To Stay or To Switch: Multiuser Multi-Channel Dynamic Access

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Abstract—In this paper we study opportunistic spectrum access (OSA) policies in a multiuser multi-channel random access cognitive radio network, where users perform channel probing and switching in order to obtain better channel condition or higher instantaneous transmission quality. Prior studies in this area include those on channel probing and switching policies for a single user to exploit spectral diversity, and those on probing and access policies for multiple users over a single channel to exploit temporal and multiuser diversity. By contrast, in this study we consider the collective switching of multiple users over multiple channels. This inevitably necessitates explicit modeling of the effect of collision. Furthermore, we consider finite arrivals, whereby users are not assumed to always have data to send and the demand for channel follows a certain arrival process. Under such a scenario, the users' ability to opportunistically exploit temporal diversity (the temporal variation in channel quality over a single channel) and spectral diversity (quality variation across multiple channels at a given time) is greatly affected by the level of congestion in the system. We investigate the associated decision process in this case, and show that the optimal policy is given by a *nested* stopping rule which may be viewed as a type of generalization to results found in existing literature in this area. We analytically and numerically evaluate the extent to which congestion affects potential gains from opportunistic dynamic channel switching.

Index Terms—Opportunistic spectrum access (OSA), cognitive radio network, diversity gain, multiuser multi-channel system, optimal stopping rule, nested stopping rule

1 Introduction

Dynamic and opportunistic spectrum access (OSA) policies have been very extensively studied in the past few years for cognitive radio networks, against the backdrop of spectrum open access as well as advances in ever more agile radio transceivers, including e.g., highly efficient channel sensing techniques [3], [16]. Within this context, a cognitive radio is capable of quickly detecting spectrum quality and performing channel switching so as to obtain good channel and transmission quality. At the heart of such opportunistic spectrum access is the idea of improving spectrum efficiency through the exploitation of *diversity*.

Within this context there are three types of diversity gains commonly explored. The first is *temporal diversity*, where the natural temporal variation in the wireless channel causes a user to experience or perceive different transmission conditions over time even when it stays on the same channel, and the idea is to have the user access the channel for data transmission when the condition is good, which may require and warrant a certain amount of waiting. Studies like [5] investigate the tradeoff involved in waiting for a better condition and deciding when is the best time to stop.

The second is *spectral diversity*, where different channels experience different temporal variations, so for a given user at any given time a set of channels present different transmission conditions. The idea is then to have the user select a channel with the best condition at any given time for data transmission, which typically involves probing multiple channels to find out their conditions. Protocols like [10] does exactly this, and studies like [2], [21] further seek to identify the best sequential probing policies using a decision making framework.

The third is *user diversity* or *spatial diversity*, where the same frequency band at the same time can offer different transmission qualities to different users due to their difference in transceiver design, geographic location, etc. The idea is to have the user with the best condition on a channel use it. This diversity gain can be obtained to some degree by using techniques like stopping rules whereby a user essentially judges for itself whether the condition is sufficiently good before transmitting, which comes as a byproduct of utilizing temporal diversity.

We note that the above forms of diversities are often studied in isolation. For instance, temporal diversity is studied in a multiuser setting but with a single channel in [19], [22]; spectral diversity is analyzed for a single user in [18], among others. More specifically, [22] developed optimal stopping policies for single-channel multiuser access, while Tan et al. [19] considered a distributed opportunistic scheduling problem for ad-hoc communications under delay constraints. In [18] authors exploited spectral diversity in OSA for a single user with sensing errors, where the multi-channel overhead is captured by a generic penalty on each channel switching. This becomes insufficient in a multiuser setting as such overhead will obviously depend on the level of congestion in the system which results in different amount of collision and the time it takes to regain access to a channel. In [10] an opportunistic auto rate multi-channel MAC protocol MOAR is presented to exploit spectral diversity for a multi-channel multi-rate IEEE 802.11-enabled wireless ad hoc network. However, this scheme does not allow parallel use of multiple channels by different users

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due to its reservation mechanism. Other works that study multi-channel access for a single user include [2], [4], [5], [12], [13], [20].

As the number of users and their traffic volume increase in such a multi-channel system, one would expect their ability to exploit the above diversity gains to decrease significantly due to the increased overhead, e.g., the time it takes to perform channel sensing or the time it takes to regain access right, or increased collision due to channel switching. As mentioned above, this overhead was captured in the form of penalty cost in prior work such as [18], but is often assumed to be independent of the traffic volume existing in the system.

Compared to the above literature, the main contribution of this paper is two-fold:

1) We present a model that captures opportunistic spectrum access policies in a multiuser multi-channel random access setting, where users are not assumed to always have data to send, while demand for channel follows a certain arrival process, and delays due to collision and contention are taken into account. We then set out to investigate the associated optimal decision process in this scenario, assuming each user follows a random sensing order.[1] We then focus on the collective effect of channel switching decisions by the users, and how their decision processes are affected by increasing congestion levels in the system.
2) For this problem we characterize the nature of an optimal access policy and identify conditions under which channel switching actually results in transmission gain (e.g., in terms of average data rate or throughput). We show that the optimal policy is given by a *nested* stopping rule involving a two-step stopping decision, which may be viewed as a type of generalization to those found in the literature and mentioned above, e.g., [22]. We also show both analytically and numerically that, unsurprisingly, with the increase in user/data arrival rate, the average throughput decreases and a user becomes increasingly more reluctant to give up a present transmission opportunity in hopes for better condition later on or in a different channel.

The remainder of this paper is organized as follows. The system model is given in Section 2. In Sections 3 and 4, we model channel evolution as IID and Markovian processes, respectively, and analyze the properties of an optimal stopping/switching rule. Numerical results are given in Section 5, and Section 6 concludes the paper.

2 Model, Assumptions and Preliminaries

2.1 Model and Assumptions

Consider a wireless system with N channels indexed by the set $\Omega = \{1, 2, \ldots, N\}$. We associate each channel with a positive reward of transmission (e.g., transmission rate) X^j, which is a positive random variable with distribution given by $f_{X^j}(x)$, assumed to have finite support with a maximum value of $\overline{X}^j$. There are m cognitive users (or radio transceivers), each equipped with a single transmitter attempting to send data to a base station. Our model also captures direct peer-to-peer communication, where m *pairs* of users communicate and each pair can rendezvous and perform channel sensing and switching together through the use of a control channel [14]. However, for simplicity of exposition, for the rest of the paper we will take the view of m users transmitting to a base station. We will assume these m users are within a single interference domain, so that at any given time each channel can only be occupied by one user. Considering spatial reuse will make the problem considerably more challenging and remains an interesting direction of future research. We consider discrete time with a suitably chosen time unit, and with all other time values integer multiples of this underlying (and possibly very small) unit. We will consider two channel models, an IID model where channel conditions over time are assumed to form an IID process defined on this time unit (in Section 3), and a Markovian model where channel conditions over time form a Markov chain (in Section 4). Different channels are in general not identically distributed but are assumed to evolve independent of each other. Strictly speaking an IID process is a special case of a Markov process. The purpose for making this distinction is to use the IID model to represent a fast-varying channel while using the Markov model for a slow-varying channel.

The system operates in a way similar to a multi-channel random access network like IEEE 802.11, with the following modifications related to dynamic and opportunistic channel access. Each user has a pre-assigned (or self-generated) random sequence of channels; this sequence determines in which order the user performs channel switching, an approach similar to that used in [18]. More on this assumption is discussed in Section 2.3. Each time a user enters a new channel, it must perform carrier sensing (CS) and compete for access (contention resolution) as in a regular 802.11 channel. As soon as it gains the right to transmit, the user reserves the channel (e.g., through the use of RTS-CTS type of handshake) and finds out the instantaneous data transmission quality (channel information could be piggybacked on these control packets) it may get if it transmits right away. Upon finding out the channel condition, this user faces the following choices:

1) Transmit on the current channel right away. Intuitively this happens if the current channel condition is deemed good enough. This action will be referred to as *STOP*. This is shown in Fig. 1, where the second RS-CP (denoting the Reservation-Channel probing process) followed by DATA indicates a STOP at the first channel (first line in the figure).
2) Forego this transmission opportunity, presumably due to poor channel condition, but remain on the same channel and compete for access again in the near future hoping to come across a better condition then. This happens if the current channel condition is poor but the average quality is believed to be good, so the user will risk waiting for possibly better condition later. This action will be referred to as *STAY*. This is

1. We discuss in much greater detail the choice of random sensing order versus optimal sensing order in Section 2.4.

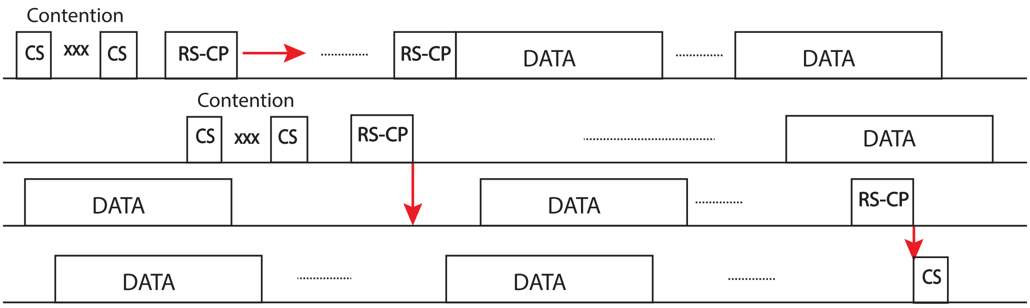


Fig. 1. System model.

illustrated by the first RS-CP on the first line (channel) followed by a horizontal arrow.

