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EFFECTS OF LOAD DYNAMICS ON POWER SYSTEM DAMPING

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Abstract

This paper explores the interaction between dynamic loads and power systems. Based on a generic model of dynamic loads, the frequency response of such load is discussed. Also, the frequency response of the system is investigated. It is shown that the dynamics of the load provide a feedback path which can influence the damping of the modal oscillations of the system. This influence is very dependent on load parameters as well as the system configuration. Under some circumstances damping can be improved, but under other conditions dynamic load may cause a decrease in damping.

Keywords: load dynamics, electromechanical oscillations, system damping

1 Introduction

Electromechanical oscillations have been, and continue to be, an ongoing source of concern in the operation of interconnected power systems [1, 2, 3]. Because of growth in demand, and the difficulty in building new transmission and generation plant, systems tend to be operating nearer to their maximum capability limits. This increase in system loading is often reflected as a decrease in the level of damping of electromechanical modes.

Electromechanical oscillations occur in interconnected power systems because of synchronous generators swinging against each other. In an n-machine system there will be (n-1) electromechanical modes. These oscillation modes result from the rotors of machines, behaving as rigid bodies, oscillating with respect to one another, using the transmission system between the machines to exchange the oscillation energy [4]. Different types of oscillating behaviour are possible. Local mode oscillations occur between a single machine, or sometimes a small group of machines, and the rest of the system. Typical oscillation frequencies range from 0.7 to 2.0 Hz [3, 5]. Another type of oscillation occurs between large groups of machines. They are called inter-area or system oscillations. Inter-area modes are usually in the range of 0.1 to 0.8 Hz [3, 5].

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While the local modes are fairly well understood and can be analyzed in a satisfactory way, the inter-area modes, and the factors influencing them, are not fully understood. There are still questions in respect of the underlying dynamic processes.

It is important that power system planners and operators are able to predict the level of damping of significant system modes. Otherwise it is possible that a system configuration could be proposed that is actually unworkable due to poorly damped oscillations [6]. Further Flexible AC Transmission System devices are being considered as a method of damping oscillations [7, 8, 9]. To properly evaluate the usefulness of these devices, and also for their tuning, planners must have confidence in damping predictions.

However, rescent studies around the world, e.g. [10], have found that the measured level of damping is often less than that predicted by studies. That is, in response to a disturbance, the system oscillates for longer than expected. It is therefore difficult to have confidence in the studies. Generally good agreement has been obtained between the measured and predicted oscillation frequencies. Where this has not been the case, the system has tended to oscillate more slowly than predicted. Studies have been corrected by modelling motor load around the system to increase the overall system inertia. This appears to work well. Unfortunately similar techniques have not been helpful in improving damping predictions. It is difficult to alter modelling to reduce damping.

Close attention has always been given to modelling of generators and associated controls, and transmission equipment. However the representation of loads has not traditionally been considered so thoroughly, even though it has been shown that loads can have a significant impact on analysis results [11]. The accurate modeling of loads is a difficult task due to several factors [12] such as: large number of diverse load components, ownership and location of load devices in customer facilities that are not directly accessible to the electric utility, changing load composition with time of day and week, seasons and weather, lack of precise information on the composition of loads, uncertainties regarding the characteristics of many load components etc.

System studies have traditionally used static load models, given by,

$$P_d = P_o(V/V_o)^{n_p} \tag{1}$$

$$Q_d = Q_o(V/V_o)^{n_q} \tag{2}$$

or combinations of constant impedance, constant current and/or constant power load models [12]. However many loads are not statically dependent on voltage, but actually have some dynamic characteristic. The generic form of response of many loads to a voltage step is shown in Figure 1. The initial power step, the final power mismatch and the rate of recovery of the load are parameters which can vary greatly across different load types. A mathematical model of this form of load response is given in [13, 14].

