. The technical difference in the multi-channel case is the selection of proper variables to construct the fixed point equation. In the single channel case, we rewrite Σ as an fixed point equation with respect to $\boldsymbol{\tau} = (\tau_i, i \in \mathcal{N})$. However, because of channelization, unknown quantities in $\Sigma^{\mathbf{g}}$ are intricately intertwined, and it turns out to be difficult to implement $\Sigma^{\mathbf{g}}$ as an equation with respect to $\boldsymbol{\tau}$ solely, where $\boldsymbol{\tau} = (\boldsymbol{\tau}^{(k)}, k \in \mathcal{C})$ and $\boldsymbol{\tau}^{(k)} = (\tau_i^{(k)}, i \in \mathcal{N})$. Nevertheless, by using auxiliary variables, namely $\hat{\boldsymbol{\rho}} = (\hat{\rho}_i, i \in \mathcal{N})$, we show in Appendix F that we can construct a fixed point equation in $[0,1]^{Kn} \times [0,1]^{Kn}$, that is,

$$(\boldsymbol{\tau}, \hat{\boldsymbol{\rho}}) = \boldsymbol{\Gamma}'(\boldsymbol{\tau}, \hat{\boldsymbol{\rho}}),$$

where $\boldsymbol{\tau}, \hat{\boldsymbol{\rho}} \in [0,1]^{Kn}$. Since $\boldsymbol{\Gamma}'$ is a continuous vector function, the existence of solutions to $\Sigma^{\mathbf{g}}$ is then established.

As to the uniqueness of solution to $(\Sigma^{\mathbf{g}}, \boldsymbol{\lambda})$. we limit our focus to a family of scheduling policies that we call the unbiased policies, and we present the result on the uniqueness of solution when the initial window size W is sufficiently large for an unbiased policy. Before we formally state our theorem, we define the the family of unbiased scheduling policies as follows.

Definition 4: A scheduling policy is *unbiased* if the stationary channel occupancy distribution is identical for every node, i.e., $q_i^{(k)} = q^{(k)}$ for all $i \in \mathcal{N}$ and $k \in \mathcal{C}$. An unbiased scheduling policy is denoted by $\mathbf{g}^U$, and the space of unbiased policies is denoted by $\mathcal{G}^U$.

We are then ready to present the theorem on the uniqueness of solution to $(\Sigma^{\mathbf{g}^U}, \boldsymbol{\lambda})$.

Theorem 2: For all sufficiently large W, $(\Sigma^{\mathbf{g}^U}, \boldsymbol{\lambda})$ admits a unique solution.

Proof: See Appendix C. ■

Combining the results of Theorem 1 and the proof of Theorem 2, we conclude the following corollary.

Corollary 1: When W is sufficiently large, $\Lambda^{\mathbf{g}^U}$ is approximated by

$$\tilde{\Lambda}^{\mathbf{g}^U} = \left\{ \boldsymbol{\lambda} \in \mathbb{R}_+^n \;\middle|\; 0 < \frac{\gamma_i^1(\lambda_i) \sum_j \gamma_j^2(\lambda_j)}{1 - \sum_i \gamma_j^1(\lambda_j)} + \gamma_i^2(\lambda_i) < \frac{2}{W+1}, \forall i \in \mathcal{N} \right\},$$

where

$$\gamma_i^1(\lambda_i) = \Big(\frac{\lambda_i T}{P} \sum_{l \in \mathcal{C}} \Big[f_l(\mathcal{Q}) h_l(\mathcal{Q}) \Big] \Big) \Big/ \Big(1 + \frac{\lambda_i T}{P} \sum_{l \in \mathcal{C}} \Big[f_l(\mathcal{Q}) h_l(\mathcal{Q}) \Big] \Big),$$

and

$$\gamma_i^2(\lambda_i) = \Big(\frac{\lambda_i T}{P} - \frac{\lambda_i (W-1)(T-\sigma)}{P(W+1)} \Big) \Big/ \Big(1 + \frac{\lambda_i T}{P} \sum_{l \in \mathcal{C}} \Big[f_l(\mathcal{Q}) h_l(\mathcal{Q}) \Big] \Big).$$

with the constants $f_l(\mathcal{Q})$ and $h_l(\mathcal{Q})$ numerically determined given $\mathcal{Q} = \{q^{(k)}, k \in \mathcal{C}\}$ in Appendix C.

C. Disccusion on the throughput-optimality within $\mathcal{G}^U$

In our random access system, the input data rates $\boldsymbol{\lambda}$ and the occupancy distribution $\mathcal{Q}^{\mathbf{g}^U}$ induced by the policy $\mathbf{g}^U$ are main sets of tunable parameters for controlling the system. However, in most application scenarios $\boldsymbol{\lambda}$ is unknown to the network manager or it is in fact regarded as private information of users. Thus, we may want to focus on designing the scheduling policy $\mathbf{g}$ (could be viewed in a higher level of mechanism design) to engineer our system for desired or even optimal performance. As to the notion of optimality and various metrics defining it, we concentrate on the so-called throughput optimality, and in the following theorem we present a result on the throughput optimality within the family of unbiased policies $\mathcal{G}^U$. We show that the equi-occupancy policy that is interpreted by its name induces throughput optimality in $\mathcal{G}^U$. This result is stated after the conventional definition of throughput optimality in the following.

Definition 5: Let $\mathcal{G}$ be a set of scheduling policies. $\mathbf{g}^* \in \mathcal{G}$ is said to be throughput-optimal in $\mathcal{G}$ if $\Lambda^{\mathbf{g}^*} \supseteq \Lambda^{\mathbf{g}}$ for all $\mathbf{g} \in \mathcal{G}$.

Theorem 3 (The unbiased equi-occupancy theorem): Consider a scheduling policy $\mathbf{g}^U \in \mathcal{G}^U$ and the associated stationary channel occupancy distribution $\mathcal{Q}^{\mathbf{g}^U}$ and stability region $\Lambda^{\mathbf{g}^U}$. For all sufficiently large initial window size W, $\mathbf{g}^U$ is throughput-optimal in $\mathcal{G}^U$ if $q^{(k)} = \frac{1}{K}$ for all k.

Proof: See Appendix D. ■

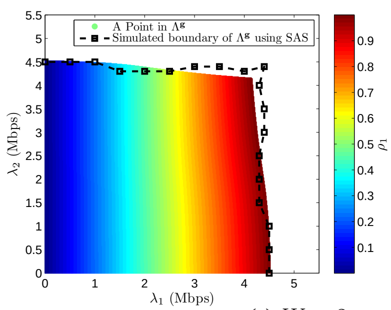
Remark 5: The above result may provide us the heuristic that given symmetric channelization and identical channel occupancy of users (or any scaled version when channels are asymmetric), traffic-load balancing optimizes the system performance in terms of expanding the stability region. However, in the context of unbiased channel occupancy distribution, the notion of balancing may also be interpreted in terms of the number of active users in each channel. In fact, balancing traffic-loads and numbers of active users in channels may have a joint effect on tuning the system, and we are still unclear how the two types of balancing affect the system performance. Also, we believe a throughput-optimal policy in the entire policy space $\mathcal{G}$, if exists, is in general a biased one. A trivial example would be that deterministically separating two users in a bi-channel system is clearly no worse than an equi-occupancy strategy. The inquires arisen above are open question to the future work.

