Analysis of Tap-Induced Oscillations Observed in an Electrical Distribution System

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***Abstract*—Slow oscillations, with a period of around 15 min, were observed in an 11-kV electrical distribution system. Initial investigations were unable to reproduce the oscillations. Through the use of hybrid system modelling and analysis concepts, however, it was determined that the oscillations resulted from interactions between tap-changing transformers and switched capacitors. The hybrid systems framework was needed to account for the nonsmooth (switched) nature of these interactions. Trajectory sensitivities were used to identify influential parameters. It was found that existence of the oscillations was dependent upon factors that included system fault level, capacitor rating, and regulator deadband limits. In all cases, grazing-type conditions separated oscillatory from steady-state behavior. A system of cascaded tap-changing transformers was also investigated, with the hybrid systems framework revealing coexisting limit cycles.**

***Index Terms*—Grazing phenomena, hybrid dynamical systems, limit cycles, postmortem analysis, tap-changing transformers, voltage oscillations.**

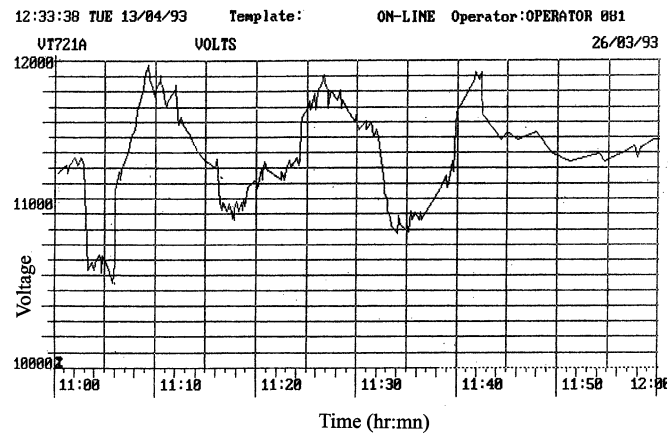

Fig. 1. Oscillations observed in a distribution system.

I. Introduction

A NUMBER of years ago, voltage oscillations were observed in an 11-kV distribution system. A chart recording taken at the time is shown in Fig. 1. Two features set this event apart from usual power system oscillations: 1) behavior was quite nonsmooth, and 2) the oscillations were slow, having a period of approximately 15 min. The event was clearly related to distribution system interactions involving tap-changing transformers, switched capacitors, and loads.

The oscillations occurred at a time when the transmission system serving the area was in a weakened state. The transmission switching procedure was not unusual though, so the local electricity utility was keen to fully understand the factors contributing to these self-excited oscillations. Why did they occur this time but not previously when conditions were apparently similar? Why did they abruptly cease? Could devices be retuned to avoid repeat occurrences?

Investigations at the time were limited by available numerical tools and were inconclusive [1]. Simulation packages were primarily tailored to smooth dynamic behavior, with little attention given to the accurate treatment of discrete events. Yet inspection of the voltage plot of Fig. 1 reveals significant nonsmoothness, due presumably to the discrete behavior of transformer tap changing and capacitor switching. *Ad hoc* event handling procedures were established, but with mixed success. Furthermore, many of the parameters describing the distribution system were uncertain. An exigent process was employed in [1] to identify parameters that were potentially influential in sustaining the oscillation. Even so, the oscillatory behavior could not be reproduced.

Recent investigations, using trajectory sensitivities [2] and grazing analysis [3], have revealed the reason for the earlier difficulties; sustained oscillations arise for only a small region of the multidimensional parameter space. The insights offered by these newer analysis techniques allow a more conclusive understanding of the behavior displayed in Fig. 1. This paper provides an overview of these investigations.

In order to understand the oscillations, modelling must take account of the discreteness of tap changing and capacitor switching. Just as importantly, the time delays and deadbands of the controllers associated with these devices must be considered. The resulting behavior involves intrinsic interactions between discrete events and the continuous dynamics of other components such as dynamic loads. Systems that exhibit such interactions have become known as *hybrid dynamical systems* [4], [5], with the oscillations of Fig. 1 providing an example of a *hybrid limit cycle*.[1] Recently, considerable attention has focussed on the analysis of hybrid systems. Accordingly, simulation tools have seen significant advances in modelling sophistication [7], [8].

[1]A formal definition of limit cycles, though in the context of smooth systems, is provided in [6].