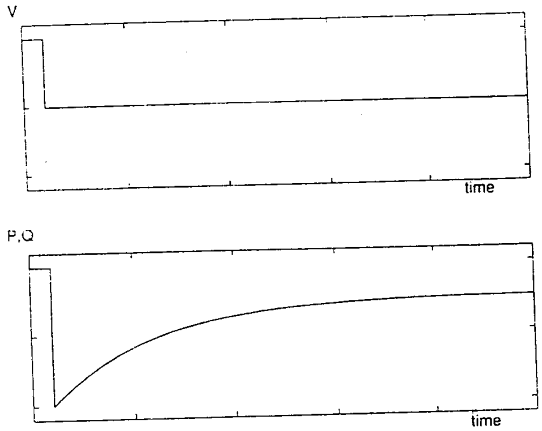

Figure 1: Typical load response to a step in voltage

In this paper the interaction between dynamic loads and the power system is explored. Intuitively it is expected that variation of the load at a bus will cause the voltage to vary. But variation of the voltage will result in a delayed variation of the load. It is shown that this feedback behaviour can have a noticeable effect on damping.

The structure of paper is as follows. Section 2 describes a nonlinear aggregate dynamic load model and its general properties. Section 3 discusses the power system properties viewed through Bode plots of system gain and phase shift for different operating points. Section 4 gives the details of dynamic load - power system interaction.

2 Nonlinear dynamic load model

Results from the measurement of actual power system loads [14] show that the response of the load to a step change in voltage is of the general form shown in Figure 1. Real and reactive power have qualitatively similar responses. They are dynamically related to the voltage. In the sequel, only the active power response will be treated as dynamic. The reactive power will be considered to be statically related to voltage.

The main characteristics of the response are primarily that a step in power immediately follows a step in voltage. The power then recovers to a new steady state value. The recovery is approximately of exponential form. The size of the step and the steady state value are nonlinearly related to voltage [14]. A model which describes this form of response was proposed in [13, 14] as:

$$T_p \dot{P}_d + P_d = P_s(V) + k_p \dot{V} \quad (3)$$

or

$$T_p \dot{P}_d + P_d = P_s(V) + T_p \sigma_p(V) \dot{V} \quad (4)$$

Let

$$P_t(V) = \int_0^V \sigma_p(\tau) d\tau + C_o \quad (5)$$

where C_o is some constant, and let

$$x_p = P_d - P_t(V) \quad (6)$$

Then the model can be written as:

$$T_p \dot{x_p} = P_s(V) - P_d \quad (7)$$

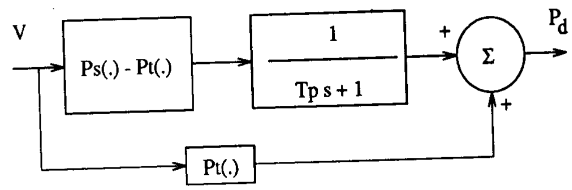

Figure 2: Block diagram representation of the dynamic load model

or

$$T_p \dot{x_p} = -x_p + P_s(V) - P_t(V) \quad (8)$$

A block diagram representation of this load model is shown in Figure 2.

The functions $P_s(V)$ and $P_t(V)$ can be defined as:

$$P_s(V) = P_o(V/V_o)^{n_{ps}} \quad (9)$$

$$P_t(V) = P_o(V/V_o)^{n_{pt}} \quad (10)$$

where V_o and P_o are nominal voltage of the bus and corresponding power of the load respectively and n_{ps} and n_{pt} are static and transient voltage exponents. They are generally in the range of 0 to 2, and 1 to 2.5 respectively [12, 14]. Time constant T_p, which characterizes the recovery response of the load, can be chosen to represent different types of loads. For loads consisting predominantly of industrial drives, such as conveyer belts, or for responses of industrial plants such as aluminium smelters, T_p is in the range of up to one second. For induction machines, T_p can be in the range of a few seconds, whilst for tap-changers and other control devices it is in the range of minutes, and for heating load in the range of hours. In this paper the predominant interest is in loads that have time constants which are the same order of magnitude as modal oscillations, i.e., up to around two seconds. Similar equations could be used to describe the behaviour of reactive power, with different time constants and voltage exponents.

This model has until now been used in the study of voltage collapse behaviour [15]. In that situation some disturbance causes a step reduction in system voltage. The recovery of load from that voltage may lead to further deterioration of the voltage, and ultimately system collapse. In the present investigation, the interest is not in step changes to the voltage, but rather in sinusoidal variation of the voltage. Referring to Figure 2, one would expect the load power P_d to vary periodically in response to sinusoidal variation of the voltage. Variation of P_d could then feedback through the system to reinforce the voltage variation. This feedback mechanism is described in Section 4. Crucial to this dynamic process though, is the gain and phase shift between V and P_d.