D. Heuristic implementation of equi-occupancy policies

In this part, we propose a simple algorithm to implement the equal occupancy policies in the bi-channel model. The proposed algorithm consists of two parts called SAS (Switching After Success) and SAC (Switching After Collision). In both parts, a switching probability is assigned to each backoff stage. In SAS (SAC), a node switches to the other channel with probability α_i after each successful transmission (resp. each collision) if it is at the ith backoff stage. In addition, in SAC, after switching to the other channel, a node does not reset its backoff stage; instead, it continues the exponential backoff due to the last collision. This algorithm heuristically implements the equi-occupancy policy in the following sense. Consider the two-dimensional Markov chains in the form of Bianchi's model [3], where each state in one channel has a mirror state in the other one. Using the argument of symmetry, the symmetric solution is one possible steady-state distribution which implements the equi-occupancy as shown in Appendix E. Yet it is indefinite that if any asymmetric solution exists while the symmetric solution is always observed in our numerical experiment. Also note that SAS satisfies the assumption **A3** but SAC does not. However, we show in Appendix E that SAC is equivalent to SAS in terms of the attempt rate in saturation in the symmetric solution. Notice that the Markov chain model implicitly assumes no empty channel for a significantly long time resulting from the policy; otherwise, the system may operate at the steady state of a single channel system for considerably long time, which undermines the assumed stationarity of the multi-channel chain, and moreover, the stationary distribution of this mean field model may not reflect the empirical long-term time average. When collision is frequent, using SAS nodes tend to cluster in one certain channel, thus incurring the above problem. Nonetheless, SAC heuristically avoids such situation in congestion. Therefore, we may combine the two simple switching strategies according to the traffic condition.

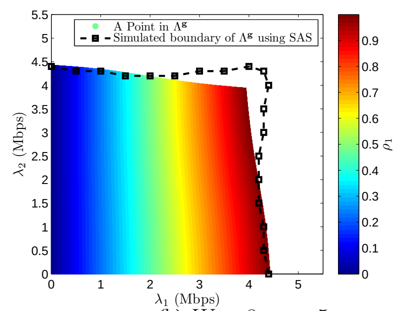
IV. Numerical Studies

In this section, we present the numerical results obtained based on the solver and simulator that we implemented on MATLAB 2008b. Specifically, we consider a system of two users with two channel. Though this scenario is a toy example, it illustrates most of the essence in our previous analysis. The specification of the test bench is reported in Table II in Appendix F. We focus on three objectives, namely, the comparison between stability regions of single and bi-channel systems, the throughput-optimality in $\mathcal{G}^U$, and the relative advantage of channelization to a single channel.



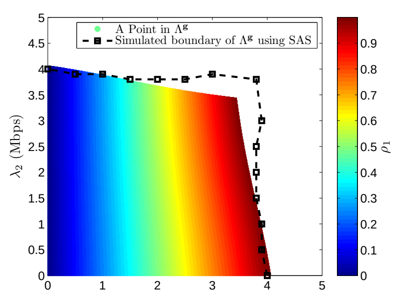
(λ_1, λ_2)	$(q_1^{(1)}, q_2^{(1)})$
(4.5, 0)	(0.5176, -)
(4.5, 0.5)	(0.4337, 0.5379)
(4.5, 1.0)	(0.5410, 0.5190)
(4.3, 1.5)	(0.4622, 0.4818)
(4.3, 2.0)	(0.4579, 0.5523)
(4.3, 2.5)	(0.4649, 0.4937)
(4.4, 3.0)	(0.4499, 0.5276)
(4.4, 3.5)	(0.4740, 0.5056)
(4.3, 4.0)	(0.4649, 0.4665)
(4.4, 4.4)	(0.4664, 0.4603)

(a) $W = 2, m = 0$



(λ_1, λ_2)	$(q_1^{(1)}, q_2^{(1)})$
(4.4, 0)	(0.4914, -)
(4.3, 0.5)	(0.5132, 0.5240)
(4.3, 1.0)	(0.5159, 0.5270)
(4.2, 1.5)	(0.4870, 0.4401)
(4.2, 2.0)	(0.4923, 0.4728)
(4.2, 2.5)	(0.4812, 0.4602)
(4.3, 3.0)	(0.4998, 0.5130)
(4.3, 3.5)	(0.5026, 0.4560)
(4.4, 4.0)	(0.4896, 0.4291)
(4.3, 4.3)	(0.5132, 0.5071)

(b) $W = 8, m = 5$

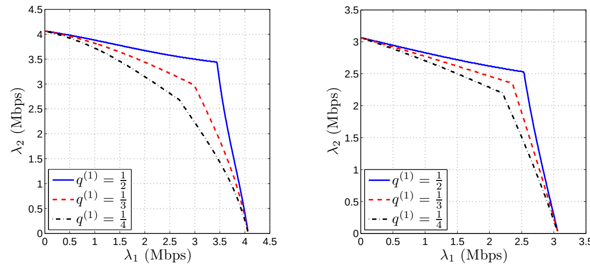


(λ_1, λ_2)	$(q_1^{(1)}, q_2^{(1)})$
(4.0, 0.0)	(0.4788, -)
(3.9, 0.5)	(0.5270, 0.5605)
(3.9, 1.0)	(0.4945, 0.4392)
(3.8, 1.5)	(0.4862, 0.5095)
(3.8, 2.0)	(0.4976, 0.5344)
(3.8, 2.5)	(0.4981, 0.5328)
(3.9, 3.0)	(0.4850, 0.4899)
(3.8, 3.8)	(0.4928, 0.5034)

(c) $W = 32, m = 5$

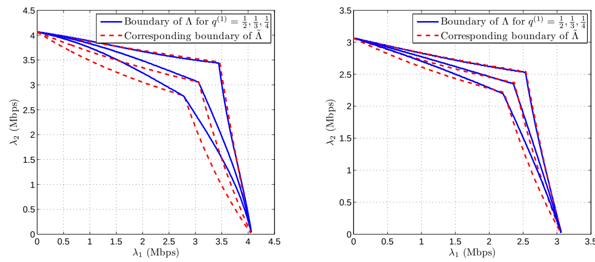
Fig. 1. The stability regions for various scenarios under $\mathbf{g}^U$ and the empirical occupancy distribution under SAS with $\alpha_i = 1$ for all i.

In Figure 1, we plot the analytical stability region for various scenarios and their simulated boundary using SAS. We also report the empirical occupancy distribution obtained aside. The method of simulation is the same as in the single channel case and is reported in [1]. Compared to results of the single channel case, convexity of the stability region is observed even with small backoff windows in the bi-channel case. Also, the numerical multi-equilibrium phenomenon [1] disappears in the bi-channel system, which is expected from the heuristical equivalence between channelization and window expansion following the discussion in [1]. To conclude, a unique convex stability region is generally expected for a mutli-channel system. Moreover, empirical data shows that SAS implements the equi-occupancy in steady state.