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Trajectory sensitivities describe the changes in a trajectory that result from perturbations in the underlying parameters and/or initial conditions [2], [9]. These sensitivities are well defined for hybrid dynamical systems and can be computed efficiently for large systems. The relative magnitudes of trajectory sensitivities enable differentiation of parameters that are influential from those that are not. Accordingly, the uncertainty associated with parameters that exhibit small sensitivities can be largely ignored. This enables attention to be focussed on the (generally much smaller) subset of parameters that exert a meaningful influence on behavior.

This paper provides, in Section II, a brief discussion of modelling features that are essential for capturing hybrid system behavior. This is followed in Section III by analysis of the limit cycle phenomenon of Fig. 1. Grazing concepts are employed in a sensitivity analysis to examine the influence of various parameters on limit cycle existence. These concepts are used to investigate a system of cascaded tap-changing transformers in Section IV. This system is known to exhibit oscillatory (limit cycle) behavior when tap-changing behavior is described by a smooth approximation. It is shown that a more accurate hybrid systems framework reveals the coexistence of numerous other limit cycles. Conclusions are provided in Section V.

II. Hybrid System Model

A. Motivation

The voltage regulator of a tap-changing transformer monitors the regulated-bus voltage for deviations from a desired deadband. Transitions outside the deadband enable (start) a timer, while transitions back to within the deadband reset and disable the timer. If the timer reaches its trigger setting, a discrete tap change occurs and the timer is reset. A detailed investigation of tap-changing control can be found in [10]. Clearly discrete dynamics play a major role. Yet voltage behavior is also influenced by the continuous dynamics of many devices, such as loads.

B. Model

Numerous formal models, such as automata [5] and Petri nets [11], exist for rigorously describing hybrid system dynamics. However, those representations are not immediately amenable to numerical implementation. Analysis of power system dynamics requires a nonrestrictive model formulation that is capable of capturing the full range of continuous/discrete hybrid system dynamics, yet is computationally efficient. It is shown in [2] and [8] that these specifications are met by a model that consists of a set of differential-algebraic equations, adapted to incorporate switching of the algebraic equations, and impulsive (state reset) action. This DA *Impulsive Switched* (DAIS) model has its genesis in the familiar DAE model

$$\dot{x} = f(x, y) \tag{1}$$

$$0 = g(x, y) \tag{2}$$

where $x \in \mathbb{R}^n$ are dynamic states, $y \in \mathbb{R}^m$ are algebraic states, $f : \mathbb{R}^{n+m} \to \mathbb{R}^n$, and $g : \mathbb{R}^{n+m} \to \mathbb{R}^m$.

When a capacitor switches, or a timer is activated as a consequence of voltage deviating beyond a regulator deadband, the algebraic equations (2) must switch to account for the system alteration. Considering a single switching event, (2) should be replaced by

$$0 = g(x, y) \triangleq \begin{cases} g^-(x, y), & s(x, y) < 0 \\ g^+(x, y), & s(x, y) > 0 \end{cases} \tag{3}$$

where the superscripts "−" and "+" index the two sets of algebraic equations. A switching event coincides with a zero crossing of the trigger function $s(x, y)$. Note that the concept of crossing is important. If the trajectory just touches (grazes) the triggering surface

$$\mathcal{S} = \{(x, y) : s(x, y) = 0\} \tag{4}$$

then behavior beyond that point is indeterminate, as switching may or may not occur [3].

The precise behavior of the model at a switching event is not completely defined by (3) and requires further explanation. Let the event occur at trigger time τ, and define τ^- as the time instant just prior to τ, and τ^+ as the instant just after τ. The limit values of the states can then be expressed as

$$\begin{aligned} x^- \equiv x(\tau^-) &:= \lim_{t \uparrow \tau} x(t) \\ x^+ \equiv x(\tau^+) &:= \lim_{t \downarrow \tau} x(t) \\ y^- \equiv y(\tau^-) &:= \lim_{t \uparrow \tau} y(t) \\ y^+ \equiv y(\tau^+) &:= \lim_{t \downarrow \tau} y(t) \end{aligned}$$

where $t \uparrow \tau$ implies $t < \tau$ approaches τ from below, and $t \downarrow \tau$ implies $t > \tau$ approaches τ from above. Two sets of variables (x^-, y^-) and (x^+, y^+) are required to fully describe behavior at an event [7].

