Analysis of the Nordel Power Grid Disturbance of January 1, 1997 Using Trajectory Sensitivities

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Abstract— **This paper uses trajectory sensitivity analysis to investigate a major disturbance of the Nordel power system which occurred on January 1, 1997. The Nordel system is described, and the details of the disturbance are presented. Background to trajectory sensitivity analysis is also provided. Results of the investigation indicate the usefulness of trajectory sensitivities for exploring the influence of various system parameters on the large disturbance behaviour of the system. The trajectory sensitivities provide a way of judging the relative importance of various factors which affected behaviour.**

Keywords: Disturbance analysis, trajectory sensitivities, security assessment.

I. Introduction

From most points-of-view, large disturbances on power systems are undesirable. However they do provide power system analysts, in particular operators and planners, with an opportunity to explore the stressed behaviour of their actual power system, rather than just the behaviour of models [1]. This often results in a better understanding of the real system, and consequently leads to improved models. Unfortunately though, tools for systematically exploring large disturbance behaviour are quite limited.

Recent work has shown that trajectory sensitivity analysis can provide valuable insights into the security of power systems [2], [3], [4]. This paper therefore proposes the use of trajectory sensitivities as a way of better understanding the dynamics of disturbed power systems. In the paper, these analysis ideas are used to investigate an incident which occurred on the Nordel power system on January 1, 1997.

In general terms, the Nordel disturbance was typical of many power system incidents, where cascaded events progressively weaken the system [5]. The initiating event was an earth fault on a 400kV busbar. Fault clearing tripped a heavily loaded transmission line, with subsequent large power and voltage swings across the system. That in turn led to the energizing of a shunt reactor and the tripping of a nuclear power station. More complete details are given in Section II-B.

PE-334-PWRS-0-11-1997 A paper recommended and approved by the IEEE Power System Dynamic Performance Committee of the IEEE Power Engineering Society for publication in the IEEE Transactions on Power Systems. Manuscript submitted July 7, 1997; made available for printing November 7, 1997.

Post-mortem analysis of this disturbance raised a number of questions. In particular, later investigations of protection flags indicated that the timer on the distance protection of a second major 400kV transmission line had started. The system stabilized before that protection had timed out. However tripping of the second transmission line could have resulted in further cascaded line tripping and possibly significant loss of supply. It was therefore important to determine how close the system was to losing the second line. Other questions regarding the influence of parameters on the large disturbance (nonlinear and nonsmooth) behaviour of the system were also raised. These are addressed in Section IV.

The influence of parameters on the nonlinear, nonsmooth behaviour exhibited by a disturbed power system is difficult to explore. Normal linearization techniques, involving linearization of the system model about an operating point, are not applicable. However trajectory sensitivity analysis offers a rigorous approach to exploring the effects of parameters [6]. This analysis is based on linearizing the system around a trajectory, rather than around an equilibrium point [7], [8]. Therefore it is possible to determine directly the change in the trajectory due to a (small) change in parameters. The ideas extend naturally through discontinuities, provided a few technical conditions are satisfied [9]. An overview of these ideas is given in Section III.

The paper is structured as follows. Section II provides a description of the Nordel power system, and the disturbance of interest. An overview of trajectory sensitivity analysis is then given in Section III. Results of the investigations are presented and discussed in Section IV, and conclusions are drawn in Section V.

II. The Nordel System

A. System description

The Nordel system consists of the interconnected power systems of Finland, Norway, Sweden and parts of Denmark. Through HVDC links it also connects to the European and the Russian power grids. The peak power of the Nordel system is around 55GW and it serves 16 million people. Figure 1 shows the Nordel 400kV network as it was in 1996.

The power generation differs between the countries. The production in Norway is nearly all hydro power. In Finland and Sweden there is a mixture of half nuclear power and half hydro power, whereas Denmark uses fossil fired power stations.