(a) $W = 32, m = 5$ (b) $W = 128, m = 5$

Fig. 2. Throughput optimality of equi-occupancy distribution.



(a) $W = 32, m = 5$ (b) $W = 128, m = 5$

Fig. 3. Approximated stability regions $\tilde{\Lambda}^{\mathbf{g}^U}$.

In Figure 2, we plot the analytical boundaries of stability regions corresponding to different unbiased policies in two scenarios for instance. As can been seen therein, the equi-occupancy policy results in the stability region that is the superset of those of other unbiased policies. It is also worth noting that as the backoff window enlarges, the gap between the superset region and other inferior regions shrinks, as the reciprocal of the window size becomes the dominant factor in upper bounding the attempt rate.

In Figure 3, we plot the approximated stability region $\tilde{\Lambda}^{\mathbf{g}^U}$ that are computed using the result of Corollary 1. Specifically, the constants $f_k(\mathcal{Q})$ and $h_k(\mathcal{Q})$ are determined using the iterative approach presented in Appendix C. Compared to solving the original system of equations $(\Sigma^{\mathbf{g}^U}, \mathrm{C}, \boldsymbol{\lambda})$, solving the approximated system is of significantly reduced computational complexity. Also, as can be seen in the plots, the resulting approximation is accurate especially when the policy is the equi-occupancy one or the backoff window is large.

V. Conclusion

In this paper, we identified the stability region of 802.11 DCF from a multi-channel perspective. Compared to the single channel case, the primarily novel problem is the characterization of the effects of channel-switching scheduling policies on the system stability. We showed that the main effect can be heuristically understood as the expansion of backoff window. Also, primary results on the throughput optimality are shown in the context of unbiased polices.

Appendix A
Computation of $\widehat{\mathcal{Q}}$ and $\widetilde{\mathcal{Q}}$

We first define the following stochastic processes generated by the queueing process at node i.

$$N_{+i}^{(k)}(t)\left(N_{-i}^{(k)}(t)\right) := \text{the total number of c-slots in channel } k \text{ that node } i \text{ is present at its beginning up to time } t;$$

$$S_{+i}^{(k)}(j)\left(S_{-i}^{(k)}(j)\right) := \text{the length of the } j\text{th c-slot in channel } k \text{ given the presence (resp. absence) of node } i \text{ at its beginning;}$$

$$T_{+i}^{(k)}(t) := \text{the total length of real time periods that node } i \text{ is present in channel } k \text{ up to time } t.$$

Note that the above processes are well-defined on the same space Ω. As we argued before, when a channel switching is scheduled, the edges of c-slots of the two channels that a node switches between may not be aligned. Hence, there may be a period of unsynchronized time of the nodal backoff timer, as shown in Figure 4. If omitting the unsynchronized time, we have

$$\sum_{l\in\mathcal{C}} \sum_{j=1}^{N_{+i}^{(l)}(t)} S_{+i}^{(l)}(j) = t = \sum_{j=1}^{N_{+i}^{(k)}(t)} S_{+i}^{(k)}(j) + \sum_{j=1}^{N_{-i}^{(k)}(t)} S_{-i}^{(k)}(j),$$

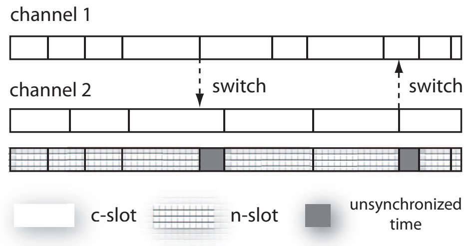


Fig. 4. Illustration of channel switching and timer synchronization.

and then

$$\sum_{\substack{l\neq k\\ l\in\mathcal{C}}}\sum_{j=1}^{N_{+i}^{(l)}(t)} S_{+i}^{(l)}(j)=\sum_{j=1}^{N_{-i}^{(k)}(t)} S_{-i}^{(k)}(j),$$

or equivalently,

$$\sum_{\substack{l\neq k\\ l\in\mathcal{C}}} N_{+i}^{(l)}(t)\frac{\sum_{j=1}^{N_{+i}^{(l)}(t)} S_{+i}^{(l)}(j)}{N_{+i}^{(l)}(t)}=N_{-i}^{(k)}(t)\frac{\sum_{j=1}^{N_{-i}^{(k)}(t)} S_{-i}^{(k)}(j)}{N_{-i}^{(k)}(t)}.$$

Using then the ergodicity assumption, $\hat{q}_i^{(k)}$ can be expressed alternatively for all $\omega\in\Omega$ as

$$\begin{aligned}
\hat{q}_i^{(k)} &= \lim_{t\to\infty}\frac{N_{+i}^{(k)}(\omega,t)}{N_{+i}^{(k)}(\omega,t)+N_{-i}^{(k)}(\omega,t)}\\
&= \lim_{t\to\infty}\frac{\frac{N_{+i}^{(k)}(\omega,t)}{N_i(\omega,t)}}{\frac{N_{+i}^{(k)}(\omega,t)}{N_i(\omega,t)}+\sum_{\substack{l\neq k\\ l\in\mathcal{C}}}\frac{N_{+i}^{(l)}(\omega,t)}{N_i(\omega,t)}\frac{\sum_{j=1}^{N_{+i}^{(l)}(\omega,t)} S_{+i}^{(l)}(\omega,j)}{N_{+i}^{(l)}(\omega,t)}\Big/\frac{\sum_{j=1}^{N_{-i}^{(k)}(\omega,t)} S_{-i}^{(k)}(\omega,j)}{N_{-i}^{(k)}(\omega,t)}}\\
&= \frac{q_i^{(k)}}{q_i^{(k)}+\sum_{\substack{l\neq k\\ l\in\mathcal{C}}}\left(\frac{q_i^{(l)}\mathbb{E}[slot_{+i}^{(l)}]}{\mathbb{E}[slot_{-i}^{(k)}]}\right)}.
\end{aligned}$$

We define a few more processes generated by the queueing process at node i.

