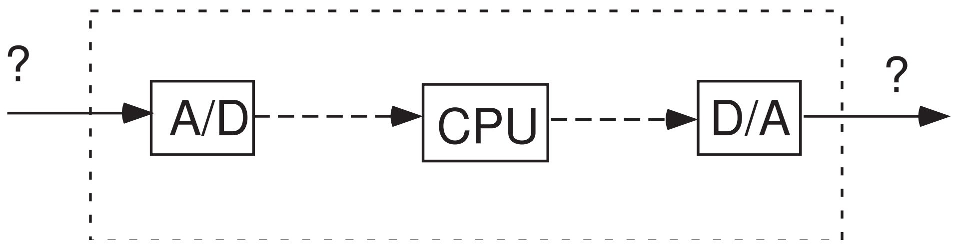
Interfacing a Microprocessor to the Analog World

In many systems, the embedded processor must interface to the non-digital, analog world.

The issues involved in such interfacing are complex, and go well beyond simple A/D and D/A conversion.



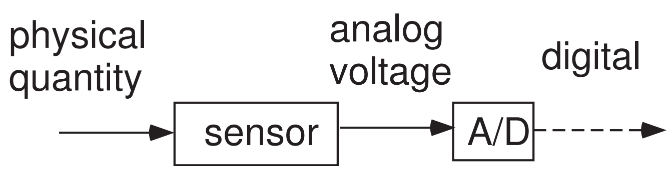
Two questions:

1. How do we represent information about the analog world in a digital microprocessor?
2. How do we use a microprocessor to act on the analog world?

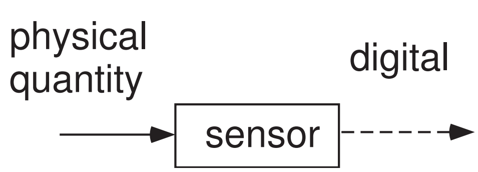
We shall explore each of these questions in detail, both conceptually in the lectures, and practically in the laboratory exercises.

Sensors

- Used to measure physical quantities such as
  - position
  - velocity
  - temperature
  - sound
  - light
- Two basic types:
  - sensors that measure an (analog) physical quantity and generate an analog signal, such as a voltage or current



    * tachometer
    * potentiometer
  - sensors that directly generate a digital value



    * digital camera
    * position encoders

Sensor Interfacing Issues

- Shall focus on issues that involve
  - *loss* of information
  - *distortion* of information

- Such issues include
  - quantization
  - sampling
  - noise

- Fundamental difference between quantization and sampling errors:
  - *Quantization errors* affect the precision with which we can represent a *single analog value* in digital form.
  - *Sampling errors* affect how well we can represent an *entire analog waveform* (or time function) digitally.

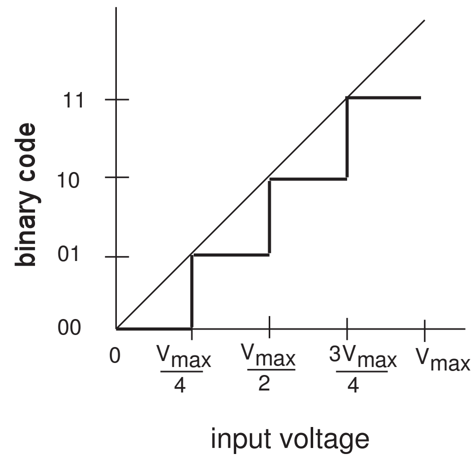
Quantization

Digital representation of an analog number [2, 3, 6]

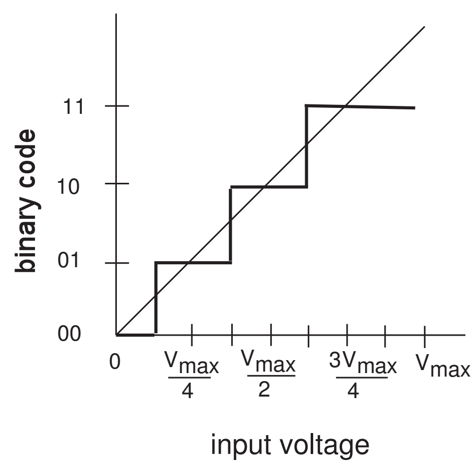
- Issue:
  - an analog voltage can take a continuum of values
  - a binary number can take only finitely many values
- Binary representation of (unsigned or signed) real number
  - unipolar coding
  - unipolar coding with centering
  - offset binary coding
  - two's complement
- Resolution [2, 3]
  - Idea: two analog numbers whose values differ by $< 1/2^n$ may yield the same digital representation
  - an n-bit A/D converter has a resolution equal to 2^{-n} times the input voltage range, $v \in [0, V_{max}]$
  - least significant bit (LSB) represents $V_{max}/2^n$

Quantization: Example

- Suppose we quantize an analog voltage in the range $(0, V_{\max})$ using a two bit binary number.
- The LSB thus represents $V_{\max}/4$

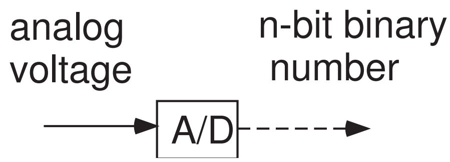


- Quantization error: from 0 to 1 LSB (e.g., 01 represents any voltage from $V_{max}/4$ to $V_{max}/2$)
- For 11 to uniquely represent V_{max}, divide voltage range into $2^n - 1$ intervals. LSB $= V_{max}/(2^n - 1)$
- Centering: 01 represents $V_{max}/8$ to $3V_{max}/8$



- Quantization error: $\pm\frac{1}{2}$LSB

A/D Conversion



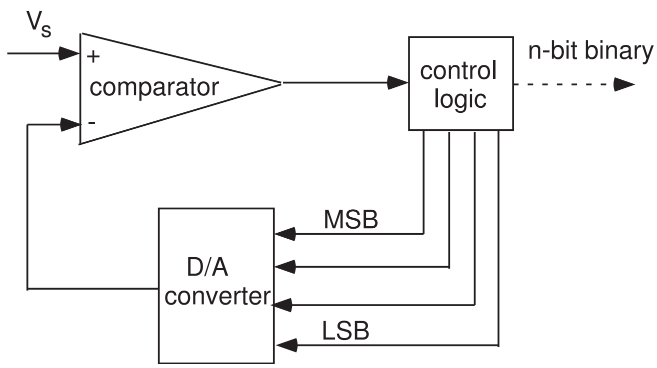
- Types of A/D converters [2]:
  - flash
  - successive approximation (MPC555)
  - single-slope (or dual-slope) integration
  - sigma-delta converters
  - redundant signed digit (RSD) [5] (MPC5553)