Sweden has twelve nuclear units, located at four power

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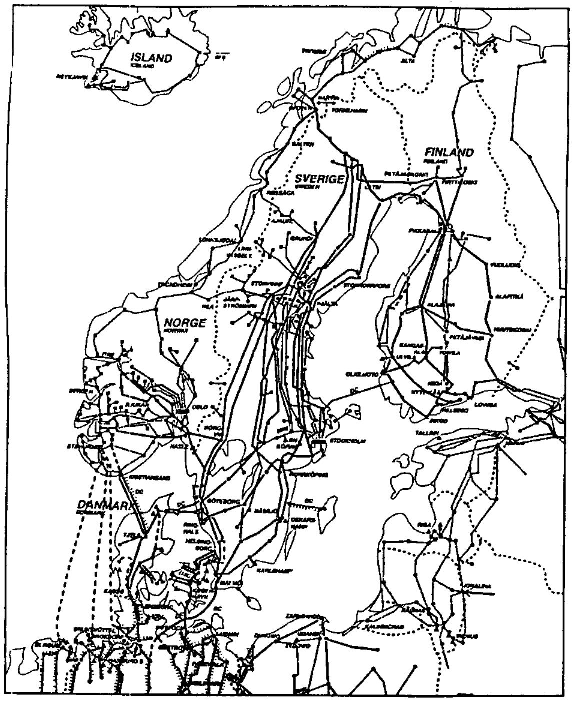

Fig. 1. The Nordel system.

stations. They are close to the major load areas in the southern part of Sweden. Most of the Swedish hydro power is located in the north. Energy is transferred to the south over relatively long transmission lines.

During years of normal rain and snow, Norway has an excess capacity of hydro power. On average, eight out of ten years Norway is a net exporter of power. During dry years though, Norway must import power from other countries in the Nordel system. Often Denmark provides this power.

B. The Disturbance of January 1, 1997

B.1 Background

A severe disturbance occurred in the Nordel system on the night of January 1, 1997. The system was heavily loaded as it was winter and electricity is used for heating. The weather was quite normal for that time of the year. The interchange between the countries consisted of export from Denmark, through Sweden, to Norway. This loading condition was unusual.

B.2 The sequence of events

Primary disturbance (1–60ms): The main cause of the disturbance was a large icicle that came in contact with one phase and caused an earth fault on the 400kV busbar at Stenkullen. The busbar protection operated correctly, tripping all connected lines. The fault was cleared after 60ms.

Damped oscillations (60ms – 4sec): One of the heavily loaded lines transferring power from Denmark to Norway was tripped. Because of this, the power flow changed and the load on the remaining lines increased. One of these lines was overloaded and subsequently tripped 4 seconds after the initial disturbance.

Undamped oscillations (4–10sec): All lines to the north from Ringhals units 1 and 2 were now disconnected. These nuclear units were then only weakly connected to the grid by two relatively long radial lines. The power from the two nuclear units oscillated with growing amplitude, and the units were ultimately tripped from the grid. During this oscillation the voltage in the 400kV grid in southern Sweden deviated from 20% under to 10% over nominal voltage. At a voltage peak, one shunt reactor was connected at a neighbouring busbar. The automatic equipment for connection worked in accordance with settings. However it is doubtful whether the connection of the shunt reactor was actually necessary (or helpful) in this case.

Damped oscillations, frequency drop and recovery (10sec–1min): After the two nuclear units were disconnected, the power oscillations decreased. The tripped units produced 1700 MW, so after the trip there was a corresponding power deficit in the Nordel system.

B.3 Recordings from the disturbance

Figure 2 shows the recorded voltage at Helsingborg in southern Sweden. We clearly see the damping of the oscillation, and observe that the oscillation was stable for the first 4 seconds. However for the interval 4–10 seconds the amplitude was growing, indicating an unstable oscillation. After the trip of the two nuclear unit at around 9–10 seconds, the oscillation amplitude decreased.