Switching events cannot efficiently capture all forms of discrete behavior. Activities such as transformer tap changing or protection timer resetting [12] are best modelled by impulsive action that introduces discrete jumps into the dynamic x-states.[2] Such behavior has the form of an impulse, which can be described by a reset equation

$$x^+ = h(x^-, y^-), \quad \text{when } s(x, y) \text{ crosses } 0. \tag{5}$$

Event triggering occurs when the trigger function $s(x, y)$ encounters zero, or steps instantly through zero, i.e., undergoes an instantaneous sign change, due to a preceding event. The values of x and y just prior to the reset event are denoted by x^- and y^-, while x^+ refers to the value of x just after the reset event. Away from this zero crossing condition, the evolution of the dynamic x-states is described by the differential equations (1).

This overview of the DAIS model has neglected some technical details, which can be found in [8]. It should be emphasized that the DAIS model is nothing more than a formalization of switching/reset models found in commercial power system

[2]Impulsive differential equations are common in the modelling of mechanical systems; an interesting example is provided in [13].

simulators. This formalization does, however, facilitate the efficient computation of trajectory sensitivities that were vital for investigating the oscillatory behavior of Fig. 1.

C. Example: Tap-Changing Transformer

The behavior of a tap-changing transformer voltage regulator, as described in Section II-A, can be modelled in the DAIS formulation as

$$\dot{x}_t = y_1 \quad (6)$$
$$\dot{n} = 0 \quad (7)$$
$$0 = nV_1 - V_2 \quad (8)$$
$$0 = y_2 - (V_2 - V_{\min}) \quad (9)$$
$$0 = y_3 - (T_{\text{tap}} - x_t) \quad (10)$$
$$0 = y_1 \qquad y_2 > 0 \quad (11)$$
$$0 = y_1 - 1 \quad y_2 < 0 \quad (12)$$

together with reset equations

$$\left.\begin{array}{r} x_t^+ = 0 \\ n^+ = n^- \end{array}\right\} \text{when } y_2 \text{ crosses } 0 \quad (13)$$
$$\left.\begin{array}{r} x_t^+ = 0 \\ n^+ = n^- + n_{\text{step}} \end{array}\right\} \text{when } y_3 \text{ crosses } 0. \quad (14)$$

For clarity, only those equations that are active during low voltage excursions have been included. The deadband lower threshold is given by $V_{\min}$, and the timer setting by T_{tap}.

III. Distribution System Analysis

A. Background

The voltage oscillation of Fig. 1 occurred in a section of the power system that consisted primarily of three 132-kV substations: Kemp, PtMq, and Tar. A simplified representation of this subsystem is shown in Fig. 2. The measurement of Fig. 1 was recorded on the Tar feeder, at the location identified by V_{meas}. The 132-kV substations were normally connected to the rest of the grid via feeders 65, 63, and 6F. Each substation included tap-changing transformers for supplying the 66/33/11-kV sub-transmission and distribution systems. Switched capacitors were installed on the distribution system for supporting the voltage.

A thorough data collection process pieced together a consistent overview of the prevailing system conditions [1]. However information describing transformer voltage regulators, capacitor switching controls, and load characteristics was extremely limited. The need for such a detailed investigation of distribution system dynamics had not previously arisen, so many records were unavailable. A sensitivity analysis was therefore conducted to identify which parameters were most influential. Section II-B provides a discussion.

At the time of the voltage oscillation event, feeder 6F was out of service for maintenance. The loads being supplied by the three substations were each approximately $(10 + j2)$ MVA.

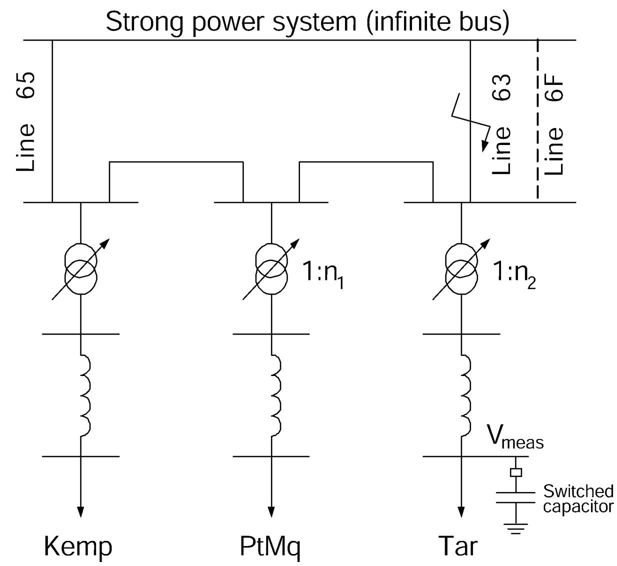

Fig. 2. Supply system (simplified) one-line diagram.