- Design issues
  - precision
  - accuracy
  - speed
  - cost
  - relative amount of analog and digital circuitry

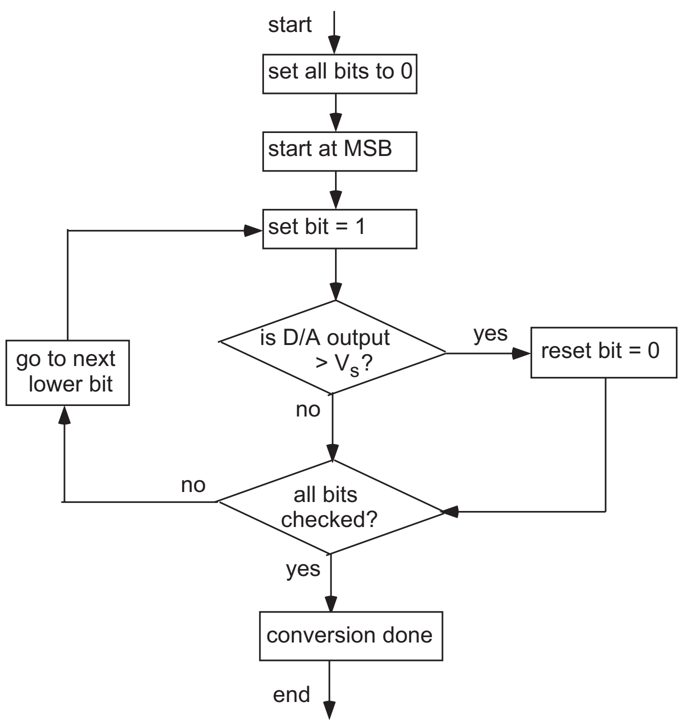
- Performance Metrics [4]
  - quantization error
  - offset and gain error
  - differential nonlinearity
  - monotonicity
  - missing codes
  - integral nonlinearity

Successive Approximation A/D Converter

- Used on the Freescale MPC555
- Bits set in succession, from most to least significant



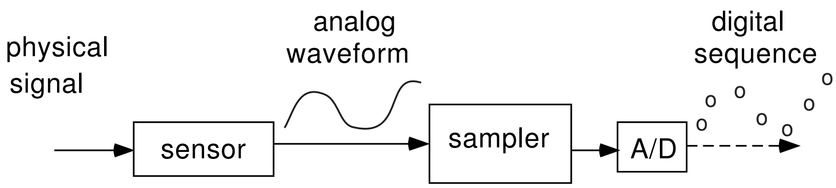
- Control logic [1]



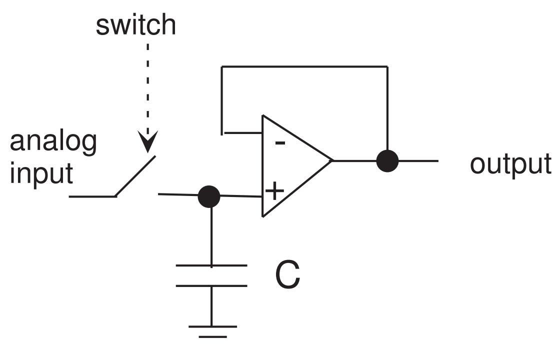
- Issues
  - timing (bits set one at a time)
  - signal to noise ratio (lower bits based on small signals)
  - cannot correct for wrong decisions on a given bit

Sampling

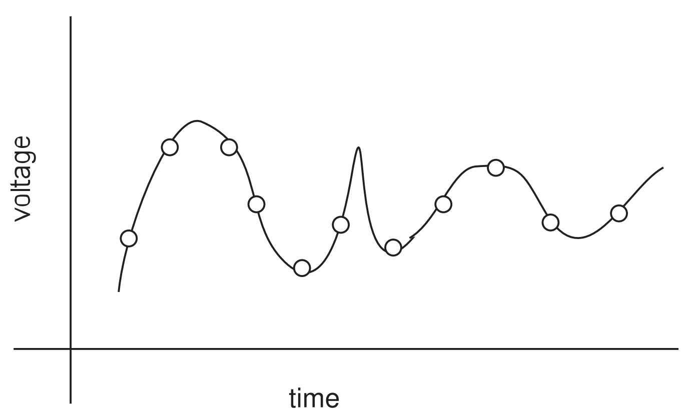
- Convert an analog function of time into a sequence of binary numbers



- sampler [3]

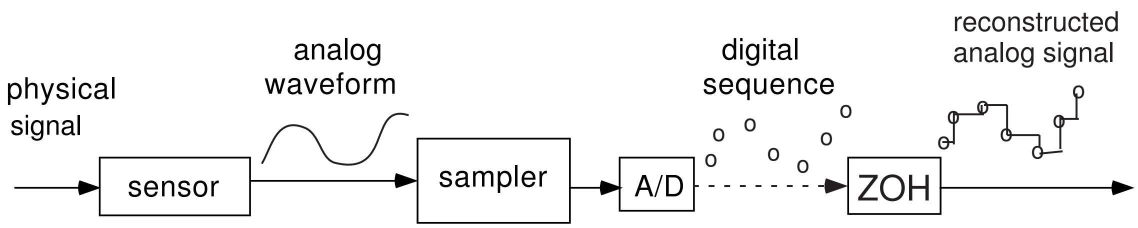


- *Information loss* in representing an analog function as a discrete sequence [2, 3, 8, 6]

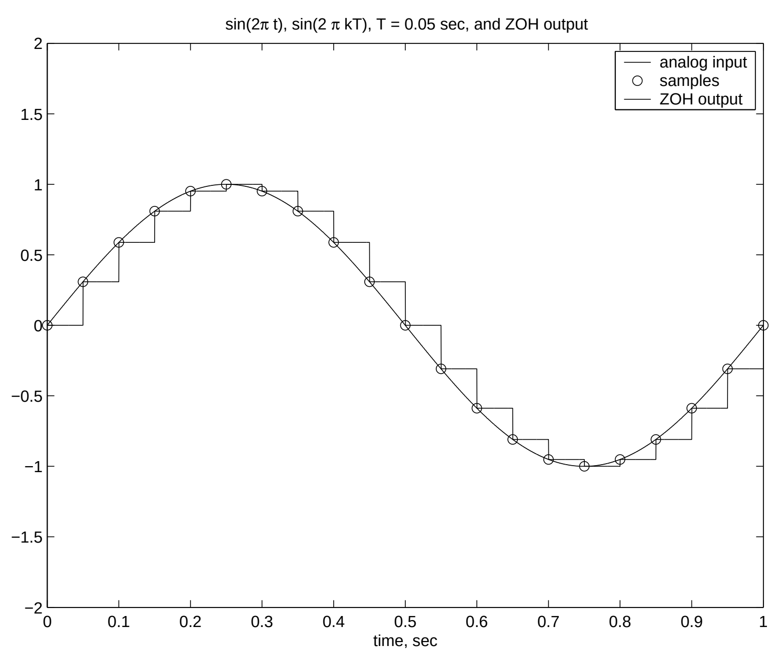


Information Loss in Sampling

- How to describe information loss?
- Idea: Try to reconstruct the analog signal from its digital representation. This may be done by a D/A converter.