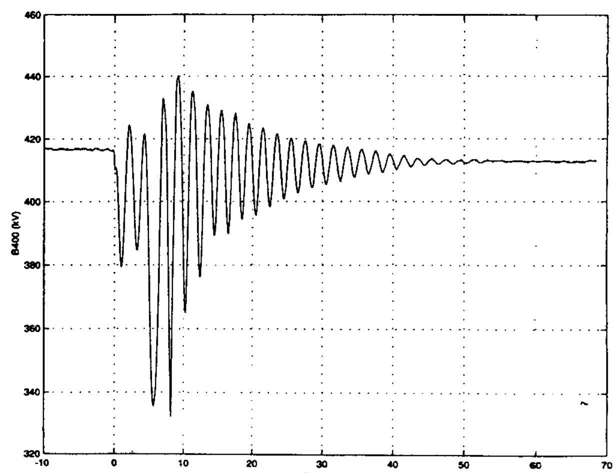

Fig. 2. Voltage recorded at Helsingborg.

Figure 3 shows the recorded frequency at Helsingborg. The oscillations have the same characteristic behaviour as for the voltage. Moreover, notice that the frequency dropped by 0.6Hz, to 49.4Hz, when units 1 and 2 at Ringhals tripped. The frequency recovery was relatively fast. After 30 seconds the frequency had recovered to a steady-state value.

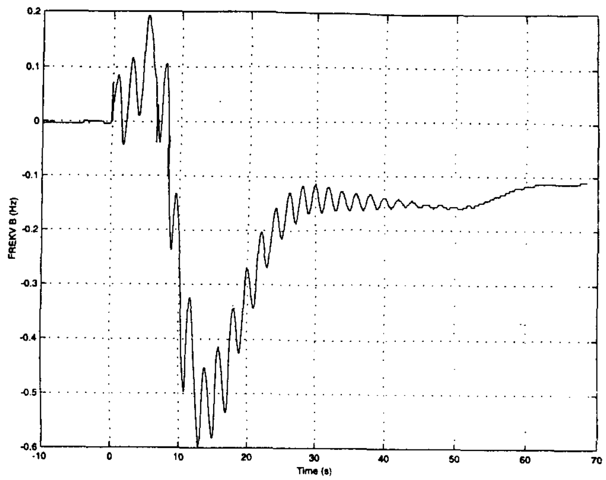

Fig. 3. Frequency recorded at Helsingborg.

B.4 Questions for further investigation

The disturbance was quite severe. Some questions raised in the analysis work afterwards were:

- Which lines were the most critical in the disturbance?
- Did the connection of the shunt reactor have a large influence on the disturbance?
- How important was the timing of the disconnection of the two nuclear units?

These questions are considered in Section IV.

III. Trajectory Sensitivity Analysis

A. Theoretical background

Power systems can be generically modelled as switched differential-algebraic systems [10]. This model has the form

$$\dot{x} = f(x,y;\lambda) \quad (1)$$

$$0 = \begin{cases} g^{(1)}(x,y;\lambda) & h(x,y;\lambda) < 0 \\ g^{(2)}(x,y;\lambda) & h(x,y;\lambda) > 0. \end{cases} \quad (2)$$

where x are the dynamic states, for example generator angles, velocities and fluxes, y are algebraic states, e.g., load bus voltage magnitudes and angles, and λ are parameters such as line reactances or switching times. For clarity, the model (1),(2) captures only a single switching decision, or discontinuity. Real power systems involve many switching decisions, of course. However the model can be easily extended to cater for the more general situation [9].

Away from discontinuities, system dynamics evolve continuously according to the familiar differential-algebraic model

$$\dot{x} = f(x,y;\lambda) \quad (3)$$

$$0 = g(x,y;\lambda). \quad (4)$$

We shall define the flows of x and y respectively as

$$x(t) = \phi_x(x_0,t,\lambda) \quad (5)$$

$$y(t) = \phi_y(x_0,t,\lambda) \quad (6)$$

where $x(t)$ and $y(t)$ satisfy (3),(4), along with initial conditions,

$$\phi_x(x_0,0,\lambda) = x_0 \quad (7)$$

$$g\left(x_0,\phi_y(x_0,0,\lambda);\lambda\right) = 0. \quad (8)$$

To obtain the sensitivities of the flows ϕ_x and ϕ_y to parameter variations, we form the Taylor series expansions of (5),(6). For $x(t)$ this gives

$$\begin{aligned} \Delta x(t) &= \Delta\phi_x(x_0,t,\lambda) \\ &= \frac{\partial\phi_x(x_0,t,\lambda)}{\partial\lambda}\Delta\lambda + \text{higher order terms.} \end{aligned}$$