In these investigations of system damping, only small disturbances are considered. Therefore the load model (6), (8) can be linearized. Linearization yields:

$$T_p \Delta \dot{x_p} = -\Delta x_p + (P_o/V_o)(n_{ps} - n_{pt}) \Delta V \quad (11)$$

$$\Delta P_d = \Delta x_p + (P_o/V_o) n_{pt} \Delta V \quad (12)$$

Introducing the Laplace operator, (11) can be written,

$$(T_p s + 1) \Delta x_p = (P_o/V_o)(n_{ps} - n_{pt}) \Delta V \quad (13)$$

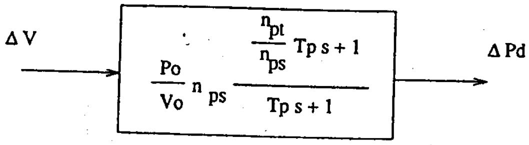

Figure 3: Block diagram representation of linearized load model

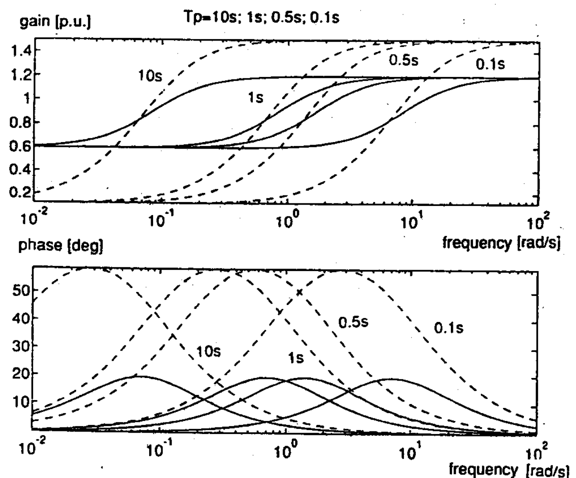

Figure 4: Bode plots of magnitude and phase of dynamic load for different time constants and voltage exponents; dashed line-n_{pt}/n_{ps}=15; solid line-n_{pt}/n_{ps}=2

Substituting (12) into (13) and manipulating yields,

$$\Delta P_d = (P_o/V_o)\frac{(n_{pt}T_p s + n_{ps})}{(T_p s + 1)}\,\Delta V \qquad (14)$$

or

$$\Delta P_d = (P_o/V_o)n_{ps}\frac{((n_{pt}/n_{ps})T_p s + 1)}{(T_p s + 1)}\,\Delta V \qquad (15)$$

This relationship is shown in block diagram form in Figure 3. It can be modelled as a lead/lag block.

From (14) or (15) it can be seen that the relationship between ΔP_d and ΔV is influenced by the load time constant T_p and the voltage exponents n_{pt} and n_{ps}. Bode plots of this relationship, for two different values of the ratio n_{pt}/n_{ps} and a number of different values of T_p are given in Figure 4. Notice from (15) and Figure 4 that at low frequencies, the gain approaches $(P_o/V_o)n_{ps}$, i.e., the steady state load characteristic dominates. At high frequencies, the gain approaches $(P_o/V_o)n_{pt}$, so the transient load characteristic dominates. This is consistent with our understanding of the load response. At intermediate frequencies, both the characteristics have an influence on behaviour. It is then that there is some phase shift through the load. The maximum phase shift is dependent on the ratio n_{pt}/n_{ps}. As the ratio becomes larger, so does the maximum phase shift.

The effect of the variation of the time constant T_p is also interesting. For a very large T_p, i.e., a load which responds slowly, the transient load characteristic has the predominant influence on behaviour. For small values of T_p, i.e., a fast load, the steady state characteristic is dominant. This can again be explained from the general form of behaviour exhibited by the load model, see Figure 1. If T_p is large, then the load will take a long time to recover from its transient value to the steady state value. Therefore, except at very low frequencies, the load will never have time to approach the steady state characteristic. For small T_p, the load will recover very quickly. So except for very high frequency variation of voltage, the load will always have time to recover to the steady state characteristic.