3) Give up the current channel and switch to the next one on its list/sequence of channels. This happens if the current channel condition is poor, and the prospect of getting better conditions later by staying on the same channel is not as good as by switching to the next channel. This action will be referred to as *SWITCH*. An example is shown by the RS-CP on the second line (channel) followed by a vertical arrow indicating a *SWITCH* action.

Note that option (2) above allows the system to exploit both multiuser diversity (the transmission opportunity is given to another user under the random access) and temporal diversity (the user in question waits for better condition to appear in time), while option (3) allows the system to exploit spectral diversity as users seek better conditions on other channels. Options (1) and (2) are similar to those used in existing stopping time frameworks, see e.g., [22].

In the above decision process once a user decides to leave a channel it cannot use the channel for transmission without going through carrier sensing and random access competition again. More importantly from a technical point of view, this assumption means that the user cannot claim the same channel condition at a later time. Once a user gets the right to transmit on a certain channel, it can transmit for a period of T time units, which is a constant. For simplicity a single time unit is assumed to be the amount of time to transmit a control packet (e.g., RTS/CTS type of packets.).

2.2 Capturing the Level of Congestion

As mentioned earlier our focus in this paper is on understanding how the users' channel access decision process is affected by increasing traffic load or congestion in the system. To model this we will first take the view of a single user, and introduce user arrival rates in each channel as well as the amount of delay involved in STAY and SWITCH as parameters that need to be taken into consideration in its decision process. Note that these parameter values are the result of the collective switching actions of all users, and therefore cannot be obtained prior to defining the switching policies. We will however assume that these parameters have well-defined averages to facilitate our analysis. Later we show that the system under the optimal switching policy converges and that these parameters indeed have well-defined averages, thereby justifying such an assumption. In other words, policies derived under the assumption that these parameters have well-defined averages lead to a stable system with well-defined averages for these parameters. This is not unlike a mean-field approach where a single user operates against a background formed by other users in a system over which this single user has no control or influence. In practice these values may be obtained through learning.

Specifically, we assume that the total packet arrivals to a channel, including external arrival, retransmission, as well as arrivals switched from other channels, form a Poisson process, with the attempt rate vector given by $\mathbf{G} = [G_1, G_2, \ldots, G_N]$ and a sum rate $\sum_{i=1}^{N} G_i = G$. These quantities will also be referred to as the *load* or *traffic load* on a channel. We will not directly deal with the external arrival processes as our analysis entirely depends on the above "internal" offered load. However, we will assume that the external arrivals are such that the system remains stable.

The level of congestion on any channel is captured by two parameters. The first is an average *contention delay* on channel j denoted by t_j^c; this is the average time from carrier sense to gaining the right to transmit on channel j. The more competing users there are on channel j, the higher this quantity is. The second is an average *switching delay* of channel j, denoted by t_j^s; this is the time from a user switching into channel j (from another channel) to its gaining the right to transmit on channel j. Compared to t_j^c the switching delay includes the additional time for the radio to perform channel switching and additional waiting time in the event that the switching occurs during an active transmission. In our characterization of t_j^s below, however, we will ignore the hardware switching delay as it simply adds a constant, which is very small compared to contention delay, and will not affect our subsequent analysis.

For a packet arriving at channel i (from an external arrival process or by switching from another channel), the delay it experiences between arrival and successful transmission consists of two parts, the average time it takes for the channel to become idle if it happens to arrive during an active transmission (including its associated control packet exchange), denoted by t_i^w, and the average time it takes to compete for and gain the right to transmit, given by t_i^c. We thus have $t_i^s = t_i^w + t_i^c$.

Denote by Y the random variable representing the time between a new arrival and the completion of the current transmission. Following results in [14], we have $f_Y(y) = S_i e^{-S_i y}$, where S^i is the success rate of channel contention given by

$$S_i = \frac{G_i e^{-2G_i}}{1 + (1+T)G_i e^{-2G_i}}. \tag{1}$$

t_i^w is then calculated as follows:

$$\begin{aligned} t_i^w &= \int_0^{1+T} f_Y(y)(1/\zeta + y)dy \\ &= \frac{1}{S_i} + \frac{1}{\zeta} - \left(T + 1 + \frac{1}{S_i} + \frac{1}{\zeta}\right) e^{-(T+1)S_i}, \end{aligned} \tag{2}$$

where $1/\zeta$ is the expected random backoff time. For t_i^c, since a competition succeeds with probability e^{-2G_i} we have

$$t_i^c = (e^{2G_i} - 1) \cdot (1/\zeta + 2) + 2. \tag{3}$$

Using the above expressions, it is not difficult to establish the following results.

Proposition 2.1. *Both t_j^c and t_j^s are non-decreasing functions of arrival rate G_j, $\forall j \in \Omega$.*

Proposition 2.2. *Both t_j^c and t_j^s are non-decreasing functions of the data transmission time T, $\forall j \in \Omega$.*

The decision process we introduce next is a function of t_i^c and t_i^s, so a user needs to know these parameter values in order to compute the optimal policy. In practice, this information may be obtained through measurement and empirical means.

2.3 Problem Formulation

For simplicity and without loss of generality, for the single user under consideration we will relabel the channels in its sequence in the ascending order: $1, 2, \dots, N$. We now define the following rate-of-return problem with the objective of maximizing the effective data rate over one successful data transmission.

Specifically, let π denote a policy $\pi = \{\alpha_1, \alpha_2, \dots \alpha_{\gamma(\pi)}\}$ which specifies the sequence of actions leading up to a successful transmission, with α_k denoting the kth action, $\alpha_k \in \{\mathrm{STAY}, \mathrm{SWITCH}\}$, $k = 1, \dots, \gamma(\pi) - 1$, and $\alpha_{\gamma(\pi)} = \mathrm{STOP}$. An action is only taken upon gaining the right to transmit in a channel, and $\gamma(\pi)$ denotes the stopping time at which the process terminates with a transmission action.

Let $X^{\pi}_{\gamma(\pi)}$ denote the data rate obtained at the last step when the process terminates. Then the total reward the user gets is $X^{\pi}_{\gamma(\pi)} \cdot T$, the total amount of data transmitted. A natural goal would be to maximize the ratio between this reward and the total amount of time spent in the decision process (summing up the delays involved in switching and contention as a result of the actions), i.e., the effective or average throughput or data rate. While this appears to be a standard rate-of-return problem, an inherent difficulty arises from the fact that different channels have different statistics, and thus the rewards generated and the delays experienced, respectively, are not independent across channels. This prevents the use of the renewal theorem to turn the expectation of the aforementioned ratio (average throughput) into a ratio of expectations as is commonly done.

To address this difficulty, we will make the following simplification: instead of maximizing the overall rate of return for each successful transmission over the entire decision process, we will seek to maximize the rate of return over the *remaining* decision process given the current state of the process. This may be viewed as a "no-recall" approximation to the original goal by ignoring the history or past decisions in the same process. This objective can be represented by the following dynamic program, noting that the user goes through the channels in the order $1, 2, \dots, N$

$$\begin{aligned} V_N(x) &= \max\left\{x,\ \frac{T}{T + t_N^c} E\{V_N(X^N \mid x)\}\right\}, \\ V_i(x) &= \max\left\{x,\ \frac{T}{T + t_i^c} E\{V_i(X^i) \mid x\},\right. \\ &\qquad \left. \frac{T}{T + t_{i+1}^s} E\{V_{i+1}(X^{i+1}) \mid x\}\right\}, \quad i < N, \end{aligned} \tag{4}$$

where $V_i(x)$ is the value function at stage i (in channel i) of the decision process when the observed channel state is x; this is also the maximum average throughput obtainable given current state x (transmission rate) in channel i: In the above equation, the first term is the reward (current transmission rate) if we STOP, the second the expected reward if we STAY, and the last the expected reward if we SWITCH.

2.4 Critique on the Model

The model given above captures the multiuser, multichannel, and random access nature of the problem. The optimal decision process defined by (4) appears to be a finite horizon problem, i.e., the process stops at channel (or stage) N. However, this would only be partly true, as (4) actually illustrates a *two-dimensional* decision problem, where there is a finite number of steps (N) along the spectral dimension (the channels), but within each channel (for each i) the decision process is over an infinite horizon along the time dimension, i.e., the decision process may go on indefinitely within a particular channel. This will be seen more clearly in Section 3.

The reason we have limited the horizon to be finite along the spectral dimension—the infinite horizon version would be where the user can continue to switch channels for an indefinite number of times, including revisiting channels it has visited in the past—has to do with the IID assumption on the channels. Since channel state realizations are independent over time (for the same channel), the second and third terms in (4) are both independent of the current state x. In other words, the comparison between the second and the third terms is independent of the current state x, suggesting that under the same contention level $\mathbf{G}$ if the second term is larger than the third term, then it will always be larger regardless of the current state. The interpretation of this observation is that if we ever decide to STAY (the second term is larger) on the same channel, then we will never SWITCH later. The opposite is also true: if we ever decide to SWITCH away from a channel (the third term is larger), then under the optimal policy we will never come back to the same channel even if we are allowed to. This means that under the objective of maximizing the future rate of return, a channel is never visited more than once, resulting in the finite horizon formulation along the spectral dimension given above. In other words, there is no need to allow the user to revisit a channel it has visited before but switched away from.