- "Staircase" output[1]: $\sin(2\pi t)$ sampled with sampling period $T = 0.05$ seconds (sampling frequency $f = 20$ Hz)

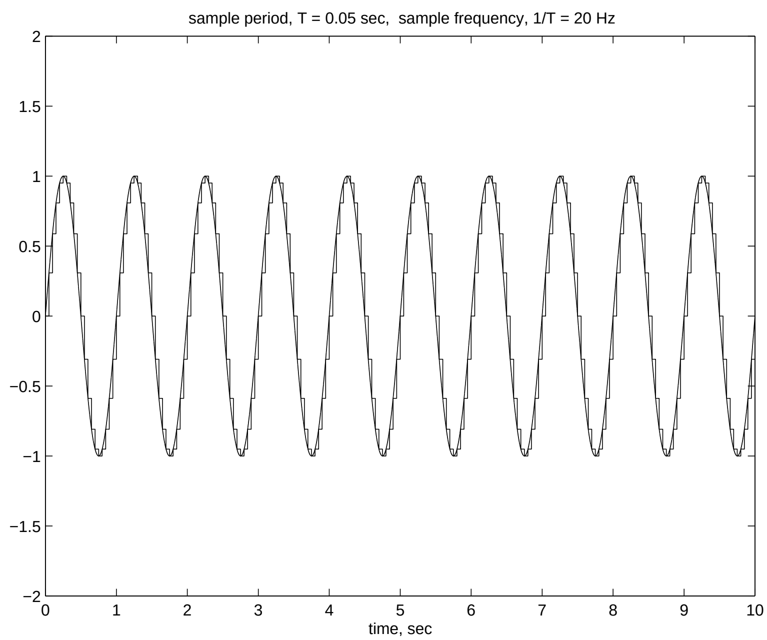


- a staircase approximation of the input delayed by $T/2$ seconds

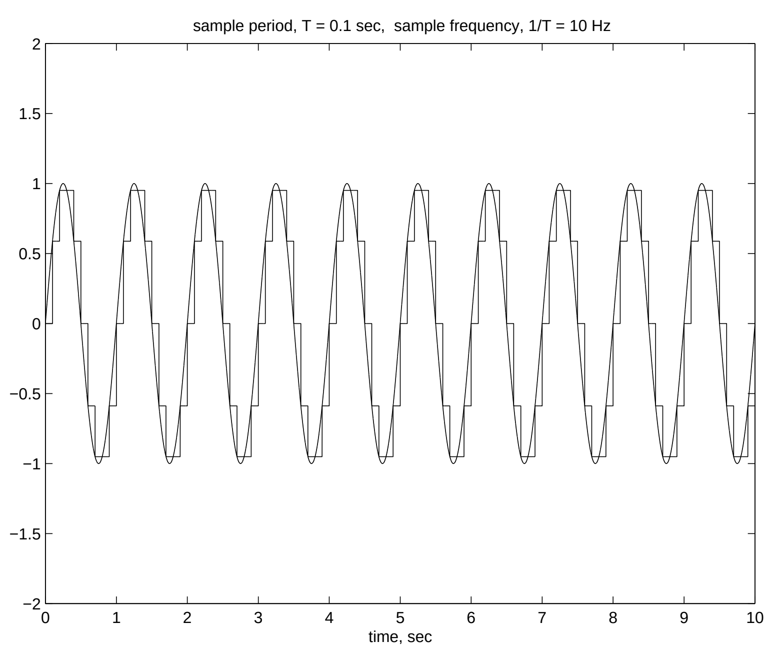
[1] created with MATLAB files staircase_approx.m and simulate_ZOH.mdl

Fast Sampling

- Compare fast and slow sampling[2]
- Input: a 1 Hz sinusoid, $\sin(2\pi t)$, sampled at 20 Hz



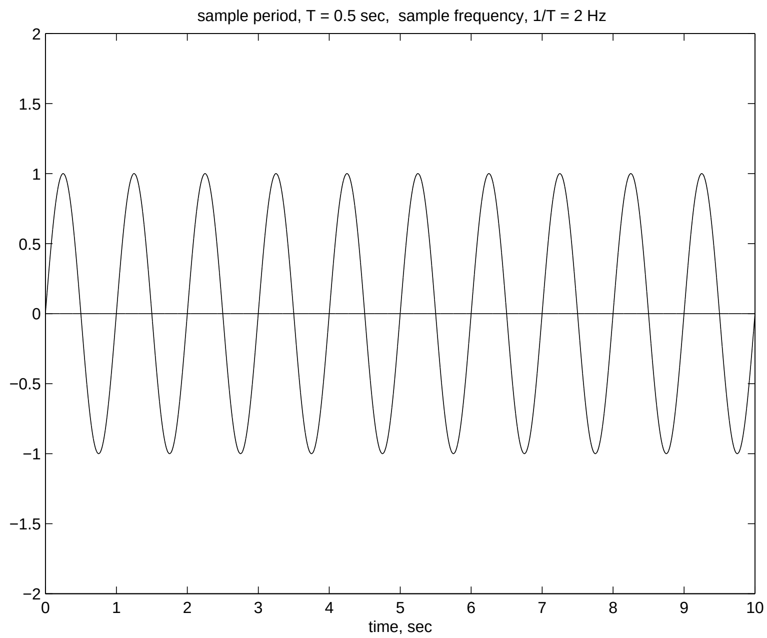
- Input: a 1 Hz sinusoid, $\sin(2\pi t)$, sampled at 10 Hz