$$\begin{aligned}
P_i^{(k)}(t) &:= \text{the total number of packets transmitted by node } i \text{ in channel } k \text{ up to time } t;\\
B_i^{(k)}(t) &:= \text{the number of busy slots in the service circle of the } j\text{th packet;}\\
I_i^{(k)}(t) &:= \text{the number of idle slots in the service circle of the } j\text{th packet,}
\end{aligned}$$

where a service circle is the period in slots between the beginning of service processes of two successive packets. A busy slot refers to a slot in the service process and a idle slot is a slot when the queue at the node is empty. Then, $q_i^{(k)}$ can also be expressed alternatively as

$$\begin{aligned}
q_i^{(k)} &= \lim_{t\to\infty}\frac{N_{+i}^{(k)}(t)}{\sum_{l\in\mathcal{C}} N_{+i}^{(l)}(t)}\\
&= \lim_{t\to\infty}\frac{\sum_{j=1}^{P_i^{(k)}(t)}(B_i^{(k)}(j)+I_i^{(k)}(j))}{\sum_{l\in\mathcal{C}}\sum_{j=1}^{P_i^{(l)}(t)}(B_i^{(l)}(j)+I_i^{(l)}(j))}\\
&= \lim_{t\to\infty}\frac{\frac{P_i^{(k)}(t)}{\sum_{h\in\mathcal{C}}P_i^{(h)}(t)}\frac{\sum_{j=1}^{P_i^{(k)}(t)}B_i^{(k)}(j)}{P_i^{(k)}(t)}\frac{\sum_{j=1}^{P_i^{(k)}(t)}(B_i^{(k)}(j)+I_i^{(k)}(j))}{\sum_{j=1}^{P_i^{(k)}(t)}B_i^{(k)}(j)}}{\sum_{l\in\mathcal{C}}\frac{P_i^{(l)}(t)}{\sum_{h\in\mathcal{C}}P_i^{(h)}(t)}\frac{\sum_{j=1}^{P_i^{(l)}(t)}B_i^{(l)}(j)}{P_i^{(l)}(t)}\frac{\sum_{j=1}^{P_i^{(l)}(t)}(B_i^{(l)}(j)+I_i^{(l)}(j))}{\sum_{j=1}^{P_i^{(l)}(t)}B_i^{(l)}(j)}},
\end{aligned}$$

where we have suppressed the reference to a sample point ω in all involved processes for brevity in notation, or interpret the equalities as with probability one. Note that the limits of all ratios are well defined, and we obtain

$$q_i^{(k)} = \frac{\tilde{q}_i^{(k)}\overline{B}_i^{(k)}\frac{1}{\hat{\rho}_i^{(k)}}}{\sum_{l\in\mathcal{C}}\tilde{q}_i^{(l)}\overline{B}_i^{(l)}\frac{1}{\hat{\rho}_i^{(l)}}} = \frac{\tilde{q}_i^{(k)}\frac{\overline{W}_i^{(k)}}{1-p_i^{(k)}}\frac{1}{\hat{\rho}_i^{(k)}}}{\sum_{l\in\mathcal{C}}\tilde{q}_i^{(l)}\frac{\overline{W}_i^{(l)}}{1-p_i^{(l)}}\frac{1}{\hat{\rho}_i^{(l)}}} = \frac{\tilde{q}_i^{(k)}\frac{\hat{q}_i^{(k)}}{\tau_i^{(k)}(1-p_i^{(k)})}}{\sum_{l\in\mathcal{C}}\tilde{q}_i^{(l)}\frac{\hat{q}_i^{(l)}}{\tau_i^{(l)}(1-p_i^{(l)})}}.$$

APPENDIX B
COMPUTATION OF $\mathbb{E}[slot_{\{\cdot\}}]$ AND RELATED QUANTITIES

Given an event $\{\cdot\}$, let $P_{idle;\{\cdot\}}^{(k)}$, $P_{succ;\{\cdot\}}^{(k)}$ and $P_{coll;\{\cdot\}}^{(k)}$ be the conditional probabilities that this slot is idle, that it contains a successful transmission, and that it has a collision, respectively. Notice that $P_{coll;\{\cdot\}}^{(k)} = 1 - P_{idle;\{\cdot\}}^{(k)} - P_{succ;\{\cdot\}}^{(k)}$. Then, $\mathbb{E}[slot_{\{\cdot\}}]$ is given by

$$\mathbb{E}[slot_{\{\cdot\}}] = P_{idle;\{\cdot\}}^{(k)} \cdot \sigma + P_{succ;\{\cdot\}}^{(k)} \cdot T_s + \left(1 - P_{idle;\{\cdot\}}^{(k)} - P_{succ;\{\cdot\}}^{(k)}\right) \cdot T_c.$$

where σ, T_s and T_c are the lengths of an empty system slot, a successful transmission, and a collision, respectively. Given $\tau_{i,\{\cdot\}}^{(k)}$, the conditional attempt rate of node i in channel k, we further have

$$P_{idle;\{\cdot\}}^{(k)} = \prod_i (1 - \tau_{i,\{\cdot\}}^{(k)}),$$

and

$$P_{succ;\{\cdot\}}^{(k)} = \sum_i \tau_{i,\{\cdot\}}^{(k)} \prod_{j\neq i}(1 - \tau_{j,\{\cdot\}}^{(k)}).$$

Explicit expressions for varieties of $\mathbb{E}[slot_{\{\cdot\}}]$ that are used throughout the paper are reported in Table I.

	$P_{idle;\{\cdot\}}^{(k)}$	$P_{succ;\{\cdot\}}^{(k)}$
$\mathbb{E}[slot_{i,\overline{Q}}^{(k)}]$	$\prod_{j\neq i}(1-\tau_j^{(k)})$	$\sum_{j\neq i}\tau_j^{(k)}\prod_{l\neq i,j}(1-\tau_l^{(k)})$
$\mathbb{E}[slot_{i,Q}^{(k)}]$	$(1-\tau_{i,Q}^{(k)})\prod_{j\neq i}(1-\tau_j^{(k)})$ where $\tau_{i,Q}^{(k)} = \frac{\hat{q}_i^{(k)}}{\overline{W}_i^{(k)}}$	$\sum_l t_l^{(k)}\prod_{j\neq l}(1-t_l^{(k)})$, where $t_j^{(k)} = \begin{cases}\tau_{i,Q}^{(k)}, & j=i\\ \tau_j^{(k)}, & j\neq i\end{cases}$
$\mathbb{E}[slot_{i,Q,\overline{Tx}}^{(k)}]$	$\prod_{j\neq i}(1-\tau_j^{(k)})$	$\sum_{j\neq i}\tau_j^{(k)}\prod_{l\neq i,j}(1-\tau_l^{(k)})$
$\mathbb{E}[slot_{+i}^{(k)}]$	$(1-\tilde{\tau}_i^{(k)})\prod_{j\neq i}(1-\tau_j^{(k)})$ where $\tilde{\tau}_i^{(k)} = \frac{\hat{\rho}_i^{(k)}}{\overline{W}_i^{(k)}}$	$\sum_l t_l^{(k)}\prod_{j\neq l}(1-t_l^{(k)})$, where $t_j^{(k)} = \begin{cases}\tilde{\tau}_i^{(k)}, & j=i\\ \tau_j^{(k)}, & j\neq i\end{cases}$
$\mathbb{E}[slot_{-i}^{(k)}]$	$\prod_{j\neq i}(1-\tau_j^{(k)})$	$\sum_{j\neq i}\tau_j^{(k)}\prod_{l\neq i,j}(1-\tau_l^{(k)})$

TABLE I
SUMMARY OF COMPUTATION OF $\mathbb{E}[slot_{\{\cdot\}}^{(k)}]$.