The reason why we limit the user to a pre-determined sequence (randomly chosen) of channels has to do with the multiuser scenario we aim to analyze. If there is only a single user, then obviously a reasonable thing to do is to also optimize the sequence/order of channel sensing, together with optimizing the switching and transmission decisions. Indeed there has been a large volume of study on determining optimal sensing orders, see e.g., [5], [6], [7], [9], with the main idea being that upon switching, a user should switch to a channel believed to present the best transmission opportunity. A related problem is channel assignment, see e.g., [1] that studied such a problem under stochastic uncertainty and with adjacent channel interference. However, contention among users is not taken into consideration in [1]; furthermore, to achieve globally optimal assignment the approach either assumes static assignment that does not respond to random realization of channel conditions, or employs a central controller. A two-user model in a similar context was introduced and analyzed in [7], but beyond two users the problem remains open. Compared to [1], the contention and the opportunistic exploitation of time-varying channel conditions are key aspects of the model we study in this paper.

An optimal sensing order becomes elusive in a multiuser setting because the above type of optimization relies on known statistics of the channel dynamics. However, this is no longer applicable when there are multiple competing users: one's previously optimal sensing order may no longer be optimal depending on what order the other users adopt. Consequently this needs to be either treated as a centralized multiuser optimization problem, where the jointly optimal sensing orders are computed simultaneously for all users, or treated as a game-theoretic problem where each user selfishly determines its sensing order to maximize its own utility. A recent study [17] adopts an approach close to the first one with a joint design framework of sensing order and channel switching decision. The model in [17] however focuses on the interaction between a secondary user and a primary user, rather than on the contention relationship among competing secondary users (which our model captures), thus it does not jointly design channel switching decisions for multiple users.

The second, game-theoretic approach is largely an open area as it involves the equilibrium analysis of complex decisions (not only the sensing order of channels but also the stopping decisions on any given channel). While this remains an interesting direction of future research, in the present study we adopt the assumption that a user simply follows a pre-defined (can be randomly chosen) sequence of channels and focus our attention on the switching decisions instead. In Section 5 we compare the results between randomly selecting these sequences and users optimally selecting these sequences individually without considering other users' behavior.

For the remainder of our presentation, we will use the terms *stages* and *steps* to describe the two time scales of decision making along the two dimensions described above. Movement along the spectral dimension (i.e., switching from one channel to the next) occur in stages; stage i means channel i and this is indexed by the subscript in the value function $V_i(x)$. The decision process within the same stage (or in the same channel) occurs in steps; the decision to remain on the same channel or switch away occurs at the boundary of a step. The indexing of steps is not explicit in the expression given in (4) but will be made explicit in our subsequent analysis.

3 Optimal Access Policy under the IID Channel Model

In this section, we model the channels as fast changing, IID processes, where successive observations of the state of the same channel are independent.

3.1 An Optimal "Nested" Stopping Rule

Since successive channel states are independent, the value function (4) is simplified:

$$V_i(x) = \max\left\{x,\ \frac{T}{T+t_i^c}E\{V_i(X^i)\},\ \frac{T}{T+t_{i+1}^s}E\{V_{i+1}(X^{i+1})\}\right\}. \tag{5}$$

The above three-way comparison suggests the following. If the current state x is sufficiently high then the optimal decision is STOP. The comparison between the second and the third terms is more interesting: both terms are independent of the state x, so if the second term is larger then it will always be larger. As previously mentioned, this implies that if we ever decide to STAY, then we will never SWITCH later. The reverse is also true: if we ever decide to SWITCH then we will never return to the same channel. These observations can lead to a concrete proof of the existence and uniqueness of a threshold rule but in general cannot produce a closed form for the computation of the threshold. Below we will instead use results from optimal stopping theory [8] to obtain not only the existence but also a closed form for the threshold. Consider the following substitution,

$$\hat{X}^i(x) = \max\left\{x,\ \frac{T}{T+t_{i+1}^s}E\{V_{i+1}(X^{i+1})\}\right\} \tag{6}$$

with the value function subsequently re-written as

$$V_i(x) = \max\left\{\hat{X}^i(x),\ \frac{T}{T+t_i^c}E\{V_i(X^i)\}\right\}. \tag{7}$$

This substitution reduces the decision process to a two-way comparison, and more importantly, a one-dimensional decision process. Specifically, since the state x is IID over the same channel/stage i, the first term $\hat{X}^i(x)$ as defined in (6) is also IID over the *same stage* i while encoding the information on other channels/stages. Therefore, if we view $\hat{X}^i(x)$ as the reward of a (meta) stopping action of state x and t_i^c as the cost for continuing, then the value function given in (7) represents a standard stopping time rate-of-return problem with two possible actions in each step, (meta) stopping and continuation, respectively, and this process concerns only a *single stage/channel*. The switching to the next stage occurs when the (meta) stopping action is taken (which essentially ends the above one-dimensional stopping time problem), and it is determined that SWITCH is a better action than STOP.

The following theorem characterizes the property of the optimal decision for the problem given in (5) or equivalently (7).

Theorem 3.1. *The optimal action at stage i of deciding between {STOP, SWITCH} and STAY is given by a stopping rule: the state space of the channel condition can be divided into a stopping set Δ_i^s and continuation set Δ_i^c, such that whenever the channel condition is observed to be in either set, the corresponding action (STOP/SWITCH versus. STAY) is taken.*[2] *Furthermore, these two sets are given by the following threshold property:*

$$\Delta_i^s = \{x : \hat{X}^i(x) \geq \lambda^*\}, \forall i, \tag{8}$$

where the threshold λ^* at the ith stage is given by the unique solution to

$$E[\hat{X}^i(x) - \lambda]^+ = \frac{\lambda \cdot t_i^c}{T}. \tag{9}$$

Proof. We first prove the existence of an optimal stopping rule. Define the reward function associated with step k of the stopping decision process at stage i as

$$Z_i^k(\lambda, x) = \hat{X}^i(x)T - \lambda\big(k \cdot t_i^c + T\big), \tag{10}$$

where λ is a positive finite valued variable. From [Theorem 1, Chapter 3, 8] we know that an optimal stopping rule exists if the following two conditions are satisfied[3]:

$$\begin{aligned} &(\text{C1}) \quad E\Big\{\sup_k Z_i^k(\lambda, X^i)\Big\} < \infty, \\ &(\text{C2}) \quad \lim_{k\to\infty} Z_i^k(\lambda, X^i) \leq Z_i^\infty(\lambda, X^i), a.s. \end{aligned} \tag{11}$$

Since we have a finite number of channels and the channel state realization is finite, $\hat{X}^i(x)$ is finite. Therefore $Z_i^\infty(\lambda, X^i) = -\infty$. Since $\hat{X}^i(x)$, $\frac{T}{T+t_{i+1}^s} E\{V_{i+1}(X^{i+1})\}$ and T are all finite, (C2) is easily satisfied. Next define

$$Z_i(\lambda, x) = \hat{X}^i(x)T - \lambda T, \tag{12}$$

which is again finite. Therefore we have $E\{Z_i(\lambda, X^i)\} < \infty$, and $E\{(Z_i(\lambda, X^i))^2\} < \infty$. Also noting that $Z_i(\lambda, X^i)$ is IID since X^i is IID, by the dominated convergence theorem we have $E\{\sup_k Z_i^k(\lambda, X^i)\} < \infty$, verifying (C1). The existence is thus established.

Next we prove that the optimal stopping rule is given by a threshold. Using the principle of optimality [Chapter 2, 8] and the results from [Section 4.1, 8] (we refer the reader to [Example 6.2, 8] for further detail), our problem as expressed in (7) is equivalent to a rate-of-return problem with a reward of stopping given by $Z_i(\lambda, x)$ and a cost of continuation given by λt_i^c. The optimal stopping rule at step k is given by

$$\Delta_i^s = \{x : Z_i(\lambda^*, x) \geq 0\} = \{x : \hat{X}^i(x) \geq \lambda^*\}, \tag{13}$$

where λ^* is such that the function $V_k^*(\lambda)$, defined recursively as [Chapter 6, 8] $V_k^*(\lambda) = E\{\max\{Z_i(\lambda, x) - \lambda t_i^c, V_k^*(\lambda) - \lambda t_i^c\}\}$, is evaluated to be zero, i.e., $V_k^*(\lambda^*) = 0$. To obtain λ^*, we take $V_k^*(\lambda^*) = 0$ into the above definition and get $E\{\max\{Z_i(\lambda, x), 0\}\} = \lambda^* t_i^c$, or equivalently, λ^* is such that it satisfies

$$E[\hat{X}^i(x)T - \lambda^* T]^+ = \lambda^* t_i^c, \tag{14}$$

which is the same as (9). This completes the proof of the form of the threshold. It remains to show that a unique solution exists to (9). Denote by $\mathcal{D}(\lambda) = E[\hat{X}^i(x) - \lambda]^+ - \frac{\lambda t_i^c}{T}$. It is not hard to verify that $\mathcal{D}(\lambda)$ is a continuous and strictly decreasing function of λ. Furthermore, we have $\mathcal{D}(\lambda = 0) = E[\hat{X}^i(x)]^+ > 0$ since all channel states are positive, and $\mathcal{D}(\lambda) \to -\infty$ as $\lambda \to \infty$. Therefore there is a unique solution to $\mathcal{D}(\lambda) = 0$, i.e., the threshold exists and is unique, completing the proof. □

In practice, to calculate this threshold, we define

$$c_i = \frac{T}{T + t_{i+1}^s} E\{V_{i+1}(X^{i+1})\}. \tag{15}$$

Re-writing (9) in the original random variables, we have

$$\begin{aligned} &E\bigg[\max\bigg\{X^i, \frac{T}{T+t_{i+1}^s} E\{V_{i+1}(X^{i+1})\}\bigg\}T - \lambda T\bigg]^+ \\ &= E\big\{\max\{X^i T - \lambda T, 0\} \mid X^i > c_i\big\} \cdot P(X^i > c_i) \\ &\quad + E\big\{\max\{c_i T - \lambda T, 0\} \mid X^i \leq c_i\big\} \cdot P(X^i \leq c_i) \\ &= \lambda t_i^c. \end{aligned} \tag{16}$$

If the solution $\lambda^* < c_i$, then it has to satisfy $\int_{c_i}^{\overline{X}^i} (x - \lambda) f_{X^i}(x)dx + (c_i - \lambda) \cdot P(X^i \leq c_i) = \lambda t_i^c / T$, and thus can be obtained by $\lambda^* = \frac{\int_{c_i}^{\overline{X}^i} x f_{X^i}(x)dx + c_i \cdot P(X^i \leq c_i)}{1 + t_i^c/T}$ and verifying that the resulting $\lambda^* < c_i$. If the solution $\lambda^* \geq c_i$, then it must satisfy $\int_{\lambda}^{\overline{X}^i} (x - \lambda) f_{X^i}(x)dx = \lambda t_i^c / T$, and the solution may be obtained using $\lambda^* = \frac{\int_{\lambda^*}^{\overline{X}^i} x f_{X^i}(x)dx}{P(X^i \geq \lambda^*) + t_i^c/T}$ and verifying[4] that the resulting $\lambda^* \geq c_i$.