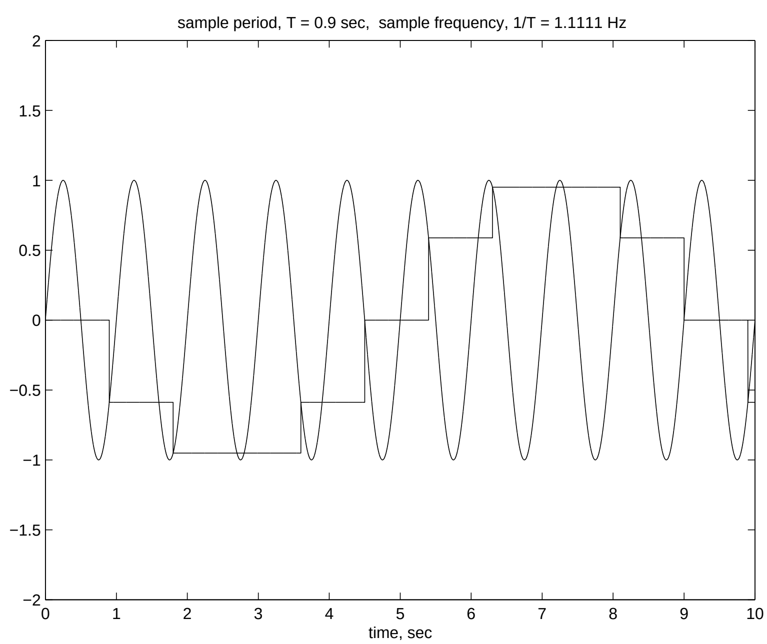
[2] MATLAB m-file fast_slow_sampling.m

Slow Sampling

- Input: a 1 Hz sinusoid, $\sin(2\pi t)$, sampled at 2 Hz



- Input: a 1 Hz sinusoid, $\sin(2\pi t)$, sampled at 1.11 Hz



Observations on Sampling

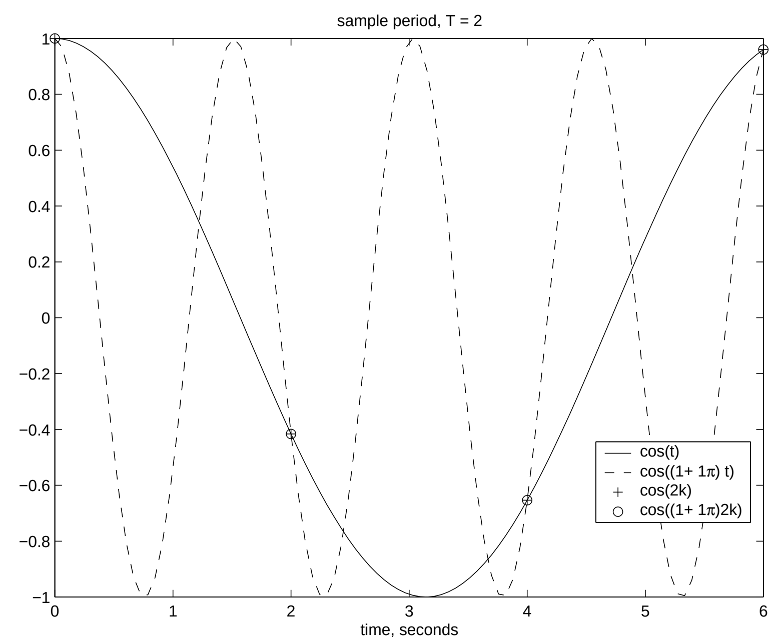
- If sampling period is fast with respect to period of the signal, then the reproduced signal approximates the original signal.
  - slight staircase effect
  - slight time delay

- If sampling period is relatively slow, then there are the reproduced signal may differ significantly from the original signal.
  - It may equal zero!
  - It may look like a periodic signal of equal amplitude but longer period.

- Other issues[6]
  - irregular sampling interval
  - synchronizing sampling with the signal

Aliasing

- Suppose we have two analog signals whose values are identical at the sample points. *Then their digital representations will also be identical.*
  - Impossible to reconstruct original signal from its digital representation.
  - Any algorithm on the CPU will be unable to distinguish between signals.
  - Especially problematic when sampling noisy analog signals.
  - $\sin(2\pi t)$ sampled at $0, 0.5, 1, 1.5, \ldots$ seconds is indistinguishable from zero!
  - $\cos(t)$ and $\cos((1 + \pi)t)$ are identical at $0, 2, 4, \ldots$ seconds[3]!



- A higher frequency signal that “masquerades” as a low frequency signal after sampling is said to be *aliased*.

[3]MATLAB m-file aliasing.m

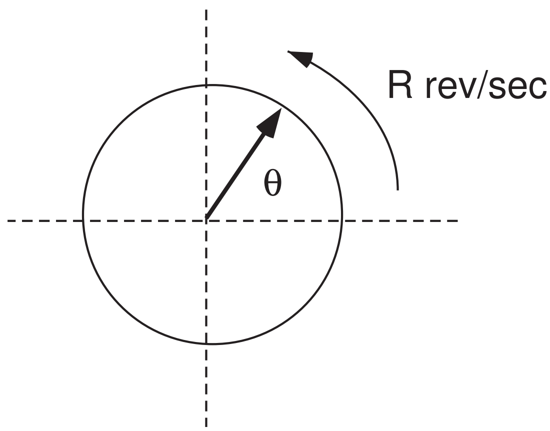
Effects of Aliasing

- Aliasing is a type of information distortion that results from *undersampling*.
- Questions:
  1. How fast must one sample an arbitrary signal to avoid aliasing?
  2. When is aliasing likely to be a problem in sensor interfacing?
  3. How does one minimize the effects of aliasing?
- We shall return to these questions after an example and a review of some ideas from signals and systems.

Example: Consider a video of a rotating wheel marked with an arrow, and made with a camcorder at a rate of 30 frames/second [8]...

Aliasing and the Wheel,I

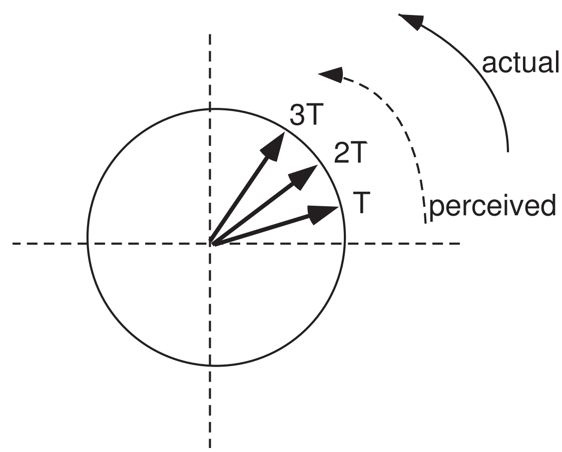
- The effects of aliasing can be striking...
- Consider a wheel rotating counterclockwise (CCW) at R rev/seconds.
- Suppose we
  - View the wheel with a strobe light every T seconds, or
  - Use a camcorder to make a video with one frame every T seconds.