(Note that sensitivity to initial condition variation can also be developed. Full details are given in [9].) Neglecting higher order terms and using (5) gives

$$\Delta x(t) = \frac{\partial x(t)}{\partial\lambda}\Delta\lambda \equiv x_\lambda(t)\Delta\lambda. \quad (9)$$

Similarly for y we obtain

$$\Delta y(t) = \frac{\partial y(t)}{\partial\lambda}\Delta\lambda \equiv y_\lambda(t)\Delta\lambda. \quad (10)$$

The sensitivities x_λ and y_λ are obtained by differentiating (3),(4) with respect to λ. This gives

$$\begin{aligned} \dot{x}_\lambda &= \frac{\partial f}{\partial x}(t)x_\lambda + \frac{\partial f}{\partial y}(t)y_\lambda + \frac{\partial f}{\partial\lambda}(t) \\ &\equiv f_x(t)x_\lambda + f_y(t)y_\lambda + f_\lambda(t) \quad (11) \end{aligned}$$

and

$$\begin{aligned} 0 &= \frac{\partial g}{\partial x}(t)x_\lambda + \frac{\partial g}{\partial y}(t)y_\lambda + \frac{\partial g}{\partial\lambda}(t) \\ &\equiv g_x(t)x_\lambda + g_y(t)y_\lambda + g_\lambda(t) \quad (12) \end{aligned}$$

where $x_\lambda \equiv \frac{dx}{d\lambda}$ and $y_\lambda \equiv \frac{dy}{d\lambda}$ are generally matrices. Note that f_x, f_y, f_λ, g_x, g_y, g_λ are evaluated along the trajectory, and hence are all time varying matrices.

Initial conditions for x_λ are obtained from (7) as

$$x_\lambda(0) = 0 \quad (13)$$

and for y_λ from (12),

$$0 = g_y(0)y_\lambda(0) + g_\lambda(0). \quad (14)$$

Equations (9),(10) provide the change $\Delta x(t)$ and $\Delta y(t)$ in a trajectory, at time t along the trajectory, for a given (small) change in parameters $\Delta\lambda$.

At a discontinuity, the sensitivities x_λ and y_λ are generally not continuous. It is necessary to calculate the *jump condition* describing the step change in x_λ and y_λ. Let $(x(\tau), y(\tau))$ be the point where the trajectory encounters the hypersurface $h(x,y;\lambda) = 0$, i.e., the point where switching occurs. This point is called the *junction point* and τ is the *junction time*.

Further, assuming nonsingularity of the model, we can use the Implicit Function Theorem to obtain y (locally) as an explicit function of x and λ. Therefore, at points along the trajectory up to the junction point $y = \varphi^{(1)}(x;\lambda)$, where $0 = g(x, \varphi^{(1)}(x;\lambda);\lambda)$. Likewise, after the junction point $y = \varphi^{(2)}(x;\lambda)$. It follows that

$$y(\tau^-) = \varphi^{(1)}\left(x(\tau^-);\lambda\right) \tag{15}$$

just prior to the switching, and

$$y(\tau^+) = \varphi^{(2)}\left(x(\tau^+);\lambda\right) \tag{16}$$

just after switching.