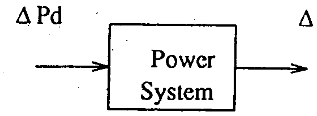

Figure 5: System representation

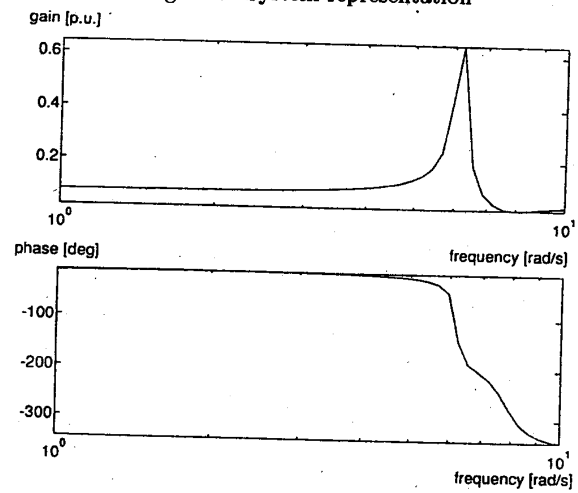

Figure 6: Bode plots of magnitude and phase for the system with P_g=0.9 p.u.;P_d=0.6 p.u. and trans. line length 200km

3 Power System description

It is well known that a change in load demand causes a change in voltage of the system. To study the effects of dynamic loads, one needs to be able to quantify the relationship between load changes and voltage variations. To do this the frequency response of the system can be used. Consider the representation of the system shown in Figure 5, where load power is the input and voltage of the load bus is the output. The frequency response of the system in the form of Bode plots, which is an appropriate way of looking at this characteristic, can be easily obtained. Typical Bode plots for two different operating points are given in Figures 6 and 7.

This approach to looking at the system is applicable for any sized system. However Figures 6 and 7 were produced for a single machine infinite bus system of the form shown in Figure 8. This simple system was used as it allowed easier exploration of the system–load interaction. The generator was represented by a third order machine

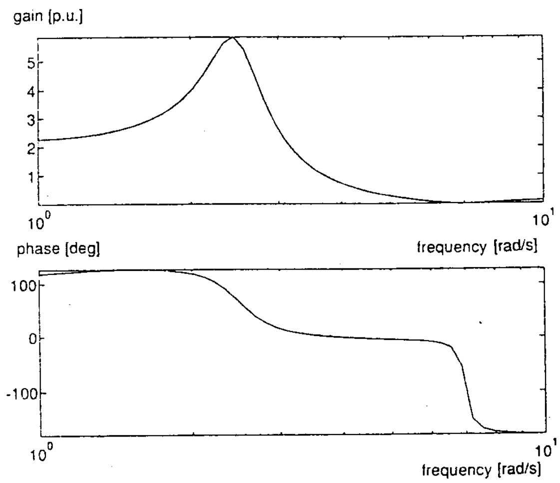

Figure 7: Bode plots of magnitude and phase for the system with P_g=-0.3 p.u.;P_d=0.6 p.u. and trans. line length 300km

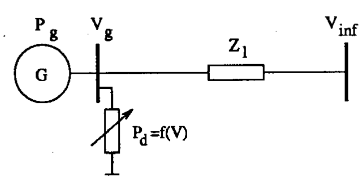

Figure 8: Single-machine infinite-bus system

model, and the transmission line was assumed to be lossless. Only the active power load was considered as having dynamics. Reactive power load was set to zero. This allowed a modified Heffron - Phillips model [1, 16] to be developed. The detailed derivation of the model is given in [17].

It can be seen from Figures 6 and 7, that Bode plots can be quite different for different operating points. These plots clearly highlight the natural resonant frequency of the system. Also it can be seen that the system which is more heavily loaded, Figure 7, has a larger peak in gain. This indicates that the system is more sensitive to load variations. It is reflecting the fact that as systems become more heavily stressed, they become more sensitive to parameter changes. There is also a noticeable move of the peak toward lower frequencies. That is another observed characteristic of heavily loaded systems. The effect of an increase in load on the frequency response is shown in Figure 9. This figure shows the trend toward higher peak gains at lower frequency as the system becomes more heavily loaded.

The traditional view of the sensitivity of system voltage to load changes relates to step changes in load. Such sensitivity is primarily determined by the fault level of the bus. In terms of the frequency response of the system in Figure 5, it should be noted that that sensitivity corresponds to the DC component of the responses, given in Figures 6, 7 and 9. If load variation is oscillatory, then dynamics of the system will also influence the sensitivity. Hence the peak at the system resonant frequency. Notice that resonance can greatly amplify the sensitivity.