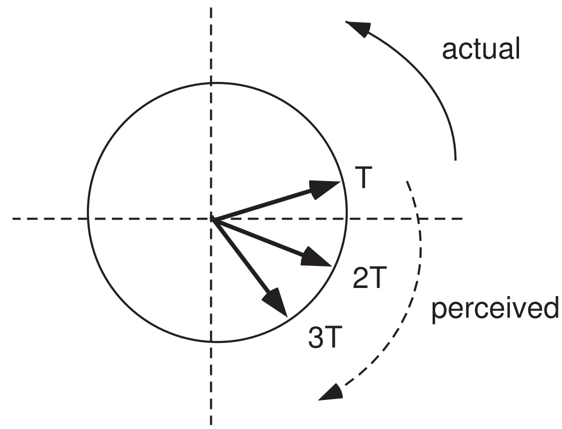
- Depending upon the relative values of T and R, the wheel may appear to be
  - rotating CCW – as we expect to see
  - stationary – not moving!
  - rotating clockwise (CW) – backwards!

Aliasing and the Wheel, II

- We visually determine the direction of motion by noting the difference between consecutive measurements of the position of the arrow.
- If T is *fast* with respect to the speed of rotation, then motion appears to be CCW:

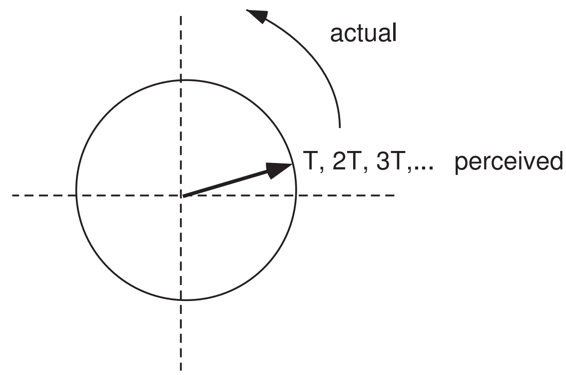


- If T is *slow* with respect to the speed of rotation, then motion appears to be CW:

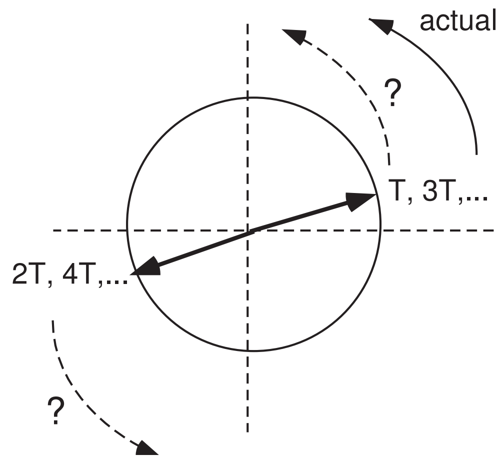


Aliasing and the Wheel, III

- At an even slower value of T, wheel appears to be stationary:

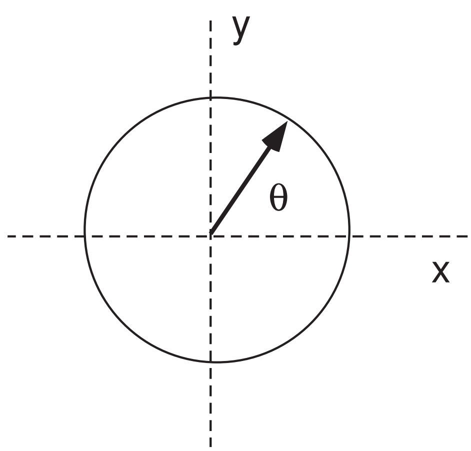


- At an intermediate value of T, we are only confused:



Aliasing and the Wheel, IV

- Suppose the wheel rotates CCW at a fixed rate R rev/sec. Can we determine the maximum value of T so that the wheel always seems to be rotating (and rotating CCW)?
- Terminology
  - sample period, T seconds
  - sampling frequency, $f = 1/T$ Hz or $\omega_s = 2\pi/T$ rad/sec
  - rotation rate, R rev/sec, or $2\pi R$ rad/sec
- Position of wheel in (x, y) coordinates is given by



$$x(t) = \cos(2\pi Rt)$$

$$y(t) = \sin(2\pi Rt)$$

$\Rightarrow$ Taking a picture of the wheel every T seconds is equivalent to "sampling" a sine wave every T seconds

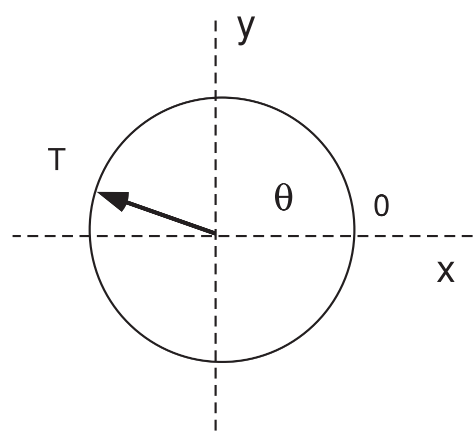
Aliasing and the Wheel, V

- It takes $1/R$ seconds for the wheel to make a complete revolution.
- Suppose that initially $\theta(0) = 0^\circ$. Hence if we sample at $T = 1/R$ samples/second, then the position coordinates at the sample times kT, $k = 1, 2, 3...$ satisfy

$$x(kT) = \cos(2\pi k) = x(0) = 1$$

$$y(kT) = \sin(2\pi k) = y(0) = 0$$

  $\Rightarrow$ the wheel looks as though it were stationary
- To determine the correct direction of rotation, we need to take *at least one* sample before it reaches the halfway point:



- The wheel reaches $\theta = 180^\circ$ in $1/2R$ seconds, hence we require
  - sample period $T < 1/2R$ sec
  - sample frequency $\omega_s > 4\pi R$ rad/sec ($f > 2R$ Hz)
- Later we shall rederive this result from the *Shannon sampling theorem* [6, 8]

Fourier Series

- Consider a periodic time signal $f(t), t \geq 0$, with period T: $f(t) = f(t + kT), k = 0, 1, 2, \ldots$.
- Examples:
  - sine wave
  - square wave
  - sawtooth wave
- Then $f(t)$ may be expressed as a sum of (possibly infinitely many) sines and cosines.
- Terminology
  - T: period of signal
  - $\omega_0 = 2\pi/T$: frequency in rad/sec
  - $f = 1/T$: frequency in Hz
- Then

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t))$$

- More terminology
  - Fourier coefficients: a_i, b_i
  - DC term: a_0
  - fundamental: $n = 1$, sinusoids of frequency ω_0
  - harmonics: $n > 1$, sinusoids of frequency $> \omega_0$