The jump condition corresponding to the sensitivity of x to a particular parameter λ_i is then given by

$$\begin{aligned} x^+_{\lambda_i}(\tau) &= x^-_{\lambda_i}(\tau) + \left\{ f\left(x(\tau), \varphi^{(2)}(x(\tau);\lambda);\lambda\right) - f\left(x(\tau), \varphi^{(1)}(x(\tau);\lambda);\lambda\right)\right\} \times \\ & \frac{\left(h_x - h_y \left(g_y^{(1)}\right)^{-1} g_x^{(1)}\right)\Big|_\tau x^-_{\lambda_i}(\tau) + \left(h_\lambda - h_y \left(g_y^{(1)}\right)^{-1} g_\lambda^{(1)}\right)\Big|_\tau}{\left(h_x - h_y \left(g_y^{(1)}\right)^{-1} g_x^{(1)}\right)\Big|_\tau f\left(x(\tau), \varphi^{(1)}(x(\tau);\lambda);\lambda\right)} \end{aligned} \tag{17}$$

with the y sensitivity following from

$$\begin{aligned} y^+_{\lambda_i}(\tau) &= -\left(g_y^{(2)}(\tau)\right)^{-1} g_x^{(2)}(\tau) x^+_{\lambda_i}(\tau) \\ &\quad - \left(g_y^{(2)}(\tau)\right)^{-1} g_\lambda^{(2)}(\tau). \end{aligned} \tag{18}$$

Following switching, i.e., for $t > \tau$, calculation of the sensitivities proceeds in the normal manner, using (11),(12). The jump condition provides the initial conditions for the post-switching calculations.

B. Numerical approximation

The sensitivities x_λ and y_λ can be calculated using a simple numerical procedure which is based on the approximations

$$\begin{aligned} x_\lambda \equiv \frac{\partial x}{\partial \lambda} &\approx \frac{\Delta x}{\Delta \lambda} \\ &= \frac{\phi_x(x_0, t, \lambda + \Delta\lambda) - \phi_x(x_0, t, \lambda)}{\Delta \lambda} \end{aligned}$$

and

$$\begin{aligned} y_\lambda \equiv \frac{\partial y}{\partial \lambda} &\approx \frac{\Delta y}{\Delta \lambda} \\ &= \frac{\phi_y(x_0, t, \lambda + \Delta\lambda) - \phi_y(x_0, t, \lambda)}{\Delta \lambda} \end{aligned}$$

when $\Delta\lambda$ is small. The procedure involves simulation of the system model for the parameter values λ and $\lambda + \Delta\lambda$. The sensitivities are then given by subtracting the simulations and dividing by the parameter change $\Delta\lambda$.

This procedure involves greater computations than direct calculation of the sensitivities using (11),(12) and (17),(18). However it is easier to implement. In the paper, simulations and sensitivities were obtained using Version 2 of the Power System Toolbox [11] for Matlab [12].

IV. Results

A. Simulation model

The Nordel system consists of several hundred generators, ranging from small windmills of 0.2MW up to nuclear power units of 1100MW. The total length of the 400kV transmission system is over 10 000km. A simplified model has therefore been used for sensitivity analysis. It is shown in Figure 4. The model has 10 generators and 27 buses. Each generator represents several lumped generators. For example, units 1 and 2 at the Ringhals nuclear station have been lumped together to form one single generator in the simulation model. The load buses are more detailed around the disturbance area and less detailed in areas remote from the disturbance. Selected buses and generators are identified in Table I.

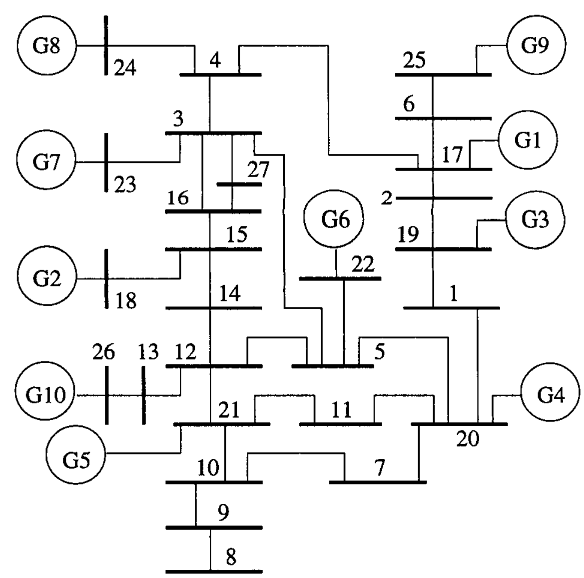

Fig. 4. Simulation model.