Remark 3.2. The quantity c_i defined above is the expected reward of SWITCH, while λ^* is the threshold for making a decision between the set {STOP, SWITCH} and STAY. The optimal policy given in the above theorem is illustrated in Fig. 2, which can be viewed as a sequence of two YES/NO questions used in decision making involving two thresholds. (1) If $\lambda^* < c_i$, then the optimal decision is either STOP or SWITCH depending on whether $x > c_i$. If the current condition is very good ($x > c_i$) then the decision is STOP; otherwise SWITCH. In this case the reward from switching is sufficiently good that we will never consider STAY. (2) If $\lambda^* > c_i$, then the optimal decision is either STOP or STAY depending on whether

2. The word “continuation” in this context refers to continuing on the same channel, whereas “stopping” (or the term (meta) stopping used earlier) refers to no longer staying on the same channel either by a transmission or by switching away.

3. The interpretation of these two conditions is that even if we know the future the maximum expected reward, or the reward approaching the supremum, is finite.

4. This function is a fixed point equation which could be solved by iterative methods as in [22].

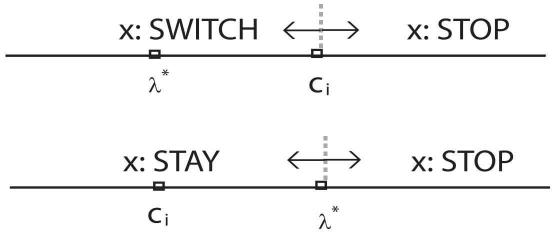


Fig. 2. Illustration of the decision process.

$x > \lambda^*$. In this case the reward from switching is inferior so that SWITCH is not an option. This policy will be referred to as a *nested stopping* policy.

3.2 Properties of the Nested Stopping Policy

We next investigate a number of properties of the multiuser multi-channel system as a result of the above nested stopping policy. We start by examining its effect on the traffic load vector $\mathbf{G}$. Unless otherwise noted, all proofs can be found in the appendix.

Lemma 3.3 (Monotonicity of the value function). *Consider two traffic load vectors* $\mathbf{G}$ *and* $\mathbf{G}^{'}$ *where* $G_i \leq G_i^{'}, \forall i \in \Omega$. *Denote the corresponding sets of value functions by* V *and* $V^{'}$, *respectively. Then we have* $E\{V_i\} \geq E\{V_i^{'}\}, \forall i \in \Omega$.

This lemma conveys the intuition that when the load increases, competition increases leading to longer delays. Thus the expected throughput decreases in general. We next establish the stability of the system under the nested stopping policy, starting with an assumption.

Assumption 1. *No channel is dominant, i.e., there is no single channel that will attract all arrivals under the nested stopping policy.*

This assumption excludes the extreme case where a single channel is of far better quality (e.g., very high data rate) that even considering the cost in competition it is beneficial to always switch to this channel, regardless of the conditions observed in the other channels.

Lemma 3.4 (Ergodicity of the arrival process). *The arrival processes are ergodic under the nested stopping policy and Assumption 1.*

Lemma 3.5 (Load balance). *We have* $\frac{\partial G_i}{\partial G} \geq 0, \forall i \in \Omega$ *under the nested stopping policy.*

In other words, if the total traffic load increases, the input/load to each channel is non-decreasing. This property combined with the monotonicity (Lemma 3.3) leads to the following stronger monotonicity result on the value function; the proof is trivial and thus omitted.

Lemma 3.6 (Strong monotonicity). $E\{V_i\}, i \in \Omega$, *are all non-increasing functions of* G.

We next analyze the impact of transmission time T and address the question whether by reserving more time for a single transmission users gain in average throughput.

Lemma 3.7 (Impact of T). $E\{V_i\}, i \in \Omega$, *are all non-decreasing functions of* T.

This result reflects the intuition that once a user finds a good transmission condition, it is beneficial for it to be able to use it for a longer period of time. However, practically T cannot be made too large due to the channel coherence time: the channel condition will likely change over a large period T.

3.3 Discussion

The performance a user obtains as a result of the preceding decision process depends on the accuracy of its measurement over the level of contention in the system, i.e., t_j^c and t_j^s. Fortunately, the performance loss due to errors in these measurements can be bounded as shown below. For simplicity we have assumed that the errors across all measurements are uniformly given by a small quantity Δ; if the errors are different for different measurements, Δ may be taken as the maximum measurement error.

Theorem 3.8. *For a given user, the change of its value function at the ith stage* ($1 \leq i \leq N$) *as a result of a small* ($\boldsymbol{\Delta}$) *change to* $\mathbf{t^c}, \mathbf{t^s}$ *is bounded as follows:*

$$|V_i(\mathbf{t^c} + \boldsymbol{\Delta}, \mathbf{t^s} + \boldsymbol{\Delta}) - V_i(\mathbf{t^c}, \mathbf{t^s})| \leq |\Delta| \cdot \sum_{j=i}^{N} \sum_{o \in \{s,c\}} \frac{T}{\left(T + t_j^o\right)^2} \cdot C_j, \tag{17}$$

where C_j *is a positive constant.*

Our model so far has assumed that each user can only access a single channel at a time. If parallel transmissions are enabled (e.g., as in an OFDM system), a single user can access multiple channels simultaneously. Under the most relaxed setting, we can model each user as having k independent antennas with no inter-channel interference. This then allows us to model the decision process of each user as k (or any number $k^{'} \leq k$ depending on how many packets it has to transmit) separate decision processes, each being the same as that presented earlier in this section. In other words, without restriction on the use of multiple interfaces, a single user is now equivalent to k different users and the subsequent analysis will remain the same.

If the use of these antennas are more restrictive, e.g., that a user may only use these interfaces concurrently, and that in doing so must access contiguous blocks of channels, or that there is throughput loss due to simultaneous channel access, then the resulting decision process becomes quite different and combinatorial. Specifically, due to this coupling, the resulting access decision is over "bundles" of channels rather than individual channels. A user may have up to $\binom{N}{k}$ choices of such channel bundles. If the user can sense either sequentially or simultaneously the channel condition in each channel within a bundle, it can then estimate the transmission reward from using this bundle. Conceptually, a similar decision process can be formulated where the user try to decide whether to switch to a different bundle or use the current bundle for transmission, or wait. A practical difficulty, however, lies in the random access nature of a channel: if the user needs to gain access in each channel in order to use the bundle then this could entail very significant amount of delay (thus the cost of delaying or switching), unless the traffic is extremely light. This remains a very relevant and interesting problem of future research.

4 Optimal Access Policy under the Markovian Channel Model

This section presents a parallel effort to the previous section, under the assumption that the channel conditions evolve over time as a Markov chain.

4.1 Uniqueness of the Optimal Strategy

Denote the state space of channel i by S_i, and the single-step (over one unit of time) state transition probability by $\mathcal{P}_i(y\,|\,x)$, $x, y \in S_i$. The k-step transition probability is denoted by $\mathcal{P}_i^k(y\,|\,x)$. The value function representing the maximum average throughput given the current condition at stage i is given by the following:

$$V_i(x) = \max\left\{\hat{X}^i(x), \frac{T}{t_i^c + T} \cdot \sum_{y \in S_i} \mathcal{P}_i^{t_i^c}(y\,|\,x) \cdot V_i(y)\right\}, \tag{18}$$

where $\hat{X}^i(x)$ follows the same definition as in the IID case. We make the following approximation. When $1/T$ is sufficiently small,[5] we have $\frac{T}{t_i^c + T} = \frac{1}{1 + t_i^c/T} \approx (\frac{1}{1+1/T})^{t_i^c}$. Denote $\beta = \frac{1}{1+1/T}$ and we arrive at the following approximated value function

$$V_i(x) = \max\left\{\hat{X}^i(x), \beta^{t_i^c} \cdot \sum_{y \in S_i} \mathcal{P}_i^{t_i^c}(y\,|\,x) \cdot V_i(y)\right\}. \tag{19}$$

Denote by $\mathcal{U} = \{\mathbf{S}, \mathbf{C}\}$ the set of two actions, stopping and continuation, where the stopping action $\mathbf{S}$ bundles STOP and SWITCH into a single action, i.e., $\mathbf{S} = \{\mathrm{STOP}, \mathrm{SWITCH}\}$ due to the definition of $\hat{X}^i(x)$ and as in the IID case, and the continuation action $\mathbf{C} = \{\mathrm{STAY}\}$. Then the above can be re-written as

$$V_i(x) = \max_{u \in \mathcal{U}}\left\{r(u, x) + \beta^{t_i^c} \cdot \sum_{y \in S_i} \mathcal{P}_i^{u, t_i^c}(y\,|\,x) \cdot V_i(y)\right\}, \tag{20}$$

where $r(\mathbf{S}, x) = \hat{X}^i(x)$, $r(\mathbf{C}, x) = 0$, $\mathcal{P}_i^{\mathbf{S}, t_i^c}(y\,|\,x) = 0$, and $\mathcal{P}_i^{\mathbf{C}, t_i^c}(y\,|\,x) = \mathcal{P}_i^{t_i^c}(y\,|\,x)$.