Examples of Fourier Series

- Example: A sine wave with period T

$$f(t) = \sin\left(\frac{2\pi}{T}t\right)$$

is its own Fourier series expansion

- Unit amplitude square wave with period T has Fourier expansion

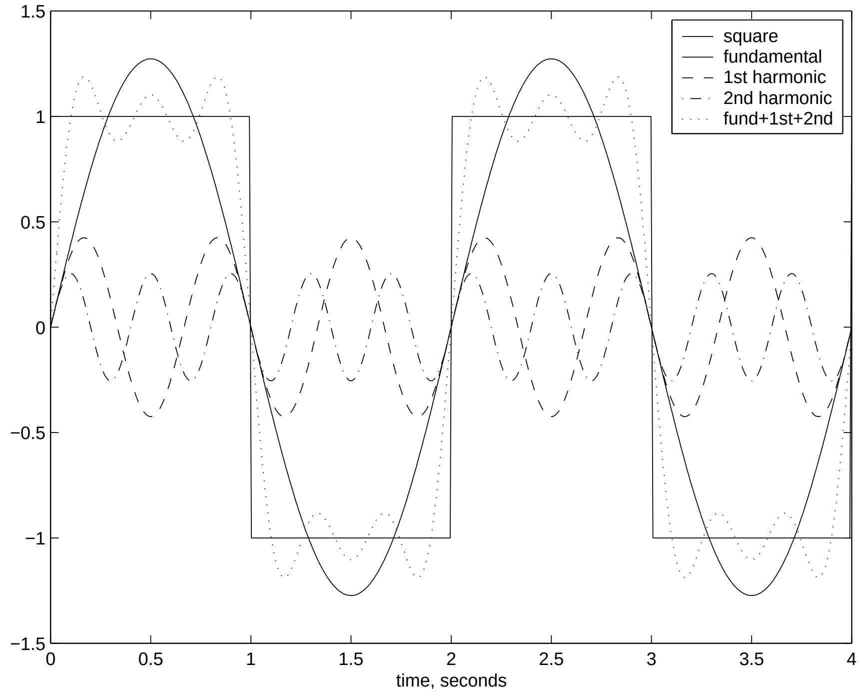
$$f(t) = \sum_{\substack{n=1 \\ n,odd}}^{\infty} \frac{4}{n\pi} \sin(n\omega_0 t)$$

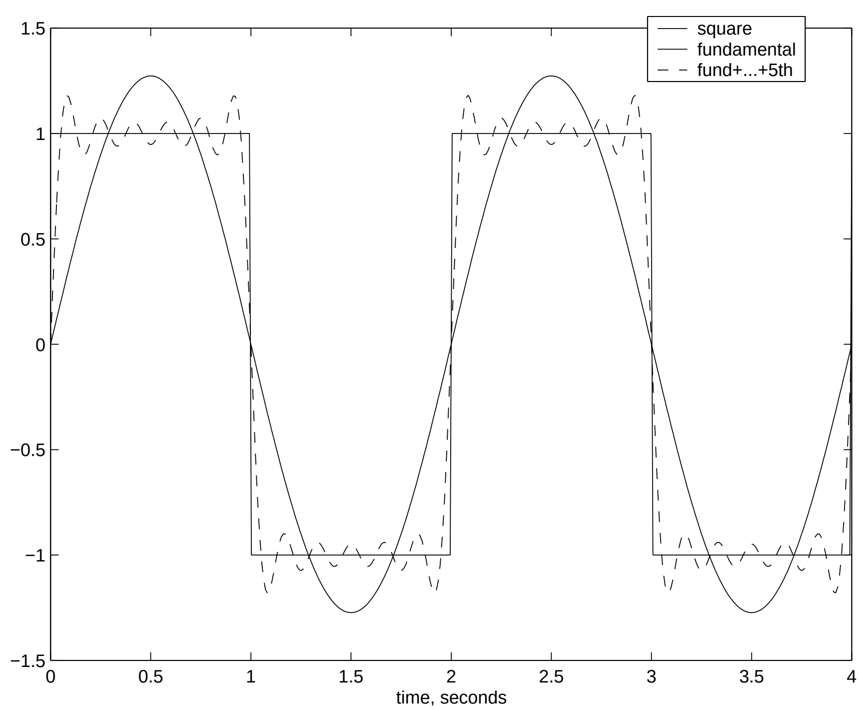
where $\omega_0 = 2\pi/T$ is the frequency of the square wave in rad/sec ($f = 1/T$ is the frequency in Hz)

- Fundamental: $n = 1$, $\frac{4}{\pi}\sin(\omega_0 t)$
- 1st harmonic: $n = 3$, $\frac{4}{3\pi}\sin(3\omega_0 t)$
- 2nd harmonic: $n = 5$, $\frac{4}{5\pi}\sin(5\omega_0 t)$

More Terms ⇒ Better Approximation

Fourier series of a square wave with period $T = 2$ seconds[4].

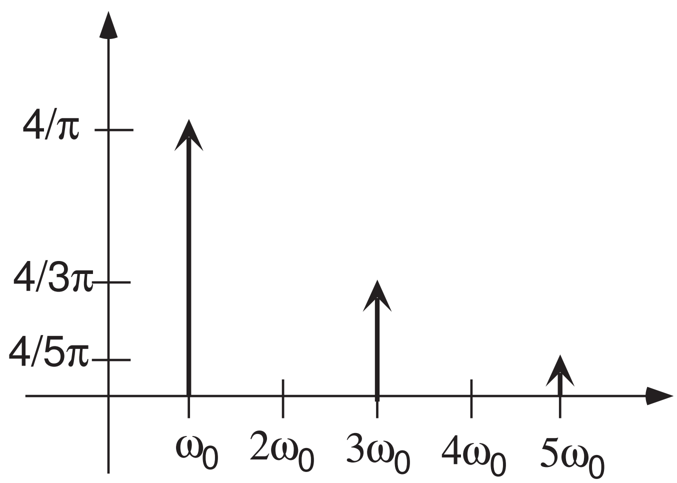




[4] Matlab m-file sq_wave.m

Frequency of a Signal

- Consider a periodic signal, such as a square wave, that has "sharp corners".
- In general, many high frequency terms are needed to construct such "sharp corners". In fact, any signal with relatively abrupt changes will contain high frequencies, even if the changes are not discontinuous.
- It is useful to sketch the location, and relative amplitude, of the various frequency components of a signal
- Example: unit amplitude square wave

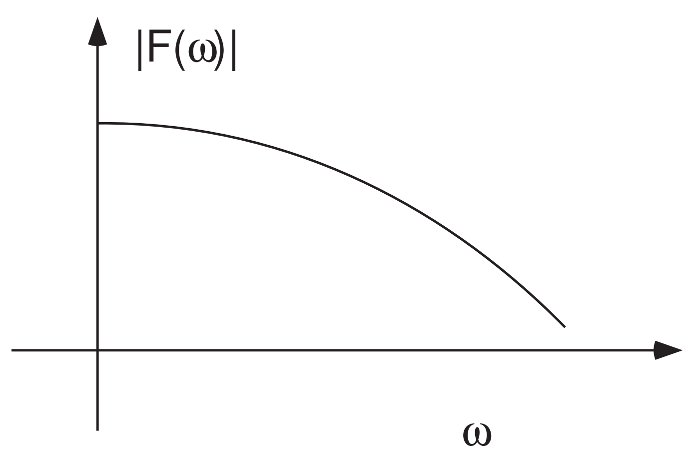


Fourier Transform

- Most signals are *not* periodic. Nevertheless, it is possible to think of "almost any" signal as the sum of sines and cosines of *all* frequencies.
- Fourier transform [7]: Under certain conditions, we can write