The initial load flow was developed using data recorded by the SCADA system at the operation center just prior to the disturbance. The model parameters were taken from [13].

Figures 5 and 6 show the simulated voltage and frequency at the faulted bus. The results are in quite close agreement with the recorded data shown in Figures 2 and 3.

Bus #	Name	Bus #	Name
3	Southern Norway	21	Barsebäck
5	Ringhals	22	Ringhals34
12	Helsingborg	26	Denmark Gen
13	Denmark Load	27	Stenkullen
18	Ringhals12		

Gen #	Name	Gen #	Name
G2	Ringhals12	G7	Southern Norway
G5	Barsebäck12	G10	Denmark
G6	Ringhals34		

TABLE I

BUS AND GENERATOR INFORMATION

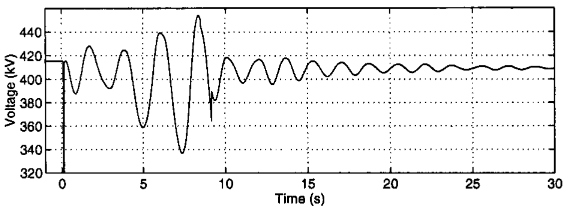

Fig. 5. Simulated voltage at Helsingborg.

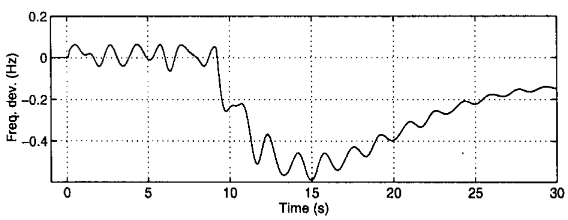

Fig. 6. Simulated frequency deviation at Helsingborg.

B. Sensitivity to line impedances

The largest angular deviation experienced during the disturbance was between Denmark and southern Norway. (Ringhals12 experienced a larger angular deviation, but subsequently tripped.) This was a result of Denmark exporting energy to the major load centres of southern Norway. We shall therefore investigate the sensitivity of that angle behaviour to various parameters. To simplify notation, the angle between Denmark and Norway shall be referred to as $\delta_{10,7}$, with southern Norway taken as the angle reference. The base case behaviour of this angle is shown in Figure 7.

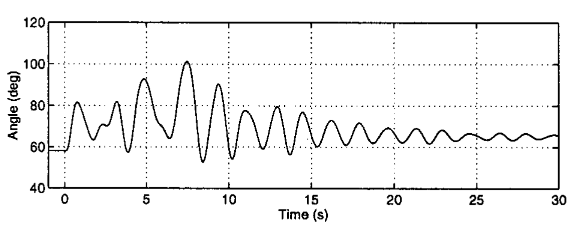

Fig. 7. Angle difference $\delta_{10,7}$ between Denmark and Norway – nominal trajectory.

Trajectory sensitivities were used to find the lines that

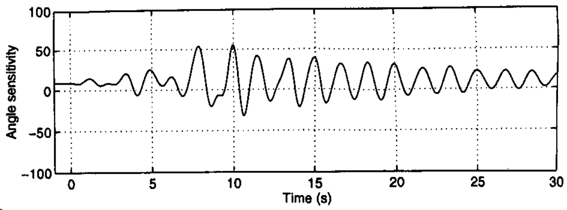

Fig. 8. Sensitivity of $\delta_{10,7}$ for line Helsingborg-Denmark.

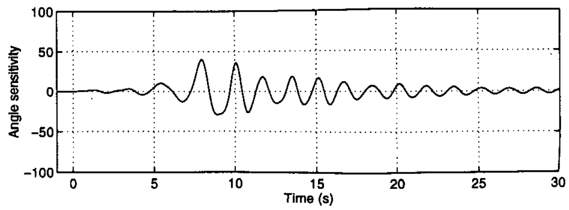

Fig. 9. Sensitivity of $\delta_{10,7}$ for line Helsingborg-Barsebäck.