Theorem 4.1. *The set of Equation (19) or equivalently (20) have a unique solution.*

Our proof is based on the contraction mapping theorem [11] and the next lemma.

Lemma 4.2. *Let $\mathcal{F}$ be the class of all functions $v : \{1, 2, \ldots, S\} \to \mathcal{R}$. Define norm $||v|| := \sum_{x \in S} |v(x)|$ and a mapping $\mathcal{T} : \mathcal{F} \to \mathcal{F}$ by*

$$(\mathcal{T}v)(x) := \max_{u \in \mathcal{U}}\left\{r(u, x) + \eta \cdot \sum_{y \in S} v(y) \cdot \mathcal{P}^u(y\,|\,x)\right\},$$

$0 < \eta < 1$; then $\mathcal{T}$ is a contraction.

The next result also immediately follows; the proof is omitted for brevity.

5. This is possible since T is an integer multiple of an arbitrary time unit, which can be made very small. The only restriction is that we have taken a single time unit to be the time it takes to transmit a control packet, so this assumption simply implies that a data transmission is much longer than a control transmission, which is typically true.

TABLE 1
Contention Levels

Load	0.1	0.2	0.3	0.4	0.5
t_i^c	10.8	13.2	14.4	15.2	15.8
t_i^s	13.1	15.8	17.3	18.4	19.4

Corollary 4.3 (Threshold policy). *The optimal stopping rule reduces to a threshold policy.*

Remark 4.4. As may be expected, this threshold policy works in a way very similar to the IID case (only the numerical calculation differs): at stage/channel i, there is a SWITCH reward c_i (expected throughput by switching away from i) and λ^* by staying on the same channel. The optimal decision is then based on the relationship between λ^* and c_i.

4.2 Properties of the Nested Stopping Policy

We can similarly obtain a number of properties for the multiuser multi-channel system as a result of the nested stopping policy under the Markovian model.

Theorem 4.5 (Monotonicity). *$E\{V_i\}, i \in \Omega$ are all non-increasing functions of G.*

Following the above result we can derive similar properties of the nested stopping policy in the Markovian case as in the IID case, including ergodicity of the arrival processes, load balance and the non-increasing value functions in T. The proof of these are omitted for brevity and due to their similarity to those in the IID case.

5 Numerical Results

5.1 The IID Channel Model

We first consider a scenario of five independent channels with their channel condition (taken to be the instantaneous transmission rate measured in bytes per time unit) exponentially distributed over a finite range, with average rates given by $\{1/0.4, 1/0.6, 1/0.5, 1/0.3, 1/0.2\}$. A single transmission period is set to $T = 40$ time units. The level of contention/congestion measured by t_i^c and t_i^s (measured in time units) as a function of load G (measured in packet per unit time) is illustrated in Table 1 for channel 1. These quantities are rounded off to the nearest integers when used in computing the optimal policy. We set packet length to be 1,024 Bytes.

In Fig. 3 we compare the nested stopping policy with the following three schemes, by measuring the average throughput across all channels.

1) A standard random access policy in which a user randomly selects a channel to use, followed by competing for channel access using IEEE 802.11 type of random access scheme.
2) A stopping rule based random access policy over temporal diversity (denoted "Temporal Diversity" in the figure) introduced in [19], [22]. In this case each user is randomly assigned a channel, and follows a stopping rule on that channel (between using the channel now or at a later time).

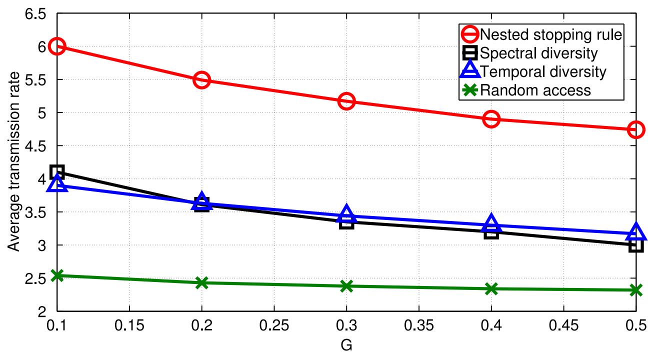


Fig. 3. Performance comparison: Exp.

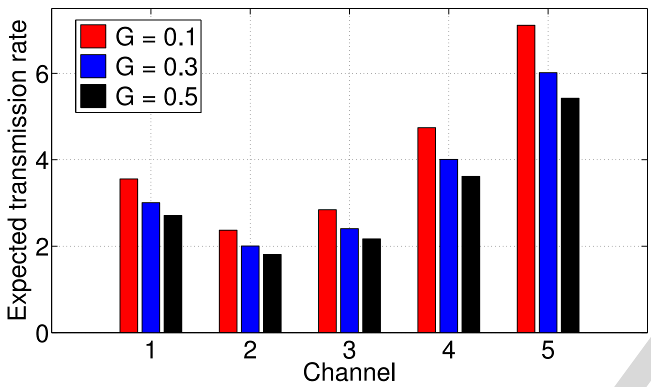


Fig. 4. Transmission rate w.r.t. G.

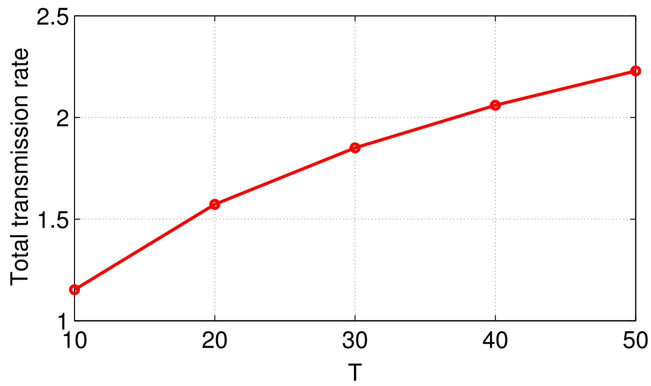
3) A stopping rule based random access policy over spectral diversity (denoted "Spectral Diversity" in the figure) introduced in [10], [18] where a user sequentially sense conditions over multiple channels to decide which channel to use for transmission.[6]

Fig. 3 shows that our nested stopping policy clearly outperforms the others. The performance gain is more prominent when the load is light. This is to be expected because when there is light congestion, waiting for better condition or switching to another channel both incur low overhead; when there is heavy congestion a user becomes more and more reluctant to wait or switch channels thereby underutilizing both types of diversity.

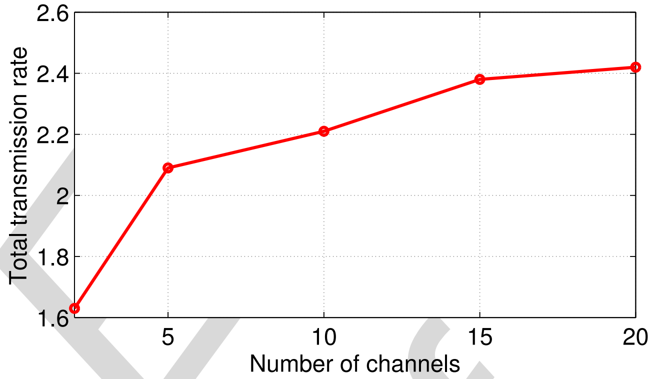
Fig. 4 shows the dampening effect of increased load G on each channel separately. Fig. 5a shows that the throughput increases in the data transmission time T as we have characterized, but this increase becomes slower since increasing T also increases the cost in channel releasing and switching. Fig. 5b shows that the throughput also increases in the number of channels (the simulation is done by adding channels with same statistics as given for the initial five), as the contention in each channel reduces.

Next in Table 2 we show the optimal decisions table for the optimal actions conditioned on continuation (STAY or SWITCH) for each channel (in this specific experiment we consider a user starts from channel 1). As can be seen, channels 2 and 3 are of low quality so the general decision is to switch away rather than waiting on the same channel

6. There are differences between these two references: [18] models sensing error and derives more structural properties, but the main idea is the same.



(a) Transmission rate w.r.t. T ($G = 0.5$)



(b) Transmission rate w.r.t. number of channels ($G = 0.5$)

Fig. 5. Performance under the IID channel model.

TABLE 2
Decision of IID Channels with Different Arrival Rate

Load	Ch 1	Ch 2	Ch 3	Ch 4	Ch 5
0.05	STAY	SWITCH	SWITCH	SWITCH	STAY
0.1	STAY	SWITCH	SWITCH	STAY	STAY
0.3	STAY	SWITCH	SWITCH	STAY	STAY
0.5	STAY	SWITCH	SWITCH	STAY	STAY

if the decision is not to transmit immediately. For channel 4, we see that the tendency to stay increases when the load is high due to the higher cost in switching than staying. The decision to stay in channel 1 is more interesting: even though better average throughput may be obtained in channels 4 and 5, the cost in doing so is considerable as it has to go through channels 2 and 3. By contrast, there is a SWITCH decision in channel 4 even though channel 4 is on average a better channel than channel 1.