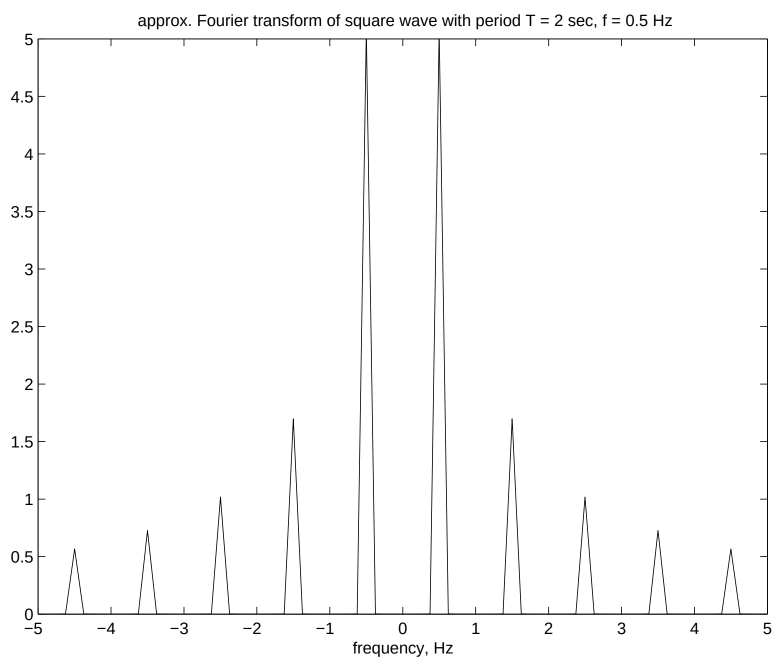
$$\begin{aligned} F(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{j\omega t}dt \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\cos(\omega t)dt + \frac{j}{2\pi}\int_{-\infty}^{\infty} f(t)\sin(\omega t)dt \end{aligned}$$

- We will not need any of the details of the Fourier transform. However, it is important to remember that time signals may be given a frequency representation.
- Can visualize the frequency content of a signal by plotting $F(\omega)$ as a function of frequency:



Fourier Transform of a Periodic Signal

- The Fourier transform of a sinusoid of frequency f Hz consists of two "delta" functions located at frequencies $\pm f$ Hz.
- The frequency response of a square wave consists of "delta" functions corresponding to all frequency components of the Fourier series expansion of the square wave.
- Example[5]: Square wave of period $T = 2$ seconds, $f = 0.5$ Hz has frequency components at $\pm f, \pm 3f, \pm 5f, \ldots$. The Fourier transform of a square wave may be approximated using algorithms from [7]



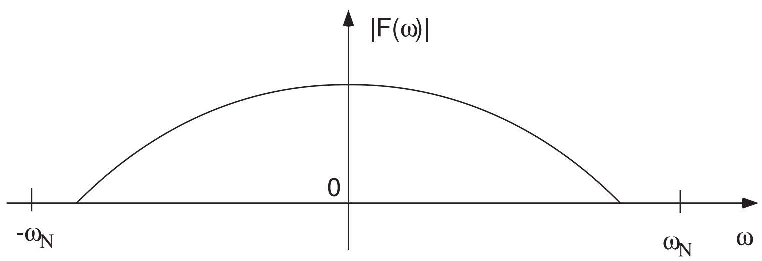
[5]MATLAB m-file sq_wave.m

Frequency Response in Embedded Systems Applications

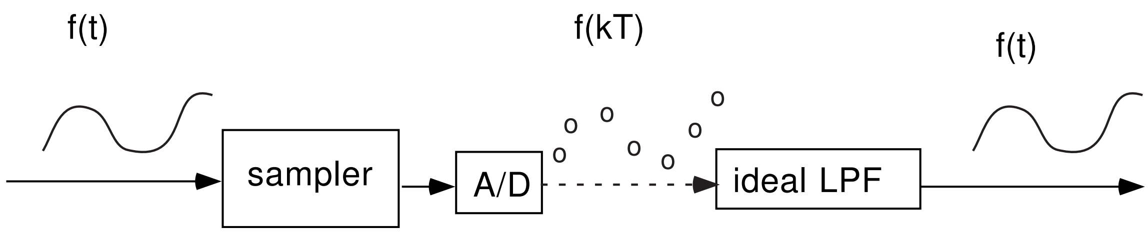
- Many embedded systems for control, communications, and signal processing applications – and anything to do with audio or video – require knowledge of *frequency content* of signals.
- an important class of embedded processors – DSP chips – has a special architecture that allows rapid computation of the frequency response of a signal using the Fast Fourier Transform (FFT) algorithm.
- Knowledge of frequency content is needed to design the interface electronics for an embedded system. For example, circuits that implement lowpass filters to remove unwanted high frequencies.
- Frequency response ideas arise in the study of sampling and aliasing, and in the use of Pulse Width Modulation (PWM) to drive a DC motor.

Shannon Sampling Theorem

- Recall Question 1: How fast must we sample to avoid aliasing?
- Shannon's Theorem [6, 8]
  - Consider a signal $f(t)$ with frequency response $F(\omega)$.
  - Suppose we sample $f(t)$ periodically, with period T sec, and define the *Nyquist frequency* $\omega_N = \pi/T$ radians/second ($f_N = 1/T$ Hz).



  - If $F(\omega) = 0$, for $|\omega| \geq \omega_N$, then it is possible to reconstruct $f(t)$ *exactly* from its samples $f(kT)$.
- Reconstruction requires an *ideal lowpass filter*:



- In practice, reconstruction can only be done approximately, because perfect reconstruction requires *all* samples of the signal, even those in the future!
- Nevertheless, this result tells how fast we must, in principle, sample to avoid aliasing: at least twice as fast as the highest frequency in the signal!

Aliasing and the Wheel, VI

- Suppose that the wheel rotates at R rev/sec, or $2\pi R$ rad/sec.
- Then position coordinates

$$x(t) = \cos(2\pi Rt)$$
$$y(t) = \sin(2\pi Rt)$$

  are sinusoids with frequency $\omega_0 = 2\pi R$.
- Nyquist says that to avoid aliasing we sample fast enough that

$$\omega_0 < \omega_N = \frac{\pi}{T} \quad \text{rad/sec} \qquad \Rightarrow \quad T < \frac{1}{2R} \quad \text{sec}$$

- Same result as we derived before!