APPENDIX C
PROOF OF THEOREM 2

A. Main proof

As in the proof of Theorem 2 in [1], we first make several simplifying approximations due to the large window assumption. With a large backoff window, the probability of collision is small, so we have $\overline{W}_i^{(k)} \approx \frac{W+1}{2}$. We also observe that $\mathbb{E}[slot_{i,Q}^{(k)}] \approx$

$\mathbb{E}[slot^{(k)}_{i,\overline{Q}}]$ since $\tau^{(k)}_{i,Q} \leq \frac{1}{W^{(k)}_i}$ and W is large. Consequently,

$$\hat{\rho}^{(k)}_i \approx \frac{\rho_i \mathbb{E}[slot^{(k)}_{i,\overline{Q}}]}{\rho_i \mathbb{E}[slot^{(k)}_{i,\overline{Q}}] + (1-\rho_i)\mathbb{E}[slot^{(k)}_{i,Q}]} \approx \rho_i.$$

Let $T_s = T_c = T$ for the simplicity of presentation. Then, $\Sigma^{\mathbf{g}}$ is approximated by the following set of equations, that is:

$$\widetilde{\Sigma}^{\mathbf{g}}: \begin{cases} \tau^{(k)}_i = \dfrac{\hat{q}^{(k)}_i \rho_i}{\frac{W+1}{2}} & \text{(a)} \\ p^{(k)}_i = 1 - \displaystyle\prod_{j\neq i}(1-\tau^{(k)}_j) & \text{(b)} \\ \rho_i = \dfrac{\lambda_i}{P} \displaystyle\sum_{k\in\mathcal{C}} \left\{ \tilde{q}^{(k)}_i \left[\frac{\frac{W+1}{2}-1}{1-p^{(k)}_i} \left(\sigma \prod_{j\neq i}(1-\tau^{(k)}_j) + T\Big(1-\prod_{j\neq i}(1-\tau^{(k)}_j)\Big) \right) + T\frac{p^{(k)}_i}{1-p^{(k)}_i} + T \right] \right\} & \text{(c)} \end{cases}$$

Note that for a policy $\mathbf{g}^U$, we have $q^{(k)}_i = q^{(k)}$ for all $i \in \mathcal{N}$. Furthermore, we have two more approximations about $\hat{q}^{(k)}_i$ and $\tilde{q}^{(k)}_i$. We assume that $\hat{q}^{(k)}_i \approx f_k\big(\mathcal{Q}\big)$ and $\tilde{q}^{(k)}_i \approx h_k\big(\mathcal{Q}\big)$ for all i and k, where $\mathcal{Q} = \{q^{(l)}, l \in \mathcal{C}\}$, and f_k and h_k are non-user-specific functions and determined after the main proof. The rest of steps is similar to the proof of Theorem 2 in [1]. For the completeness of report, we repeat them as follows. Substituting $\widetilde{\Sigma}^{\mathbf{g}}$(b) and (c) in (a), we obtain

$$\begin{aligned} \tau^{(k)}_i &= \hat{q}^{(k)}_i \frac{2\lambda_i}{P(W+1)} \sum_{l\in\mathcal{C}} \left\{ \tilde{q}^{(l)}_i \left[\frac{W+1}{2} T \prod_{j\neq i} \frac{1}{1-\tau^{(l)}_j} - \frac{W-1}{2}(T-\sigma) \right] \right\} \\ &= \hat{q}^{(k)}_i \sum_{l\in\mathcal{C}} \left\{ \tilde{q}^{(l)}_i \left[\frac{\lambda_i T}{P} \prod_{j\neq i} \frac{1}{1-\tau^{(l)}_j} - \frac{\lambda_i (W-1)(T-\sigma)}{P(W+1)} \right] \right\}. \end{aligned}$$

Using the first-order Taylor approximation, we have $\prod_{j\neq i} \frac{1}{1-\tau^{(l)}_j} \approx 1 + \sum_{j\neq i} \tau^{(l)}_j$. Hence,

$$\tau^{(k)}_i \approx \hat{q}^{(k)}_i \sum_{l\in\mathcal{C}} \left\{ q^{(l)}_i \left[\frac{\lambda_i T}{P} \left(1 + \sum_{j\neq i} \tau^{(l)}_j \right) - \frac{\lambda_i (W-1)(T-\sigma)}{P(W+1)} \right] \right\}.$$

Let $\tau_i = \frac{2\rho_i}{W+1}$, and then notice $\tau^{(k)}_i = \hat{q}^{(k)}_i \tau_i$. Therefore, we can further reduce the dimensionality of the space of unknowns by rewriting the above equation w.r.t. τ_i's. We have

$$\hat{q}^{(k)}_i \tau_i = \hat{q}^{(k)}_i \sum_{l\in\mathcal{C}} \left\{ \tilde{q}^{(l)}_i \left[\frac{\lambda_i T}{P} \left(1 + \sum_{j\neq i} \Big(\hat{q}^{(l)}_j \tau_j \Big) \right) - \frac{\lambda_i (W-1)(T-\sigma)}{P(W+1)} \right] \right\},$$

or equivalently,

$$\begin{aligned} \tau_i &= \frac{\lambda_i T}{P} \sum_{l\in\mathcal{C}} \left[\tilde{q}^{(l)}_i \left(1 + \sum_{j\neq i} \Big(\hat{q}^{(l)}_j \tau_j \Big) \right) \right] - \frac{\lambda_i (W-1)(T-\sigma)}{P(W+1)} \\ &= \frac{\lambda_i T}{P} \left(1 + \sum_{l\in\mathcal{C}} \sum_{j\neq i} \tilde{q}^{(l)}_i \hat{q}^{(l)}_j \tau_j \right) - \frac{\lambda_i (W-1)(T-\sigma)}{P(W+1)}. \end{aligned}$$

Using the approximations of $\hat{q}$ and $\tilde{q}$, then

$$\tau_i = \frac{\lambda_i T}{P} \sum_{l\in\mathcal{C}} \Big[f_l\big(\mathcal{Q}\big) h_l\big(\mathcal{Q}\big) \Big] \cdot \sum_{j\neq i} \tau_j + \frac{\lambda_i T}{P} - \frac{\lambda_i (W-1)(T-\sigma)}{P(W+1)}$$

or equivalently,

$$\left(1 + \frac{\lambda_i T}{P} \sum_{l\in\mathcal{C}} \Big[f_l\big(\mathcal{Q}\big) h_l\big(\mathcal{Q}\big) \Big] \right) \tau_i = \frac{\lambda_i T}{P} \sum_{l\in\mathcal{C}} \Big[f_l\big(\mathcal{Q}\big) h_l\big(\mathcal{Q}\big) \Big] \cdot \sum_{j} \tau_j + \frac{\lambda_i T}{P} - \frac{\lambda_i (W-1)(T-\sigma)}{P(W+1)}.$$