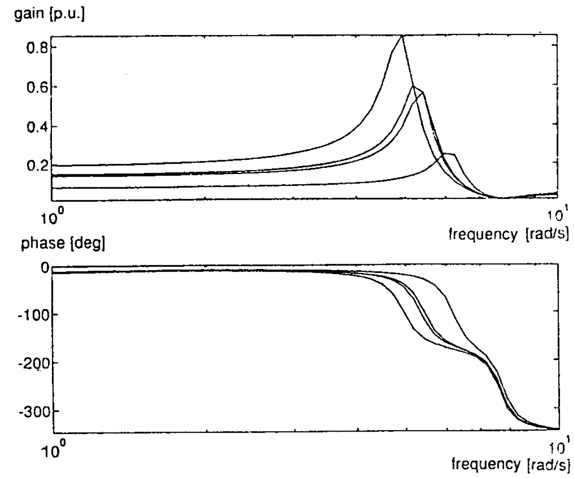

Figure 9: Bode plots of gain and phase of the system for different levels of loading

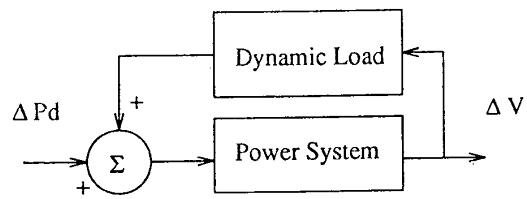

Figure 10: Load - power system interaction

4 Power System - Load Interaction

In Section 2 we focused on dynamic loads and discussed the behaviour of the load power for voltage variations. Figure 3 described this relationship. In Section 3 we focused on the power system and considered the behaviour of voltage for load variation. Figure 5 depicted this case. We are now interested in system behaviour when a dynamic load is connected to the power system. Figure 10 captures diagramatically the interaction.

It can be seen that the load provides a feedback path , and hence has the potential to alter the overall system behaviour. Depending on the load characteristic this feedback may improve damping. But it could also destabilize the system and cause a deterioration in damping. Consider the case where there was a negative phase shift θ^o through the power system, i.e., ΔV lagged ΔP_d by θ^o. From Figure 4 we can see that dynamic load can only ever have a positive phase shift. (This is based on the assumption that $n_{pt} > n_{ps}$. If $n_{pt} < n_{ps}$ phase lag would occur.) So the dynamic load must feed back a component of ΔP_d which is less than θ^o out of phase with the original oscillations. For example if the phase lead of the load was θ^o, then the load would feed back oscillations that were exactly in phase with the original oscillations. Such reinforcement would tend to decrease the damping of the system. The exact effect would depend on the gain through the load. The influence of the load would be reduced for load phase shifts which were not equal to θ^o.

Now consider the case of a positive phase shift through the power system. Because of the positive phase shift through the load, the oscillations in P_d due to the load

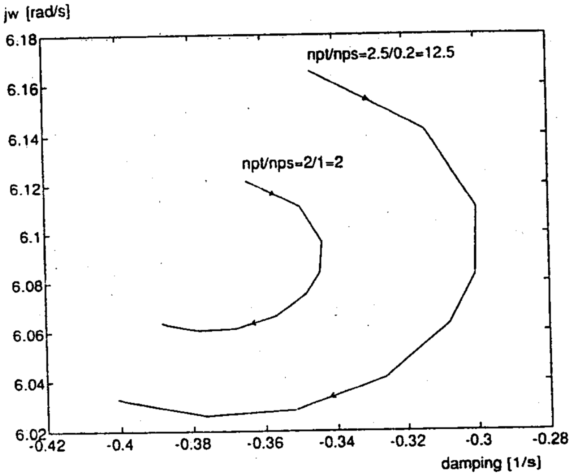

Figure 11: Root locus of the system's electromechanical mode for P_g=0.9 p.u.;P_d=0.6 p.u. and trans. line length 200km, and for different n_{pt}/n_{ps} ratio