We also consider a more practical AWGN wireless channel model considering both propagation loss and shadowing effects. The transmission rates are given by the Shannon capacity formula for AWGN channels: $R = \log(1 + \rho\,|\,h|^2)$ nats/s/Hz, where h denotes the random channel gain with a complex Gaussian distribution. Moreover, the cdf of transmission rate is given by $F_R(r) = 1 - \exp(-\frac{\exp(r)-1}{\rho}), r \geq 0$. Consider a scenario with five channels with average SNR ρ given by Table 3. Fig. 6 shows the same performance comparison as before. While the nested stopping rule continues to

TABLE 3
Parameter Table

Channels	Ch1	Ch 2	Ch 3	Ch 4	Ch 5
ρ	10	25	20	30	10

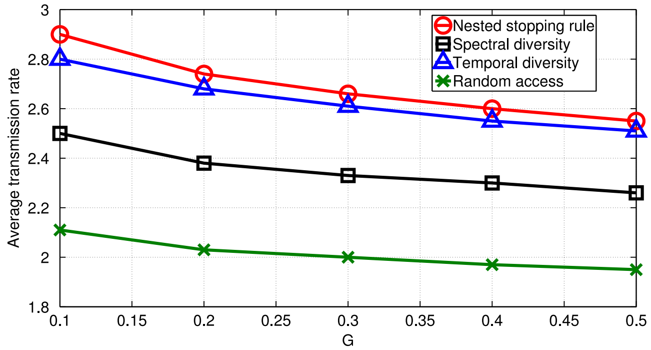


Fig. 6. Performance comparison: AWGN.

TABLE 4
Reward Table for Markovian Channels with Different States

States	Ch 1	Ch 2	Ch 3	Ch 4	Chl 5
1	10	15	5	10	5
2	20	20	10	20	10
3	30	45	15	30	15
4	40	60	20	40	20
5	50	75	25	50	25

outperform the other schemes, an interesting observation here is that the scheme based solely on temporal diversity also outperforms using only spectral diversity and it has very similar performance as the nested policy. This shows that due to the dynamic nature of AWGN channels, most of the gain is derived from exploiting temporal diversity rather than spectral diversity.

5.2 The Markovian Channel Model

We now simulate the nested stopping policy under a Markovian channel model. We model all five channels' state (again taken to be the instantaneous transmission rate in bytes per time unit) change as a birth-death chain with five states and the associated transition probabilities given as follows:

$$\begin{aligned} &\mathcal{P}_k(\min\{i+1,5\}\,|\,i) = 0.8,\\ &\mathcal{P}_k(\max\{i-1,1\}\,|\,i) = 0.2,\\ &\qquad 1 \le i \le 5, 1 \le k \le 5. \end{aligned} \tag{21}$$

For each channel the rewards increase in state indices, and are given in Table 4. Transmission time is again set to be $T = 40$ time units.

The same performance comparison is shown in Fig. 7. Taking this result together with previous results under exponential and AWGN channel models, we observe something quite revealing. In Fig. 7 we see that the performance of the temporal and spectral schemes are reversed: exploiting spectral diversity results in much higher gain than only exploiting temporal diversity. This is because this set of

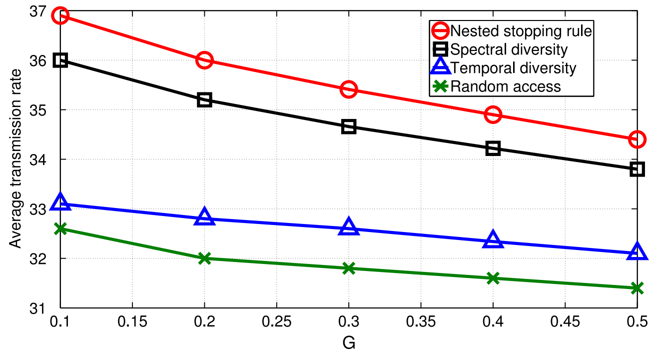


Fig. 7. Performance comparison: Markovian model.

TABLE 5
Decision Table for Markovian Channels with Different States

States	Ch1	Ch 2	Ch 3	Ch 4	Ch 5
1	SWITCH	STAY	SWITCH	STAY	STAY
2	SWITCH	STAY	SWITCH	STOP	STAY
3	SWITCH	STOP	STOP	STOP	STOP
4	STOP	STOP	STOP	STOP	STOP
5	STOP	STOP	STOP	STOP	STOP

Markovian channels are relatively slow-varying in time compared to the previous models, thus staying on the same channel waiting for better condition becomes less beneficial, while hopping through channels seeking better conditions is more effective. These results show that, compared to exploiting only one type of diversity, our policy is very robust against different dynamic properties of the channels and can extract the largest amount of performance gain.

The decision table in this case is shown in Table 5. Similar observations are made here: when the channel condition is good enough, the user would choose to transmit immediately (STOP); the SWITCH decision is associated with poor conditions and when a user hopes to get much better conditions in the next channel; the STAY decision is made on a reasonably good channel and when there is limited prospect of getting better condition in the next channel.

5.3 Channel Sensing Order and "No-Recall" Approximation

We next examine the effect of selecting different sequence of channels to use. As discussed earlier, with multiple users ($m \geq 2$) it is very challenging to either jointly determine optimal sensing orders for all users involved in a cooperative setting, or determine the equilibrium sensing orders selected by selfish individuals in a non-cooperative setting. For this reason in our analysis we have assumed that each user follows a fixed (which can be randomly chosen) order. We now compare this choice where each user randomly picks a sequence with an optimal sensing order where users sense channels ordered in an derived optimal sensing order [5] and make decisions on each channel according to the threshold decision derived in our paper;[7] as a result each

7. For certain channel quality distributions, e.g., exponential distribution, the optimal sensing order is equivalent to a greedy sensing order.

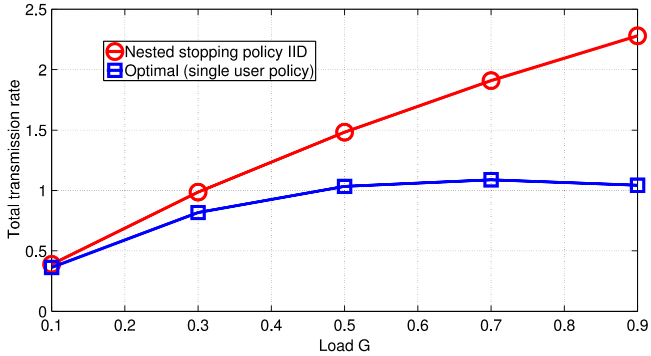


Fig. 8. Channel sensing order comparison.

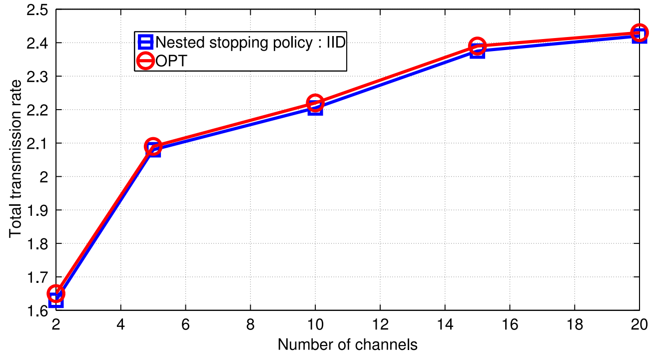


Fig. 9. Performance of approximation model.

user follows/cycles through the same sequence but the starting position for each user is randomized to avoid synchronization. This comparison is shown in Fig. 8 for the IID channel case; it is clear that it is far better for each user to sense in a different order especially when the load is high. This comparison also highlights some of the challenges mentioned earlier in employing an optimal channel sensing order in a multi-user setting. Note that we have simulated a user-homogeneous environment where all users perceive identical channel conditions. It is evident that in this case having all users follow the same optimal sensing order derived for a single user (assuming it is the only user present) is not a good strategy, while determining jointly optimal sensing orders for all users is analytically intractable and computationally prohibitive. In this sense our policy simply assumes randomization as a compromise to enable the multi-user study performed in this paper.

We end this section by investigating the effect of the "no-recall" approximation introduced in Section 2 and adopted in our analysis, by comparing it with the exact optimal solution. We show this in the IID case in Fig. 9; we see that this approximation has very little effect on the system performance.

6 Conclusion

In this paper we considered the collective switching of multiple users over multiple channels. In addition, we considered finite arrivals. Under such a scenario, the users' ability to opportunistically exploit temporal diversity (the temporal variation in channel quality over a single channel) and spectral diversity (quality variation across multiple channels at a given time) is greatly affected by the level of congestion in the system. We investigated the optimal decision process under both an IID and a Markovian channel models, and evaluate the extent to which congestion affects potential gains from opportunistic dynamic channel switching.