Therefore, let $y = \sum_j \tau_j$,

$$\gamma^1_i = \left(\frac{\lambda_i T}{P} \sum_{l\in\mathcal{C}} \Big[f_l\big(\mathcal{Q}\big) h_l\big(\mathcal{Q}\big) \Big] \right) \Big/ \left(1 + \frac{\lambda_i T}{P} \sum_{l\in\mathcal{C}} \Big[f_l\big(\mathcal{Q}\big) h_l\big(\mathcal{Q}\big) \Big] \right),$$

and

$$\gamma_i^2 = \Big(\frac{\lambda_i T}{P} - \frac{\lambda_i (W-1)(T-\sigma)}{P(W+1)}\Big) \Big/ \Big(1 + \frac{\lambda_i T}{P} \sum_{l \in \mathcal{C}} \Big[f_l\big(\mathcal{Q}\big) h_l\big(\mathcal{Q}\big)\Big]\Big),$$

and $\widetilde{\Sigma}^{\mathbf{g}^U}$ is then equivalent to

$$\widetilde{\Sigma}^{\mathbf{g}^U} : \begin{cases} \tau_i = \gamma_i^1 y + \gamma_i^2 & \text{(a}'\text{)} \\ y = \sum_i \left(\gamma_i^1 y + \gamma_i^2\right) & \text{(b}'\text{)} \end{cases}$$

which admits only one solution, namely,

$$\tau_i = \frac{\gamma_i^1 \sum_i \gamma_i^2}{1 - \sum_i \gamma_i^1} + \gamma_i^2.$$

B. Technical approximations

When the window size is sufficiently large, we can approximate $\hat{q}_i^{(k)}$ and $\tilde{q}_i^{(k)}$ by $f_k\big(\mathcal{Q}\big) = h_k\big(\mathcal{Q}\big) = q^{(k)}$. A more accurate approximation of $\hat{q}_i^{(k)}$ is given as follows. Given

$$\hat{q}_i^{(k)} = \frac{q_i^{(k)}}{q_i^{(k)} + \sum_{\substack{l \neq k \\ l \in \mathcal{C}}} \left(\frac{q_i^{(l)} \mathbb{E}[slot_{+i}^{(l)}]}{\mathbb{E}[slot_{-i}^{(k)}]} \right)},$$

we consider $\frac{\mathbb{E}[slot_{+i}^{(l)}]}{\mathbb{E}[slot_{-i}^{(k)}]}$ where $l \neq k$. Using the first-order Taylor approximation, we have

$$\begin{aligned} \mathbb{E}[slot_{\{\cdot\}}^{(k)}] &= \sigma \prod_i \left(1 - \tau_{i,\{\cdot\}}^{(k)}\right) + T\Big(1 - \prod_i \left(1 - \tau_{i,\{\cdot\}}^{(k)}\right)\Big) \\ &\approx \sigma\Big(1 - \sum_i \tau_{i,\{\cdot\}}^{(k)}\Big) + T \sum_i \tau_{i,\{\cdot\}}^{(k)} \\ &= \sigma + (T-\sigma) \sum_i \tau_{i,\{\cdot\}}^{(k)}. \end{aligned}$$

Then referring to Table I, it follows that

$$\begin{aligned} \frac{\mathbb{E}[slot_{+i}^{(l)}]}{\mathbb{E}[slot_{-i}^{(k)}]} &= \frac{\sigma + (T-\sigma)\Big(\sum_{j \neq i} \tau_j^{(l)} + \tilde{\tau}_i^{(l)}\Big)}{\sigma + (T-\sigma)\sum_{j \neq i} \tau_j^{(k)}} \\ &= \frac{\widetilde{W} + \Big(\sum_{j \neq i} \hat{q}_j^{(l)} \rho_j + \rho_i\Big)}{\widetilde{W} + \sum_{j \neq i} \hat{q}_j^{(k)} \rho_j}, \end{aligned}$$

where $\widetilde{W} = \frac{W+1}{2} \cdot \frac{\sigma}{T-\sigma}$. We then let $\rho_i = 1$ for all i following the heuristic that the accuracy of the entire approximated stability region can be reduced to that of the boundary estimated. Therefore, we obtain

$$\hat{q}_i^{(k)} \approx \frac{q_i^{(k)}}{q_i^{(k)} + \sum_{\substack{l \neq k \\ l \in \mathcal{C}}} \left(q_i^{(l)} \frac{\widetilde{W} + \left(\sum_{j \neq i} \hat{q}_j^{(l)} + 1\right)}{\widetilde{W} + \sum_{j \neq i} \hat{q}_j^{(k)}} \right)}.$$

Recalling that $\hat{q}_i^{(k)} = f_k\big(\mathcal{Q}\big)$ for all i, the above equation is in fact a fixed point equation, and we can then solve it using an iterative approach with the initial value $\hat{q}_i^{(k)} = q^{(k)}$.

Appendix D
Proof of Theorem 3

We consider the unbiased model established by $\widetilde{\Sigma}^{\mathbf{g}}$ with the approximation $\hat{q}_i^{(k)} = \tilde{q}_i^{(k)} = q^{(k)}$. Using $\widetilde{\Sigma}^{\mathbf{g}}$(a) and (b), we can rewrite $\widetilde{\Sigma}^{\mathbf{g}}$(c) as follows,

$$\rho_i = \frac{\lambda_i}{P} \sum_{k \in \mathcal{C}} \left[q^{(k)} \left(\frac{W+1}{2} T \prod_{j \neq i} \frac{1}{1 - \tau_j^{(k)}} - \frac{W-1}{2} (T-\sigma) \right) \right]$$

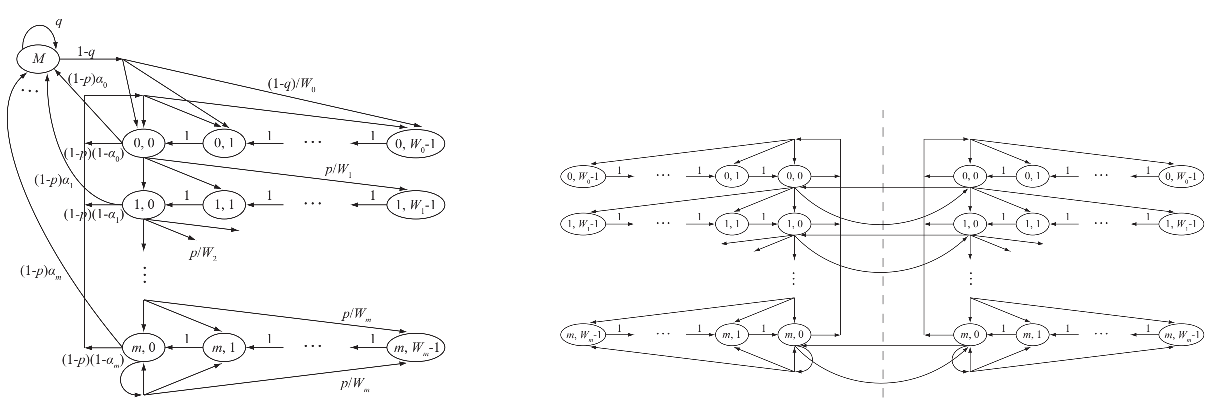
$$
\begin{aligned}
&= \sum_{k\in\mathcal{C}} \left[q^{(k)} \left(\frac{\lambda_i(W+1)T}{2P} \prod_{j\neq i} \frac{1}{1-\tau_j^{(k)}} - \frac{\lambda_i(W-1)(T-\sigma)}{2P} \right) \right] \\
&= \sum_{k\in\mathcal{C}} \left[q^{(k)} \left(\theta_i^1 \prod_{j\neq i} \frac{1}{1-\tau_j^{(k)}} + \theta_i^2 \right) \right] \\
&= \theta_i^1 \sum_{k\in\mathcal{C}} \left(q^{(k)} \prod_{j\neq i} \frac{1}{1-\tau_j^{(k)}} \right) + \theta_i^2 \\
&= \theta_i^1 \sum_{k\in\mathcal{C}} \phi_i(q^{(k)}; \rho_j, j\neq i) + \theta_i^2,
\end{aligned}
$$