These loads were modelled as voltage-dependent dynamic loads [14]. Tap positions were initialized to $n_1 = n_2 = 1$ pu. Feeder 63 was outaged at 5 s, leaving the system radially fed through feeder 65.

In subsequent analysis, tap-changers and switched capacitors are modelled as event-driven switching devices. It was found that their time-delay elements were particularly important. A discrete-event description of tap-changer controls is discussed in [10], and the DAIS model is provided in Section II-C. Switched capacitors have similar deadband/timer interactions, though typically the timer is set for a longer delay.

B. Sensitivity Analysis

Trajectory sensitivities were used to explore the influence on dynamic behavior of the many uncertain parameters. These sensitivities describe (approximately) the change in the trajectory that would result from a (small) change in parameters and/or initial conditions [2], [9]. Small sensitivities imply that a parameter has negligible effect on behavior. Conversely, large sensitivities identify influential parameters [15].

The critical unknown parameters in this investigation related to transformer and capacitor switching controls, and load dynamics. Sensitivity analysis showed, not surprisingly, that the voltage V_{meas} was strongly influenced by the tap-changing transformer regulating the voltage on feeder Tar, the switched capacitor at the measurement bus, and by the aggregate load on that feeder. Other switched capacitors on that feeder had smaller ratings, so their influence was relatively insignificant.

Sensitivity analysis also showed that the tap-changing transformer at the adjacent substation PtMq exerted a non-negligible influence on V_{meas}. The aggregate load on that feeder was also relatively significant, but capacitor switching was insignificant. The dynamics associated with feeder Kemp had negligible effect on V_{meas}.

Even though numerous parameters of the system were uncertain, trajectory sensitivities identified the crucial parameters upon which attention should focus. The model was simplified

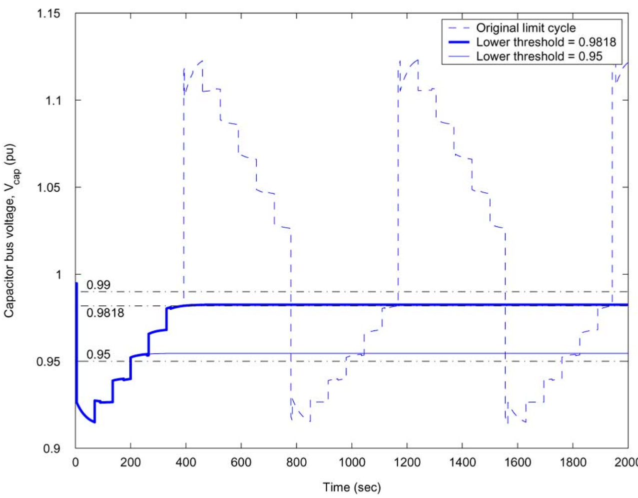

Fig. 3. Influence of deadband lower limit $V_{\min}$ on behavior.

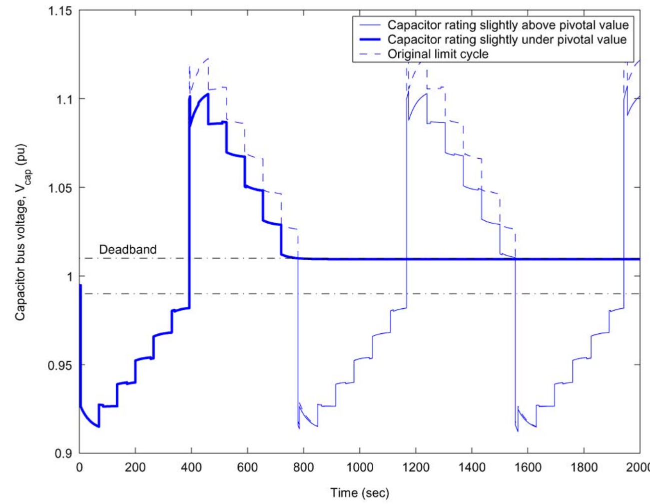

Fig. 4. Influence of capacitive susceptance on behavior.

accordingly, with tap control and load dynamics modelled on feeders PtMq and Tar, and capacitor switching modelled on feeder Tar only.

The simplified model did not immediately yield oscillatory behavior. As mentioned previously, the region of parameter space that gave rise to oscillations was small, and the transition from non-oscillatory to oscillatory behavior was abrupt. The pivotal conditions that dictated the onset of oscillations corresponded to a *grazing* phenomenon. This aspect of the analysis is explored further in Section III-C.