Appendix A
Proof of Lemma 3.3: Monotonicity of Value Function in G

We prove this by induction. When $i = N$, i.e., the last stage, we have $\lambda_N^* \bar{t}_N^c = \int_{\lambda_N^*}^{\overline{X}^N} (x - \lambda_N^*) f_{X^N}(x)dx$, where $\bar{t}_i^c = t_i^c/T$. As $\bar{t}_N^c$ is a non-decreasing function in G_N, it is also non-decreasing in $\mathbf{G}$. Thus with the increase in $\bar{t}_N^c$, the solution λ_N^* cannot be increasing, proving that λ_N^* is a non-increasing function of $\mathbf{G}$. Since our value functions $(E\{\max(X^i, \lambda_i^*)\})$ are non-decreasing functions of the thresholds λ^*s, we have now shown that they are non-increasing in $\mathbf{G}$. Next assume the non-decreasing property holds for $i = n+1, \dots, N-1$. Consider $i = n$. We prove this in the cases $\lambda_n^* < c_n$ and $\lambda_n^* \geq c_n$, respectively. For the case $\lambda_n^* \geq c_n$, we have $\lambda_n^* \bar{t}_N^c = \int_{\lambda_n^*}^{\overline{X}^n} (x - \lambda_n^*) f_{X^n}(x)dx$. Using similar argument as in the case $i = N$ we know λ_n^* is non-increasing in $\mathbf{G}$. For the case $\lambda_n^* < c_n$, $\lambda_n^* = \frac{\int_{c_n}^{\overline{X}^n} x f_{X^n}(x)dx + c_n \cdot P(X^n \leq c_n)}{1+\bar{t}_n^c}$, and we get $E\{V_n\} = \int_{c_n}^{\overline{X}^n} x f_{X^n}(x)dx + c_n \cdot P(X^n \leq c_n)$. Taking the derivative of $E\{V_n\}$ with respect to $\mathbf{G}$ we get

$$\frac{\partial E\{V_n\}}{\partial \mathbf{G}} = \left\{ \frac{\partial E(X^n)}{\partial c_n} - \frac{\partial \int_0^{c_n} x f_{X^n}(x)dx}{\partial c_n} + \frac{\partial c_n P(X^n \leq c_n)}{c_n} \right] \right\} \cdot \frac{\partial c_n}{\partial \mathbf{G}}$$

$$\frac{\partial c_n}{\partial \mathbf{G}} = \frac{\frac{\partial E\{V_{n+1}\}}{\partial \mathbf{G}} \left(T + t_{n+1}^s\right) - E\{V_{n+1}\} \frac{\partial t_{n+1}^s}{\partial G_{n+1}}}{\left(T + t_{n+1}^s\right)^2}. \tag{22}$$

By induction hypothesis we know $\frac{\partial E\{V_{n+1}\}}{\partial \mathbf{G}} \leq 0$ and $\frac{\partial t_{n+1}^s}{\partial G_{n+1}} \geq 0$. Therefore we conclude $\frac{\partial c_n}{\partial G_n} \leq 0$ $\frac{\partial E\{V_n\}}{\partial \mathbf{G}} \leq 0$, completing the induction step and the proof. □

Appendix B
Proof of Lemma 3.4: Ergodicity of G

By Assumption 1 there exists a threshold $\tilde{G}_i$ such that $E\{V_i(G_i)\} < E\{V_{-i}(G_{-i})\}$, $\forall i \in \Omega$, for all $G_i \geq \tilde{G}_i$, where G_{-i} denotes the aggregated load on all other channel except channel i, and $E\{V_{-i}(G_{-i})\}$ is defined as the average reward/rate-of-return of all other channels except i. In this case, the arrivals to all other channels except i will not switch to channel i, i.e., under loads $G_i > \tilde{G}_i$ the probability of load G_i drifting higher is 0 almost surely. Define any increasing, unbounded Lyapunov function $L(G_i)$ on $[0, G]$ (e.g., $L(G_i) = \frac{1}{G - G_i}$), we have $E_{\tilde{G}_i}[L(\tilde{G}_i) \,|\, G_i] \leq L(G_i)$. By the Foster-Lyapunov criteria [15] we establish the ergodicity of the system load vector. □

Appendix C
Proof of Lemma 3.5: Load Balance

We prove this by induction on N. When $N = 1$, i.e., the system degenerates to a single channel case, the claim holds obviously. Assume the claim holds for $N = 2, \ldots, n-1$, and now consider the case $N = n$. Suppose we increase the total load from G to G', and assume that without loss of generality the load to channel 1 decreases, i.e., $G_1^{'} < G_1$. By the induction hypothesis, the loads on all other channels have increased, i.e., $G_i^{'} > G_i$, $\forall i \neq 1$. As a result, their corresponding value functions decrease by the previous lemma, i.e., $E\{V_i^{'}\} < E\{V_i\}$, $\forall i \neq 1$. This means that the amount switching out of channel 1 must be non-increasing, due to the fact that the threshold of switching c_1 is a non-increasing function of $\mathbf{G}$, while the amount switching into channel 1 must be non-decreasing, leading to an overall non-decreasing load on channel 1, which is a contradiction. □

Appendix D
Proof of Lemma 3.7: Monotonicity of Value Functions in T

When $i = N$, i.e., the last stage, we have $\lambda_N^* \bar{t}_N^c = \int_{\lambda_N^*}^{\overline{X}^N} (x - \lambda_N^*) f_{X^N}(x)dx$. Following a similar argument as in the monotonicity in G, with the decrease in $\bar{t}_N^c$, the solution λ_N^* cannot be decreasing, proving that λ_N^* is a non-decreasing function of T. Assume now the claim holds for $i = n+1, \ldots, N-1$. When $i = n$, consider two cases. For the case $\lambda_n^* \geq c_n$, we have $\lambda_n^* \bar{t}_n^c = \int_{\lambda_n^*}^{\overline{X}^n} (x - \lambda_n^*) f_{X^n}(x)dx$. We know λ_n^* is a non-decreasing function of T. For the case $\lambda_n^* < c_n$, $\lambda_n^* = \frac{\int_{c_n}^{\overline{X}^n} x f_{X^n}(x)dx + c_n \cdot P(X^n \leq c_n)}{1 + \bar{t}_n^c}$. We have $E\{V_n\} = \int_{c_n}^{\overline{X}^n} x f_{X^n}(x)dx + c_n \cdot P(X^n \leq c_n)$ and taking the derivative of $E\{V_n\}$ w.r.t. T we have

$$\frac{\partial E\{V_n\}}{\partial T} = \left\{ \frac{\partial E(X^n)}{\partial c_n} - \frac{\partial \int_0^{c_n} x f_{X^n}(x)dx}{\partial c_n} + \frac{\partial c_n P(X^n \leq c_n)}{c_n} \right]\right\} \cdot \frac{\partial c_n}{\partial T} \tag{23}$$

$$\frac{\partial c_n}{\partial T} = \frac{\partial \frac{T}{T+t_c}}{\partial T} E\{V_{n+1}\} + \frac{T}{T+t_c} \frac{\partial E\{V_{n+1}\}}{\partial T}, \tag{24}$$

and moreover we see

$$\frac{\partial \frac{T}{T+t_c}}{\partial T} = \frac{T + t_c - T\left(1 + \frac{\partial t_c}{\partial T}\right)}{(T+t_c)^2}, \tag{25}$$

and combine with the fact $\frac{\partial E\{V_{n+1}\}}{\partial T} \geq 0$ (induction hypothesis) and $\frac{\partial t_{n+1}^s}{\partial T} \geq 0$, we conclude $\frac{\partial c_n}{\partial T} > 0$ $\frac{\partial E\{V_n\}}{\partial T} > 0$, completing the induction step and the proof. □

Appendix E
Proof of Lemma 4.2: Contraction

For $v, z \in \mathcal{F}$, we have

$$\begin{aligned} &(\mathcal{T}v)(x) - (\mathcal{T}z)(x) \\ &= \max_{u \in \mathcal{U}} \left\{ r(u,x) + \eta \cdot \sum_{y \in S} v(y) \cdot \mathcal{P}^u(y \,|\, x) \right\} \\ &- \max_{u \in \mathcal{U}} \left\{ r(u,x) + \eta \cdot \sum_{y \in S} z(y) \cdot \mathcal{P}^u(y \,|\, x) \right\}. \end{aligned} \tag{26}$$

Let $\mu = \arg\max_{u \in \mathcal{U}} \{ r(u,x) + \eta \sum_{y \in S} v(y) \cdot \mathcal{P}^u(y \,|\, x) \}$, then

$$\begin{aligned} (\mathcal{T}v)(x) - (\mathcal{T}z)(x) &= \left\{ r(\mu,x) + \eta \cdot \sum_{y \in S} v(y) \cdot \mathcal{P}^\mu(y \,|\, x) \right\} \\ &- \max_{u \in \mathcal{U}} \left\{ r(u,x) + \eta \cdot \sum_{y \in S} z(y) \cdot \mathcal{P}^u(y \,|\, x) \right\} \\ &\leq \left\{ r(\mu,x) + \eta \cdot \sum_{y \in S} v(y) \cdot \mathcal{P}^\mu(y \,|\, x) \right\} \\ &- \left\{ r(\mu,x) + \eta \cdot \sum_{y \in S} z(y) \cdot \mathcal{P}^\mu(y \,|\, x) \right\} \\ &= \eta \sum_{y \in S} [v(y) - z(y)] \cdot \mathcal{P}^\mu(y \,|\, x) \\ &\leq \eta \max_{y \in S} |\, v(y) - z(y)| = \eta ||v - z||. \end{aligned} \tag{27}$$

Similarly by reversing the order of z, v we have $(\mathcal{T}z)(x) - (\mathcal{T}v)(x) \leq \eta ||v - z||$. Therefore we reach at $||\mathcal{T}v - \mathcal{T}z|| \leq \eta ||v - z||$, i.e., $\mathcal{T}$ is a contraction. □

Appendix F
Proof of Theorem 4.5 : Monotonicity of the Value Function in G

As proved in [11], $V_i(x), x \in S, i \in \Omega$ can be interpreted as follows:

$$V_i(x) = \max_{\mathbf{u}} E \sum_{k=0}^{\infty} \beta^{k \cdot t_i^c} \cdot r_i(u_k, x_k). \tag{28}$$