where $\theta_i^1 = \frac{\lambda_i(W+1)T}{2P}$, $\theta_i^2 = -\frac{\lambda_i(W-1)(T-\sigma)}{2P}$, and $\phi_i(q^{(k)}; \rho_j, j\neq i) = q^{(k)} \prod_{j\neq i} \frac{1}{1-\tau_j^{(k)}} = \prod_{j\neq i} \frac{q^{(k)}}{1-\alpha_j q^{(k)}}$ with $\alpha_j = \frac{2\rho_j}{W+1}$ and $1-\alpha_j q^{(k)} > 0$ for all j. Notice that $\phi_i(q^{(k)}; \rho_j, j\neq i)$ is a convex function of $q^{(k)}$ given any fixed ρ_j where $j \neq i$, and it is also an increasing function of ρ_j's given any fixed $q^{(k)}$. We then have

$$
\begin{aligned}
\rho_i &= \theta_i^1 \sum_{k\in\mathcal{C}} \phi_i(q^{(k)}) + \theta_i^2 \\
&= \theta_i^1 \cdot K \sum_{k\in\mathcal{C}} \left(\frac{1}{K} \phi_i(q^{(k)}) \right) + \theta_i^2 \\
&\geq \theta_i^1 \cdot K \phi_i \left(\sum_{k\in\mathcal{C}} \left(\frac{1}{K} q^{(k)} \right) \right) + \theta_i^2 \\
&= \theta_i^1 \cdot K \phi_i \left(\frac{1}{K} \right) + \theta_i^2,
\end{aligned}
$$

where the equality holds when $q_i^{(k)} = \frac{1}{K}$. Therefore, when switching to the equi-occupancy scheduling policy from any arbitrary unbiased policy, the utilization factor of each node is always non-increasing. Hence, we conclude that equi-occupancy scheduling policy is throughput-optimal in $\mathcal{G}^U$.

APPENDIX E
SAS AND SAC



(a) SAS (b) SAC

Fig. 5. The two-dimensional Markov chain models of SAS and SAC.

Using the similar argument in [3], we can determine the stationary distribution of the Markov chain of SAS and SAC in each channel as follows. First note that the time unit in this Markov chain model is implicitly the *n-slots*, and also we solve for the symmetric solution. Consider the SAS model as shown in Figure E. Let $b_{\{\cdot\}}$ be the stationary distrbition of state $\{\cdot\}$, and hence we have

$$b_M = \frac{1}{2}, \tag{1}$$

where the M state is the aggregate of the mirror states in the other channel. By the global balance equation and the chain's regularities, we can then obtain the following results:

$$b_{i-1,0} \cdot p = b_{i,0}, 0 < i < m, \text{ and } b_{m-1,0} \cdot p + b_{m,0} \cdot p = b_{m,0}, \tag{2}$$

or equivalently

$$b_{i,0} = p^i b_{0,0}, 0 < i < m, \text{ and } b_{m,0} = \frac{p^m}{1-p} b_{0,0}, \tag{3}$$

and

$$b_{i,k} = \frac{W_i - k}{W_i} \cdot \begin{cases} (1-p)\sum_{j=0}^{m}(1-\alpha_j)b_{j,0} + (1-q)b_M, & i = 0 \\ p \cdot b_{i-1,0}, & 0 < i < m \\ p(b_{m-1,0} + b_{m,0}), & i = m \end{cases}. \tag{4}$$

We note that due to the global balance at state $(0,0)$, we have

$$b_{0,0} = (1-p)\sum_{j=0}^{m}(1-\alpha_j)b_{j,0} + (1-q)b_M. \tag{5}$$

Hence, combining Eqn. (2) and (5), Eqn. (4) gives

$$b_{i,k} = \frac{W_i - k}{W_i} b_{i,0}, 0 < i < m, 0 < k < W_i - 1. \tag{6}$$

Furthermore, by the normalization condition and Eqn. (6), we have

$$\begin{aligned} 1 &= \left(\sum_{i=0}^{m}\sum_{k=0}^{W_{k-1}} b_{i,k}\right) + b_M = \left(\sum_{i=0}^{m} b_{i,0} \sum_{k=0}^{W_{k-1}} \frac{W_i - k}{W_i}\right) + b_M \\ &= \left(\sum_{i=0}^{m} b_{i,0} \frac{W_i + 1}{2}\right) + b_M. \end{aligned} \tag{7}$$

Substituting Eqn. (1) and (3) in Eqn. (7), we get

$$\begin{aligned} b_{0,0} &= \left[W\left(\sum_{i=0}^{m-1}(2p)^i + \frac{(2p)^m}{1-p}\right) + \frac{1}{1-p}\right]^{-1} \\ &= \frac{(1-2p)(1-p)}{(1-2p)(W+1) + pW(1-(2p)^m)}. \end{aligned} \tag{8}$$

Thus, the probability that a station transmits in an arbitrary n-slot in a specific channel (say, 1), denoted by τ_{sas}, is given by

$$\tau_{sas} = \sum_{i=0}^{m} b_{i,0}. \tag{9}$$

To evaluate the summation on the right hand side of Eqn. (9), we note that, by the global balance at state M, we have

$$b_M = q \cdot b_M + (1-p)\sum_{i=0}^{m} \alpha_i b_{i,0}. \tag{10}$$

Combining Eqn. (5) (10) (9) and (8), we have

$$\tau_{sas} = \frac{b_{0,0}}{1-p} = \frac{(1-p)}{(1-2p)(W+1) + pW(1-(2p)^m)}.$$

As for SAC, we evaluate the stationary distribution in a similar manner. Let $b^{(j)}_{\{\cdot\}}$ be the stationary distribution of state $\{\cdot\}$ in channel j, where $j = 1, 2$[3]. For the symmetric solution, we have

$$b^{(1)}_{i,k} = b^{(2)}_{i,k}, \forall i \text{ and } \forall k,$$

or equivalently,

$$b^{(1)}_M = b^{(2)}_M = \frac{1}{2}. \tag{11}$$

Due to balance equations and regularities of the chain, we have

$$b^{(1)}_{i-1,0} \cdot p(1-\alpha_{i-1}) + b^{(2)}_{i-1,0} \cdot p\alpha_{i-1} = b^{(1)}_{i,0}, 0 < i < m, \tag{12}$$

and

$$b^{(1)}_{m-1,0} \cdot p(1-\alpha_{m-1}) + b^{(2)}_{m-1,0} \cdot p\alpha_{m-1} + b^{(1)}_m \cdot p(1-\alpha_m) + b^{(2)}_m \cdot p\alpha_m = b^{(1)}_{m,0}, \tag{13}$$