C. Pivotal Parameter Values

Throughout the investigation, it was found that variation of certain parameters could result in structurally different forms of behavior. These pivotal values, at which behavior changed significantly, were crucial to the development of an understanding of parametric influences, and ultimately in replicating the observed limit cycle. Such pivotal situations are closely related to *grazing bifurcations* [3], [16], where a system trajectory just touches a switching hypersurface, triggering the corresponding event.[3]

Consider for example the lower limit of the capacitor controller deadband, $V_{\min}$, with Fig. 3 providing an illustration. By lowering $V_{\min}$ to 0.95 pu, switching operations ceased after 250 s, and the capacitor voltage reached a steady-state value of $V_{\rm cap} = 0.955$ pu. Although the undesirable oscillations were suppressed, the load voltage profile was unacceptably low. Sizing the deadband is clearly a trade-off between maintaining a good post-outage voltage profile and suppressing sustained oscillations. The pivotal case is obtained by raising the value of $V_{\min}$ until capacitor switching is only just initiated. This situation corresponds to a *time-difference* form of grazing [3], with the trajectory just touching the deadband $V_{\min} = 0.9818$ pu at the exact moment the capacitor timer reaches its trigger value. This pivotal case is depicted in Fig. 3 as a thicker solid line. An incremental delay in reaching the deadband would result in capacitor energization and the consequent onset of limit cycle behavior.

[3]The implication is that for an incremental parameter change, there is no contact with the switching hypersurface, so no event triggering occurs.

The rating of the switched capacitor was also found to have an important influence on limit cycle creation. Referring to Fig. 4, a reduction in the capacitor rating led to a smaller voltage overshoot. For small values of capacitive support, transformer tapping was able to restore the voltage to within the deadband before the capacitor switched out. The pivotal value corresponds to the voltage entering the deadband at exactly the instant of capacitor switching. Fig. 4 shows the voltage response with the capacitor rating incrementally above that pivotal value (sustained oscillations) and incrementally below (oscillation extinguished.)

D. Results

The investigation found that for realistic parameter values, the outaging of line 63 could in fact initiate sustained oscillations (limit cycle behavior) in distribution system voltages. Fig. 5 shows the time response of the capacitor bus voltage, while Fig. 6 provides a phase portrait view of behavior, with tap position n_2 plotted against capacitor voltage $V_{\rm cap}$.[4] These oscillations show the same qualitative behavior as the measured response in Fig. 1. Clearly the real system exhibited much richer dynamic behavior, with many effects unmodelled in the simplified representation. Also, some features of Fig. 1 may be artifacts of the rather antiquated chart recording technology. Nevertheless, the general behavioral trends and oscillation period match well.

Referring to Fig. 5, following the outage of feeder 63, the capacitor and tap-changer voltages fell outside their deadbands. This activated all respective timers. Tap-changing transformers increased their tap positions until about 400 s, when the capacitor controller timed out and the capacitor switched into service. However, this switching action drove voltages beyond deadband upper limits. Consequently, tap-changers started decrementing their tap positions until the capacitor switched out of service at about 800 s. Although capacitor switching is meant to aid tap changing, it continually resulted in voltage over- and undershoot. Steady-state conditions could not be achieved, with the

[4]In Fig. 6, the transition from one tap position to the next is shown as a continuous line, even though taps take discrete values. The continuous line is provided to elucidate the limit cycle behavior.

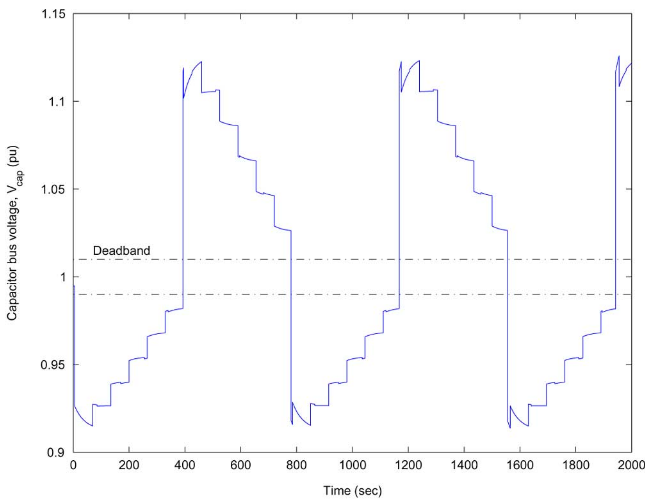

Fig. 5. Capacitor voltage trajectory.