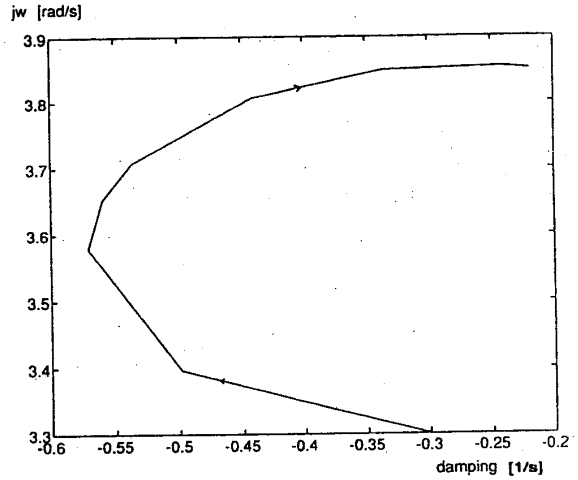

Figure 12: Root locus of the system's electromechanical mode for P_g=-0.3 p.u.;P_d=0.6 p.u. and trans. line length 300km

would be out of phase with the system oscillations by more than θ^o. The contribution of the load to reinforcement of the oscillations would therefore be greatly reduced. In fact, the load contribution could be in anti-phase to the system oscillations. The load would then have a positive effect on damping.

The systems considered in Figures 6 and 7 can be used to illustrate these effects, as well as the effects due to parameter variations. As is shown in Figure 4, variation of time constant and voltage exponents of the load results in variation of the phase shift and gain introduced by the load. It can be expected therefore that the influence of the load on system behaviour will vary with changes to these parameters. In Figures 11 and 12 the effects of time constant and voltage exponent variation are shown as root locuses of the system electromechanical mode.

Consider Figure 11 which relates to the system of Figure 6. It can be seen that as the time constant T_p increases, damping reduces, and reaches its minimum for T_p=0.1 s. After that damping increases with the increase of time constant. This behaviour can be explained by looking at the Bode plots of the load, Figure 4. For T_p=0.1 s the load has maximum phase shift near the system resonant frequency and the gain is about 0.8 p.u.. With further increase of time constant the phase shift for that particular frequency decreases and the gain is almost constant. Note that the system has negative phase shift and the load has positive, therefore the system oscillations are being reinforced by load behaviour. Obviously the level of the reinforcement is determined by the gain through the system and the gain through the load. As the gain of the system gets greater, for weaker systems, the effect of the feedback through the load increases. Also it can be seen from Figure 4 that for the larger n_{pt}/n_{ps} ratio, the load has a larger phase shift and larger gain, which has as a consequence larger variation of damping of the electromechanical oscillations of the system.

The root locus of the electromechanical mode for the weaker system, is given in Figure 12. As would be expected from the system phase and gain responses, Figure 7, the effect of load dynamics on damping is more significant in this case, and the range of changes of damping with time constant variation is greater. In this case the system has positive phase shift and the load too. Therefore the system oscillations are being first damped by load behaviour for small time constants, up to T_p=0.2 s. For this range of T_p, the load has large phase shift and smaller gain at the system resonant frequency. As T_p increases further, the oscillations begin to be reinforced, so damping deteriorates. In this case the phase shift becomes negligible, and the gain is large. In Figures 11 and 12, the arrows on the root locus plots denote the direction of movement of electromechanical mode of the system with increase of T_p from zero to 100 s. With T_p=0 s, the static load model is effectively presented. These results illustrate the discrepancies that can occur if a dynamic load is modelled as being static.

The effect of load modelling on the damping of power systems oscillations is illustrated in Figure 13. A static load model was used for the better damped case. Worse damping can be observed when the load was modelled dynamically.

5 Conclusions

It has been shown in this paper that loads which respond dynamically to voltage variations can have an influence on the damping of electromechanical oscillations. A generic nonlinear dynamic load model has been used to investigate the gain and phase shift between sinusoidal variation of voltage, and the periodic response of the load. It is shown that the load can be represented as a lead/lag block. The parameters of the load model have a significant influence on the frequency response of the load.

It is useful in the investigation of load-system interactions to treat the system as a transfer function, with load deviation as the input, and voltage deviation as the output. The interaction can then be investigated by determining the frequency response of the system and comparing it with the frequency response of the load.