[3]Since the state transition in SAC are intertwined between mirror state pairs, we use the superscript to avoid ambiguity when necessary, otherwise suppressed.

or equivalently,

$$b_{i,0} = p^i b_{0,0}, 0 < i < m, \text{ and } b_{m,0} = \frac{p^m}{1-p} b_{0,0}, \tag{14}$$

and also

$$b_{i,k} = \frac{W_i - k}{W_i} \cdot \begin{cases} (1-p)\sum_{j=0}^{m} b_{j,0}, & i = 0 \\ p(1-\alpha_{i-1}) \cdot b^{(1)}_{i-1,0} + p\alpha_{i-1} \cdot b^{(2)}_{i-1,0}, & 0 < i < m \\ p(1-\alpha_{m-1}) \cdot b^{(1)}_{m-1,0} + p\alpha_{m-1} \cdot b^{(2)}_{m-1,0} + p(1-\alpha_m) \cdot b^{(1)}_{m} + p\alpha_m \cdot b^{(2)}_{m}, & i = m \end{cases}. \tag{15}$$

Using the global balance equation at state $(0,0)$, we have

$$b_{0,0} = (1-p)\sum_{j=0}^{m} b_{j,0}. \tag{16}$$

Hence, combining Eqn. (12) (13) and (16), Eqn. (15) gives

$$b_{i,k} = \frac{W_i - k}{W_i} b_{i,0}, 0 < i < m, 0 < k < W_i - 1. \tag{17}$$

Therefore, using the normalization condition and Eqn. (17), we have

$$\begin{aligned} 1 &= \left(\sum_{i=0}^{m}\sum_{k=0}^{W_i-1} b_{i,k}\right) + b_M = \left(\sum_{i=0}^{m} b_{i,0} \sum_{k=0}^{W_i-1} \frac{W_i - k}{W_k}\right) + b_M \\ &= \sum_{i=0}^{m} b_{i,0} \frac{W_i + 1}{2} + b_M. \end{aligned} \tag{18}$$

Substituting Eqn. (11) and (14) in Eqn. (18), we get

$$\begin{aligned} b_{0,0} &= \left[W\left(\sum_{i=0}^{m-1}(2p)^i + \frac{(2p)^m}{1-p}\right) + \frac{1}{1-p}\right]^{-1} \\ &= \frac{(1-2p)(1-p)}{(1-2p)(W+1) + pW(1-(2p)^m)}. \end{aligned} \tag{19}$$

Thus, the probability that a station transmits in an arbitrary n-slot in a channel under SAC, denoted by τ_{sac}, is given by

$$\tau_{sac} = \sum_{i=0}^{m} b_{i,0}. \tag{20}$$

Combing Eqn. (16) and (20), we have

$$\tau_{sac} = \frac{b_{0,0}}{1-p} = \frac{(1-p)}{(1-2p)(W+1) + pW(1-(2p)^m)} = \tau_{sas}.$$

APPENDIX F
MISCELLANEOUS

A. Rewrite $\Sigma^{\boldsymbol{g}}$ as a fixed point equation

Let $x \leftarrow y$ mean the relation "x quantitatively depend(s) on y". We then have

$$\left.\begin{array}{rcl} \tau_i^{(k)} & \leftarrow & \hat{q}_i^{(k)}, \hat{\rho}_i^{(k)}, \overline{W}_i^{(k)} \\ \hat{q}_i^{(k)} & \leftarrow & \mathbb{E}[slot_{+i}^{(l)}], \mathbb{E}[slot_{-i}^{(k)}] \leftarrow \boldsymbol{\tau}^{(k)}, \hat{\rho}_i^{(k)}, \overline{W}_i^{(k)} \\ \overline{W}_i^{(k)} & \leftarrow & p_i^{(k)} \leftarrow \boldsymbol{\tau}^{(k)} \end{array}\right\} \Rightarrow \tau_i^{(k)} \leftarrow \hat{\rho}_i^{(k)}, \boldsymbol{\tau}^{(k)},$$

and

$$\left.\begin{array}{rcl} \hat{\rho}_i^{(k)} & \leftarrow & \rho_i, \mathbb{E}[slot_{i,\overline{Q}}^{(k)}], \mathbb{E}[slot_{i,Q}^{(k)}] \leftarrow \rho_i, \boldsymbol{\tau}^{(k)}, \hat{q}_i^{(k)}, \overline{W}_i^{(k)} \\ \rho_i & \leftarrow & \tilde{q}_i^{(k)}, \overline{W}_i^{(k)}, p_i^{(k)}, \mathbb{E}[slot_{i,Q,\overline{Tx}}^{(k)}] \leftarrow \tilde{q}_i^{(k)}, \boldsymbol{\tau}^{(k)} \end{array}\right\} \Rightarrow \hat{\rho}_i^{(k)} \leftarrow \hat{\rho}_i^{(k)}, \boldsymbol{\tau}^{(k)}.$$

Therefore,

$$(\boldsymbol{\tau}, \hat{\boldsymbol{\rho}}) = \boldsymbol{\Gamma}'(\boldsymbol{\tau}, \hat{\boldsymbol{\rho}}).$$

Total bandwidth	11 Mbps
Data packet length P	1500 Bytes
DIFS	50 μs
SIFS	10 μs
ACK packet length (in time units)	203 μs
Header length (in time units)	192 μs
Empty system slot time σ	20 μs
Propagation delay δ	1 μs
Initial backoff window size W	32
Maximum backoff stage m	5
Data rate granularity $\Delta\lambda$	100 Kbps
Instability threshold constant	1%
Total simulated time T_f	10 seconds

TABLE II
SPECIFICATIONS OF THE IMPLEMENTATION OF TEST BENCH.

B. Referential tables

See Table II.

REFERENCES

[1] Q. Wang and M. Liu, "Characterizing the Stability Region of IEEE 802.11 Distributed Coordination Function - Part I: Single Channel Analysis," tech. rep., 2010. http://www.eecs.umich.edu/~mingyan/pub.html.
[2] J. D. C. Little, "A Proof for the Queuing Formula: $L = \lambda W$," *Operations Research*, vol. 9(3), pp. 383–387, 1961.
[3] G. Bianchi, "Performance Analysis of the IEEE 802.11 Distributed Coordination Function," *IEEE Journal on Selected Areas in Communications*, vol. 18, pp. 535–547, 2000.