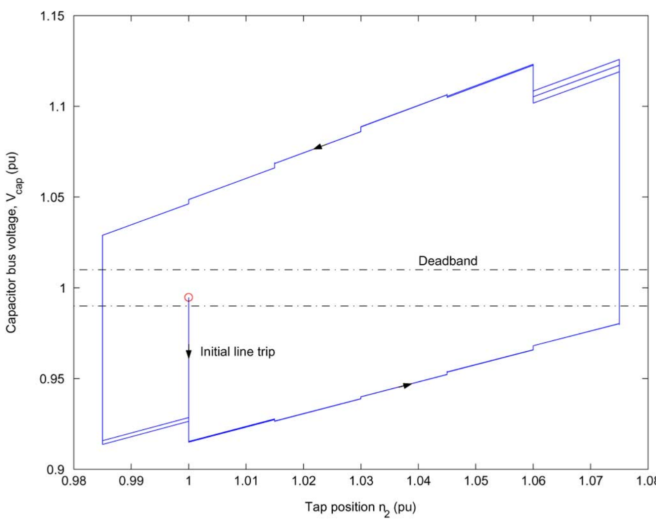

Fig. 6. Trajectory in $n_2 - V$ plane.

tap-changer/capacitor interactions giving rise to a hybrid limit cycle. This limit cycle behavior is clearly evident in Fig. 6.

The voltage over/under-shoot due to capacitor switching is a direct consequence of the reduced fault level resulting from the outage of feeders 63 and 6 F. When the system is stronger, with at least one of those feeders in service, capacitor switching causes much smaller voltage steps, eliminating the problem.

IV. System of Cascaded Transformers

A. Background

The earliest attempts [1] to reproduce the oscillations of Fig. 1 unsuccessfully made use of approximate continuous-tap models for transformer tapping. It is apparent from the results of Section III that more exact tap-changing representations give rise to richer forms of dynamic behavior. This observation is also true for the system of cascaded tap-changing transformers of Fig. 7. This system was used in [17] to illustrate various bifurcation phenomena. It was shown that transformer tap interactions induce limit cycle behavior over a wide range of parameter values.

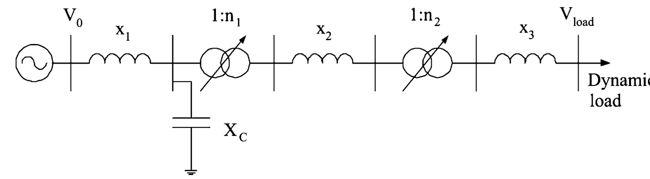

Fig. 7. System of cascaded transformers feeding a dynamic load.

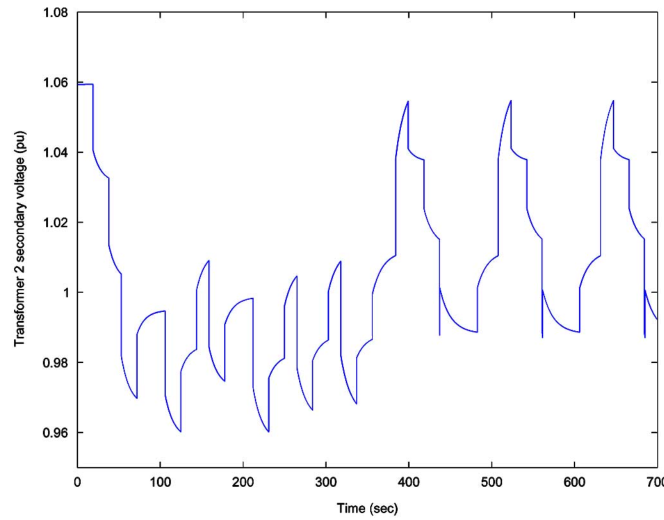

Fig. 8. Load-side voltage for the cascaded transformer system.

Network parameters for this system are given in [17]. The load was modelled with recovery dynamics [14] driving the real and reactive (dynamic) states x_p and x_q. Corresponding load time constants were $T_p = 10$ s and $T_q = 5$ s, respectively. Both transformers were represented by the discrete-event model of Section II-C, with timer settings of $T^1_{\text{tap}} = 53$ s and $T^2_{\text{tap}} = 19$ s. In both cases, the deadband upper and lower limits were 1.01 pu and 0.99 pu, respectively.