It has been found that depending on load and system parameters, a dynamic load can reinforce oscillations, and so cause a deterioration in damping. It is also possible though, that the load may oscillate out of phase with the system, and so lead to an improvement in damping. It

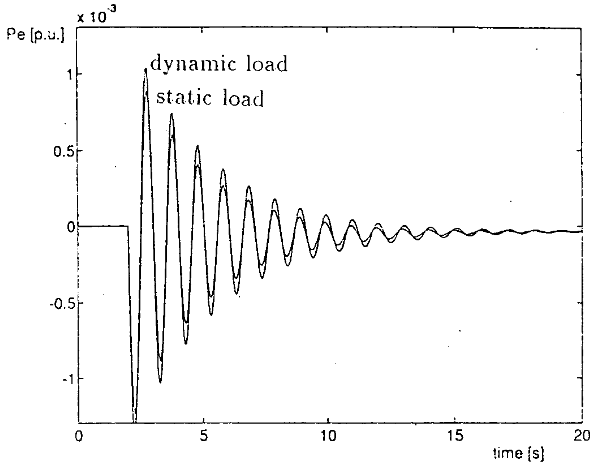

Figure 13: Power response to a step in field voltage for the system with P_g=0.9 p.u.;P_d=0.6 p.u. and trans. line length 200km, for static load and for dynamic load with T_p=0.1s and n_{pt}/n_{ps}=12.5

has also been found that as systems become weaker, they are influenced more by the dynamics of the loads.

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DISCUSSION

S.C. SRIVASTAVA, K.N. SRIVASTAVA, S.N. SINGH and S.K. JOSHI (Department of Electrical Engineering, Indian Institute of Technology, Kanpur, INDIA): The discussers would like to commend the authors for exploring the effects of load dynamics on Power system overall damping. The effect of a dynamic load model from ref.[A] has been studied by several other researchers [B,C,D]. It has been shown that under variation of a parameter such as load demand, the power system undergoes Hopf bifurcation giving rise to subcritical or a supercritical periodic orbit. These periodic orbits further undergo several other kinds of bifurcations, such as cyclic fold bifurcation, period doubling bifurcation etc, leading to chaotic oscillations. It has also been shown [B,C,D] that the system voltage may collapse as a result of boundary crisis. In view of this, the discussers would like to know the nature of dynamic instability the authors have encountered with the dynamic load model considered in their study.

The authors have considered the reactive power to be statically related to bus voltages. The reactive power balance equations will, therefore, appear as algebraic constraint to the solution trajectory defined by differential equation of the system dynamics. The discussers would like to know how the authors' finding will be affected in case of singularity induced bifurcation when the jacobian matrix of partial derivatives of reactive power bus injections with respect to bus voltages becomes singular.

The authors have studied a very simple power system model of two bus considering a linearised third order model of synchronous generator. It will be advantageous to know if the authors' finding will remain valid for larger systems. Discussers would like to request for authors' valuable comments.

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A thorough investigation of the types of dynamic instability that can occur when load is modelled as in this paper has not yet been undertaken. In the line of work reported in this paper, we have only been interested in Hopf bifurcations, i.e., oscillatory instability. The load model has also been used extensively in the analysis of voltage collapse [15]. In that case, interest has been in simple monotonic and oscillatory instability. However we have recently begun investigations of more exotic forms of behaviour. The load model of this paper is similar to that of [A], except that it includes a term to describe load recovery. There is no reason to suspect that the forms of behaviour mentioned by the discussors could not occur with this load model.

The load model used in this paper is composed of a differential equation (7) and an algebraic equation (6). Therefore each load introduces an algebraic constraint from the dynamic load model, and an algebraic constraint describing the reactive power balance. A detailed analysis of algebraic singularity conditions for this model is given in [E]. In general, operating points do not coincide with a singularity induced bifurcation. Algebraic singularity only becomes an issue as the system responds dynamically to some large disturbance. Then the trajectory may encounter the impasse surface, i.e., a surface of algebraic singularity, and terminate. In this paper we are interested in small disturbance analysis of operating points. So, in general, algebraic singularity is not an issue. It would only become an issue if the operating point lay on the impasse surface. However one would then conclude that the modelling was inadequate.

Subsequent to this paper, we have explored the behaviour of an eight bus, four generator power system [F]. The generators were modelled using a sixth order machine model, and AVR/stabilizer control loops were included. Similar findings were obtained. The significance of the machine modelling is explored further in [G].

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