Consider the expected maximum throughput at the last stage, i.e., $V_N(x) = \max_{\mathbf{u}} E \sum_{k=0}^{\infty} \beta^{k \cdot t_N^c} \cdot r_N(u_k, x_k)$. Consider a $G_N^{'} \geq G_N$ which gives us $t_N^{c'} \geq t_N^c$. Consider an arbitrary term in the above sum $\beta^{k \cdot t_N^{c'}}$, and there exists a k' such that $k' \cdot t_N^c \leq t_N^{c'} \leq (k'+1) \cdot t_N^c$. Together with the fact that $\beta^t \cdot \sum_y \mathcal{P}^t(y \,|\, x) \cdot y$ is convex w.r.t. t we know

$$\begin{aligned} \max\Big\{ &\beta^{(k'+1) \cdot t_N^c} \cdot \sum_y \mathcal{P}^{(k'+1) \cdot t_N^c}(y \,|\, x) \cdot y, \\ &\beta^{k' \cdot t_N^c} \cdot \sum_y \mathcal{P}^{k' \cdot t_N^c}(y \,|\, x) \cdot y \Big\} \\ &\geq \beta^{k \cdot t_N^{c'}} \cdot \sum_y \mathcal{P}^{k \cdot t_N^{c'}}(y \,|\, x) \cdot y \end{aligned} \tag{29}$$

$$
\begin{aligned}
V_N^{'}(x) &= \max_{\mathbf{u}} E \sum_{k=0}^{\infty} (\beta^{k \cdot t_N^{c'}})^k \cdot r_N(u_k, x_k) \\
&\leq \max_{\mathbf{u}} E \sum_{k=0}^{\infty} \beta^{k \cdot t_N^c} \cdot r_N(u_k, x_k) = V_N(x).
\end{aligned} \tag{30}
$$

Therefore as $E\{V_N\} = \sum_x \pi_x \cdot V_N(x)$, and we know $E\{V_N\}$ is a non-increasing function of $\mathbf{G}$. This establishes the induction basis. Now assume that the theorem holds for $i = n+1, \ldots, N-1$. Consider the case $i = n$. Assume $\mathbf{G}^{'} > \mathbf{G}$. As discussed in the IID section we have $r_n^{'}(u, x) \leq r_n(u, x)$. (This can be proved by taking the derivative of c_i with respect to $\mathbf{G}$ and by induction hypothesis $\frac{\partial E\{V_{n+1}\}}{\partial \mathbf{G}} \leq 0$). Therefore again similarly as argued above we have

$$
\begin{aligned}
V_n^{'}(x) &= \max_{\mathbf{u}} E \sum_{k=0}^{\infty} \beta^{k \cdot t_n^{c'}} \cdot r_n^{'}(u_k, x_k) \\
&\leq \max_{\mathbf{u}} E \sum_{k=0}^{\infty} \beta^{k \cdot t_n^c} \cdot r_n^{'}(u_k, x_k) \\
&\leq \max_{\mathbf{u}} E \sum_{k=0}^{\infty} \beta^{k \cdot t_n^c} \cdot r_n(u_k, x_k) = V_n(x)
\end{aligned} \tag{31}
$$

which completes the induction step. □

Appendix G
Backward Calculation of the Two-Dimensional Nested Stopping Policy

We describe the process of calculating the threshold for each channel. Note at the last stage of the decision process there is no more channel to switch to; therefore the dynamic program degenerates to a standard rate-of-return problem. The standard optimal stopping rule thus applies and the details are omitted. By going backward, at a subsequent stage $i < N$, the quantity $E\{V_{i+1}(X^{i+1})\}$ is available, and we have $V_i(x) = \max\{\hat{X}^i(x), \frac{T}{t_i^c + T} \cdot E\{V_i(X^i)\}\}$. We calculate c_i using $c_i = \frac{T}{T + t_{i+1}^s} E\{V_{i+1}(X^{i+1})\}$, and obtain $\frac{\int_{c_i}^{\overline{X}^i} x f_{X^i}(x) dx + c_i \cdot P(X^i \leq c_i)}{1 + t_i^c / T}$. If the latter is less than c_i, we are done and take this as the threshold λ^*. Otherwise, we proceed to a fixed-point equation $\lambda = \frac{\int_{\lambda}^{\overline{X}^i} x f_{X^i}(x) dx}{P(X^i \geq \lambda) + t_i^c / T}$ which can be solved iteratively to obtain the threshold.

Appendix H
Proof of Theorem 3.8 : Sensitivity of the Value Function in $\mathbf{t}^\mathbf{s}, \mathbf{t}^\mathbf{c}$

When Δ is small, using Taylor approximation ($f(\Delta) \approx f(0) + f'(0) \cdot \Delta$) we have

$$
|V_i(\mathbf{t^c} + \mathbf{\Delta}, \mathbf{t^s} + \mathbf{\Delta}) - V_i(\mathbf{t^c}, \mathbf{t^s})| \leq \left| \frac{\partial V_i(\Delta)}{\partial \Delta} \right|_{\Delta = 0} \cdot |\Delta|. \tag{32}
$$

We prove the result inductively. Recall at stage N we have the following fixed point equation for characterizing $E\{V_N\}$

$$
E\{V_N\} \overline{t}_N^c = \int_{E\{V_N\}}^{\overline{X}^N} (x - E\{V_N\}) f_{X^N}(x) dx, \tag{33}
$$

where $\overline{t}_i^c = t_i^c / T$. Taking the derivative on both sides w.r.t. Δ we have

$$
\begin{aligned}
&\frac{\partial E\{V_N(\Delta)\}}{\partial \Delta} \cdot \frac{t_N^c + \Delta}{T} + \frac{E\{V_N(\Delta)\}}{T} \cdot \frac{\partial (t_N^c + \Delta)}{\partial \Delta} \\
&\quad = -(1 - F(E\{V_N(\Delta)\}) \cdot \frac{\partial E\{V_N(\Delta)\}}{\partial \Delta},
\end{aligned} \tag{34}
$$

where F is the cdf of X^N. Rearranging terms we have

$$
\begin{aligned}
&\left| \frac{\partial E\{V_N(\Delta)\}}{\partial \Delta} \right|_{\Delta=0} \\
&= \left| \frac{\frac{E\{V_N(\Delta)\}}{T}}{\frac{t_N^c + \Delta}{T} + 1 - F(E\{V_N(\Delta)\}} \right|_{\Delta=0} = \left| \frac{E\{V_N\}}{t_N^c + T(1 - F(E\{V_N\})} \right| \\
&= \left| \frac{E\{V_N\}}{1 - F(E\{V_N\})} \cdot \frac{1}{t_N^c / (1 - F(E\{V_N\}) + T} \right| \\
&\leq \left| \frac{E\{V_N\}}{1 - F(E\{V_N\})} \right| \cdot \left| \frac{1}{t_N^c + T} \right| \leq C_N \cdot \frac{1}{T + t_N^c},
\end{aligned} \tag{35}
$$

with C_N being constant, where the last inequality uses the fact that $E\{V_N\}$ is bounded by a constant.

Therefore by the dynamic programming equation Eqn. (4) (the second term) we have,

$$
\begin{aligned}
\left| \frac{\partial V_N(\Delta)}{\partial \Delta} \right|_{\Delta=0} &\leq C_N^{'} \cdot \frac{T}{(T + t_N^c)^2} + \frac{T}{T + t_N^c} \cdot C_N \cdot \frac{1}{T + t_N^c} \\
&\leq \hat{C}_N \cdot \frac{T}{(T + t_N^c)^2}
\end{aligned} \tag{36}
$$

with $C_N^{'}, \hat{C}_N$ being constants. The first term comes from $\frac{\partial \frac{T}{T + t_N^c + \Delta}}{\partial \Delta}$ and the second comes from Eq. (35). This establishes the induction basis. Suppose the result holds for $i+1, \ldots, N$, consider stage i. If the decision is STAY on at same stage/channel i, then using Eqn. (4) we see this case is similar to stage N, and

$$
\left| \frac{\partial V_i(\Delta)}{\partial \Delta} \right|_{\Delta=0} \leq \hat{C}_i \cdot \frac{T}{(T + t_i^c)^2}. \tag{37}
$$

When the decision is SWITCH, we have (the third term in Eqn. (4))

$$
\left| \frac{\partial V_i(\Delta)}{\partial \Delta} \right|_{\Delta=0} \leq \frac{\partial \frac{T}{T + t_{i+1}^s + \Delta} E\{V_{i+1}(\Delta)\}}{\partial \Delta} \tag{38}
$$

By the induction hypothesis we have

$$\begin{aligned}
&\left|\frac{\partial \frac{T}{T+t_{i+1}^s+\Delta} E\{V_{i+1}(\Delta)\}}{\partial \Delta}\right|_{\Delta=0} \\
&= \left|\frac{\partial \frac{T}{T+t_{i+1}^s+\Delta}}{\partial \Delta} E\{V_{i+1}(\Delta)\}\right|_{\Delta=0} \\
&\quad + \left|\frac{\partial E\{V_{i+1}(\Delta)\}}{\partial \Delta} \frac{T}{T+t_{i+1}^s+\Delta}\right|_{\Delta=0} \\
&\leq C_i' \cdot \frac{T}{\left(T+t_i^s\right)^2} + \sum_{j=i+1}^{N} \sum_{o\in\{s,c\}} \frac{T}{\left(T+t_j^o\right)^2} \cdot C_j.
\end{aligned} \tag{39}$$

Combining Eqs. (37) and (39) completes the induction step. □

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