An initial condition was chosen with loads in steady state and transformer taps set to $n_1 = n_2 = 1$. Under these conditions, voltages at the tap-changer regulated buses were outside their respective voltage deadbands. Tap changing was therefore initiated. Fig. 8 shows the time response of the voltage on the load side of the second transformer, while Fig. 9 provides the $x_p - x_q$ phase portrait view of behavior. The initial point is marked by an "a." The system undergoes an initial transient phase before settling into the periodic response of the limit cycle. The limit cycle is a consequence of quite complicated interactions between the tap changer controls (due to their different time constants) and load dynamics. In particular, the deadbands play an important role in sustaining the oscillations.

B. Poincaré Maps

Limit cycles can be accurately and efficiently located using shooting methods that build on Poincaré map concepts [6], [18]. Such techniques extend naturally to hybrid (nonsmooth) limit cycles [16], [19]. The essential features of Poincaré maps follow from Fig. 10, where Γ denotes a limit cycle and Σ is a hyperplane transversal to Γ. The trajectory emanating from x^* will again encounter Σ at x^*. Likewise, the trajectory emanating from an arbitrary point x will encounter Σ at $P(x)$, where $P(\cdot)$

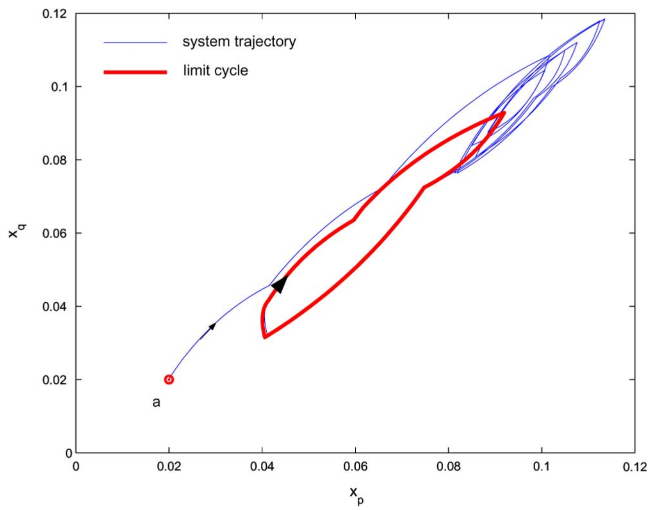

Fig. 9. State-space behavior of the cascaded transformer system, shown in the $x_p - x_q$ plane.

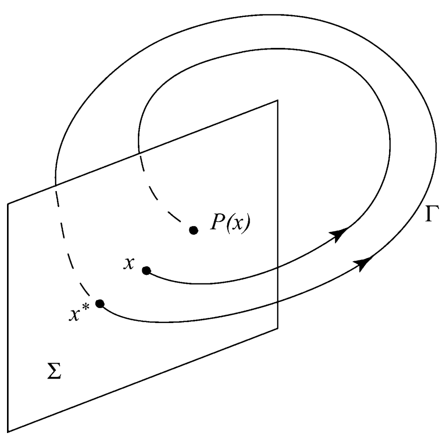

Fig. 10. Poincaré map concepts.

is referred to as the Poincaré map. Shooting methods locate the limit cycle by solving for the point x^* that satisfies

$$P(x^*) - x^* = 0. \tag{17}$$

This numerical process can reliably locate unstable limit cycles, and limit cycles that have very small regions of attraction. It can therefore reveal features that are not obvious from simulation.

With that in mind, the shooting method was used to determine whether other limit cycles coexisted with that of Fig. 9. By choosing various different Poincaré hyperplanes Σ, other limit cycles were located. Fig. 11 shows the original limit cycle from Fig. 9, together with two new ones. The three choices of Σ are also shown. Characteristic multipliers (eigenvalues) of all these limit cycles lie within the unit circle, suggesting they are all locally stable. However, limit cycles 2 and 3 have very small regions of attraction, and they could not easily be located by simulation. Various initial conditions were tried, but most trajectories converged to limit cycle 1. This emphasizes the advantage of shooting methods over simulation for locating limit cycles.