Nyquist and Embedded System Applications

- A frequency analysis is done of each analog signal that must be measured with a sensor and represented in digital form.
- Although the signals will have energy at all frequencies, usually the "information" lies in some low frequency range, say $\omega < \omega_0$.
- If possible, set the sample period T so that the Nyquist and sampling frequencies satisfy

$$\omega_N = \frac{\pi}{T} > \omega_0 \quad \Leftrightarrow \quad \omega_s = \frac{2\pi}{T} > 2\omega_0$$

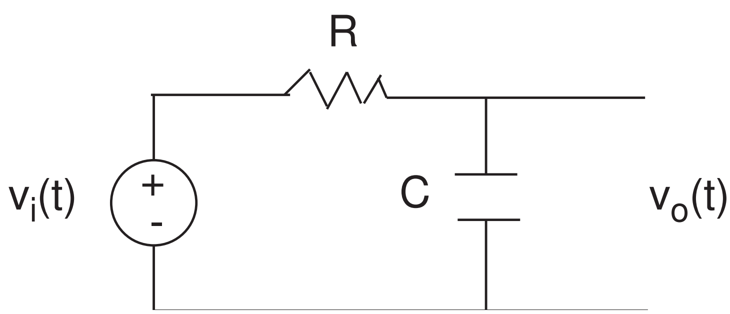
  (usually, we set sampling frequency $\omega_s > (5 - 10)\omega_0$, twice as fast is only the theoretical limit)

Problems with Aliasing

- When is aliasing likely to be a problem?
- Almost all signals are corrupted by noise
  - 60 Hz hum
  - EMI from spark ignition
  -
- Often the noise is at a higher frequency than the information contained in the signal. If the noise is at a sufficiently high frequency, it will get "aliased" to a lower frequency, and corrupt the signal we are trying to measure.
- How to resolve?

Frequency Response Functions

- a linear filter has a frequency response that determines how it responds to periodic input signals
- Example: RC circuit



  - frequency response function

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

  - magnitude, or gain

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

  - phase

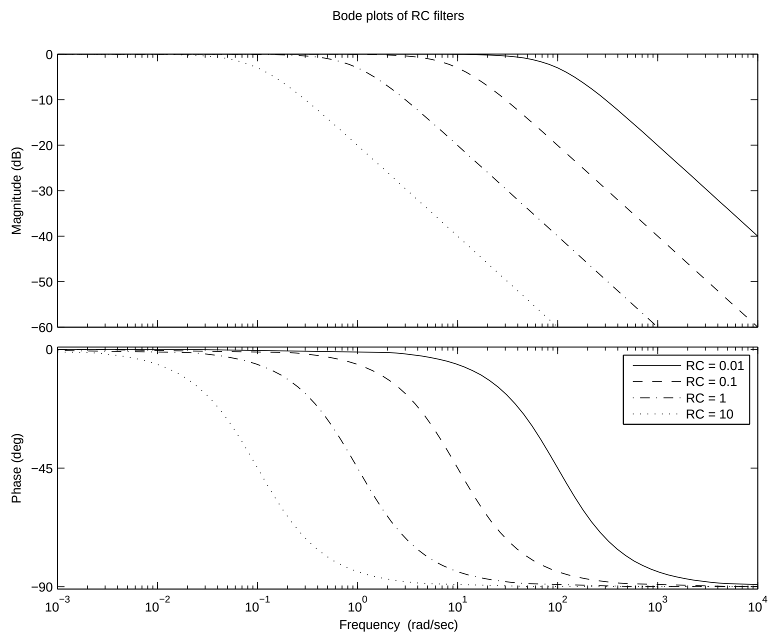
$$\angle H(j\omega) = -\tan^{-1}(\omega RC)$$

- After transients die out, the steady state response of the filter to a sinusoid is determined by it frequency response function:

$$v_i(t) = \sin(\omega_0 t) \Rightarrow v_o(t) \rightarrow |H(j\omega_0)| \sin(j\omega_0 t + \angle H(j\omega_0))$$

Gain and Phase Plots

- Bode plots: gain and phase vs frequency[6]

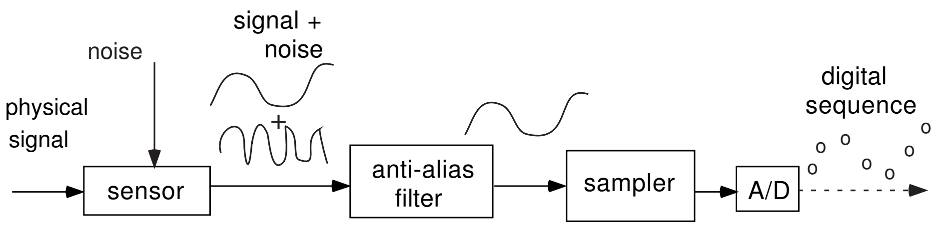


- Lowpass filter
  - passes low frequencies
  - attenuates high frequencies
  - introduces phase lag
- Bandwidth of RC filter proportional to $1/RC$

[6] MATLAB m-file RC_filter.m

Anti-Aliasing Filters

- Potential solution to aliasing problem: "anti-aliasing filters" that are inserted before the sampler to remove high frequencies



- Commercial devices often have an AA filter built in, but may need to build another one to configure the frequency response for the application.
- Problems:
  - may not have frequency separation between signal and noise
  - phase lag in control applications

References

[1] http://hyperphysics . phy-astr . gsu . edu / hbase/electronic/adc . html#c3.

[2] D. Auslander and C. J. Kempf. *Mechatronics: Mechanical Systems Interfacing*. Prentice-Hall, 1996.

[3] W. Bolton. *Mechatronics: Electronic Control Systems in Mechanical and Elecrical Engineering, 2nd ed.* Longman, 1999.

[4] J. Feddeler and B. Lucas. *ADC Definitions and Specifications*. Freescale Semiconductor, Application Note AN2438/D, February 2003.

[5] M. Garrard and P. Ryan. *Design, Accuracy, and Calibration of Analog to Digital Converters on the MPC5500 Family*. Freescale Semiconductor, Application Note AN2989, July 2005.

[6] S. Heath. *Embedded Systems Design*. Newness, 1997.

[7] E. Kamen and B. Heck. *Fundamentals of Signals and Systems using MATLAB*. Prentice Hall, 1997.

[8] J. H. McClellan, R. W. Schafer, and M. A. Yoder. *DSP First: A Multimedia Approach*. Prentice-Hall, 1998.