C. Grazing Phenomena

It was found in Section III-C that the deadband lower limit $V_{\min}$ had a significant effect on the nature of system behavior,

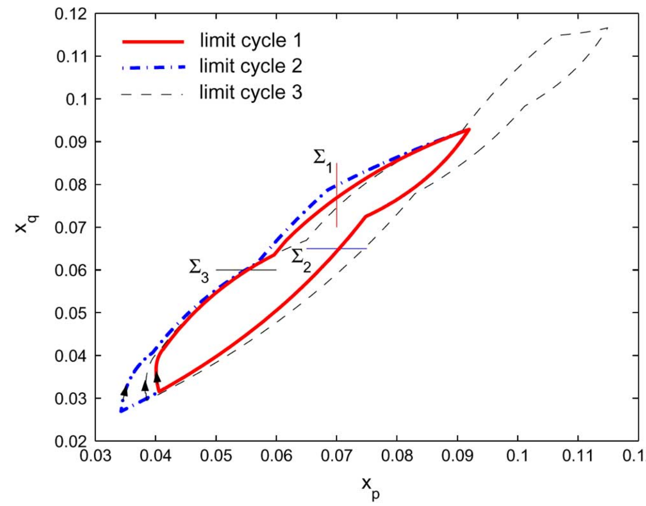

Fig. 11. Different Poincaré hypersurfaces locating different limit cycles.

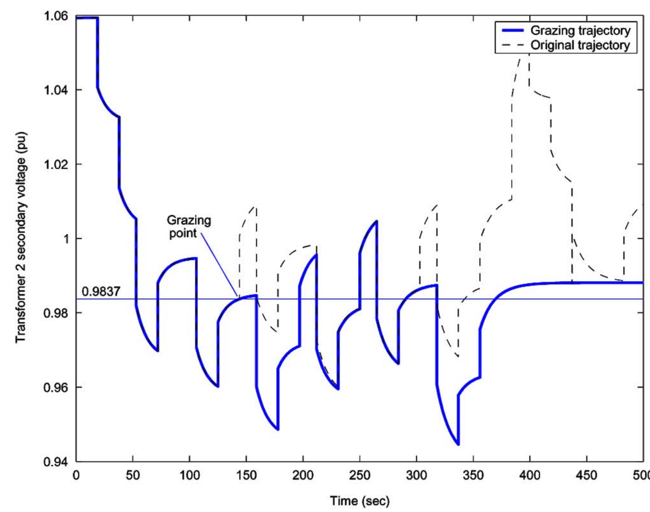

Fig. 12. Different forms of behavior arising from grazing.

and in particular on the creation of limit cycles. A similar situation arises for this system of cascaded transformers.

As the deadband lower limit is reduced from $V_{\min} = 0.99$ pu, a sequence of grazing-related bifurcations occurs. One of those bifurcations, corresponding to $V_{\min} = 0.9837$ pu, is presented in Fig. 12. The grazing point is labelled. At that point, the voltage recovers to the deadband at the exact instant the timer triggers a tap change. With a slight increase in $V_{\min}$, the voltage would still be outside the deadband when the timer triggered a tap change, leading ultimately to a limit cycle. On the other hand, a small decrease in $V_{\min}$ would enable the voltage to recover to within the deadband before a tap change could be triggered, with steady-state conditions the final outcome. This latter case is shown as the thick line in Fig. 12. The grazing bifurcation separates these two totally different outcomes.

Accurate modelling of tap-changer controls, as event-driven switching devices, reveals a richness in behavior that is not captured by simplified continuous-tap models.

V. Conclusion

The controls associated with distribution system devices, such as tap-changing transformers and switched capacitors, introduce discrete events into system dynamics. Simultaneous action of these devices may give rise to sustained oscillations, in the form of hybrid (nonsmooth) limit cycles. This is particularly so when the supply system has been weakened by feeder outages.

This paper reports on an actual oscillation that was observed in a distribution system. By careful modelling of tap-changer and switched-capacitor controls, it was possible to obtain a good match between measured and simulated behavior. In retrospect, the factors underpinning the oscillation are not unexpected. The process of identifying those factors, however, highlighted the importance of establishing appropriate models and of understanding parametric influences.

Throughout the investigation, it was found that variation of certain parameters could result in structurally different forms of behavior. These pivotal values at which behavior changed were crucial in understanding parametric influences and ultimately in replicating the observed limit cycle. Such pivotal situations are closely related to *grazing bifurcations*, where a system trajectory just touches a switching hypersurface.

Knowledge gained from investigating the observed oscillation was used to analyze the behavior of a system of cascaded transformers. Again it was found that accurate event-driven modelling revealed a richness in behavior that was not captured by simplified continuous-tap models. A shooting method, that was based on Poincaré map concepts, was used to locate coexisting limit cycles.

Numerous analytical tools underlie the investigations. Those tools have been presented here at the conceptual level, though complete details can be found in [16].

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