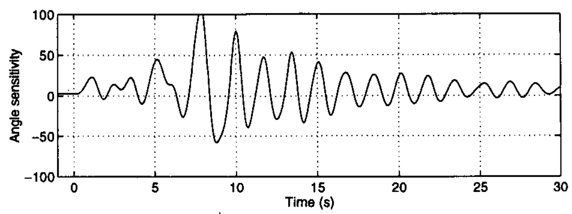

Fig. 10. Sensitivity of $\delta_{10,7}$ for line Helsingborg-Ringhals.

had the greatest influence over the behaviour of the angle $\delta_{10,7}$. Three lines were considered: Helsingborg-Denmark, Helsingborg-Barsebäck and Helsingborg-Ringhals. Sensitivity of $\delta_{10,7}$ to the impedance of each line is shown in Figures 8 to 10 respectively. Angle sensitivity in these figures has units of degrees/pu line impedance.

From the figures, it can be seen that the angle $\delta_{10,7}$ between Norway and Denmark is most sensitivity to the impedance of the line Helsingborg-Ringhals. The line Helsingborg-Denmark also seems to be important, with Helsingborg-Barsebäck having the least influence. These results are consistent with the system topology shown in Figure 4, and are in general agreement with operational experience.

Notice that the trajectory sensitivities for all three lines are relatively small over the first 4 seconds, before the second line trip. During that period, the system was stable, with oscillations tending to be damped. However oscillations increased following the second line trip, indicating that the system was (at best) marginally stable. This is reflected in the sensitivities, with angle behaviour becoming very sensitive to small parameter changes. It is argued in [2], [9] that as a system approaches the stability boundary, trajectory sensitivities will approach infinity. These results exhibit that form of behaviour. After Ringhals12 tripped, system stability was restored. The trajectory sensitivities also stabilize, though they are quite poorly damped. This tends to indicate that the system was still weak.

Comparing Figures 8 and 10, it can be seen that the line Helsingborg-Denmark is less important than Helsingborg-

Ringhals during the transient phase following the second line trip. However the line Helsingborg-Denmark has a slightly greater influence on the longer term damping of system oscillations. Such information is helpful in assessing ways of improving system behaviour.

C. Sensitivity of angles to a critical line

In Figure 8, the sensitivity of the angle between southern Norway and Denmark was presented. This sensitivity was with respect to the impedance of the line Helsingborg-Denmark. It is interesting to look at the influence of this line on other generators. Figures 11 to 13 show the angle sensitivities for generators Barsebäck12, Ringhals12 and Ringhals34, with all angles referenced to southern Norway. It can be seen that Ringhals12, which is the generator that ultimately loses stability, is very sensitive. However the other generators are relatively insensitive. It may be concluded that the line Helsingborg-Denmark has little influence on the behaviour of Ringhals34 and Barsebäck12.

This information could again be very useful, for example in studies aimed at choosing effective locations for FACTS devices.

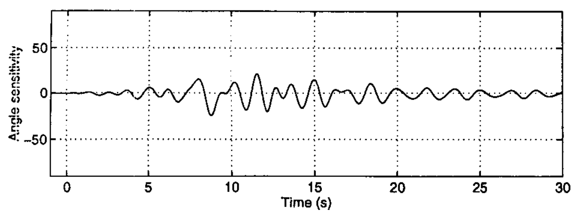

Fig. 11. Angle sensitivity for generator Barsebäck12.

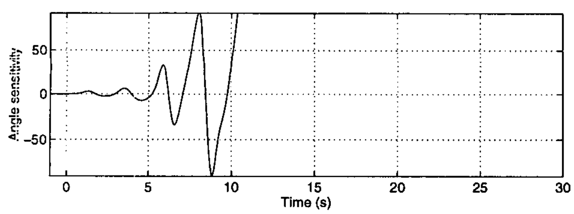

Fig. 12. Angle sensitivity for generator Ringhals12.

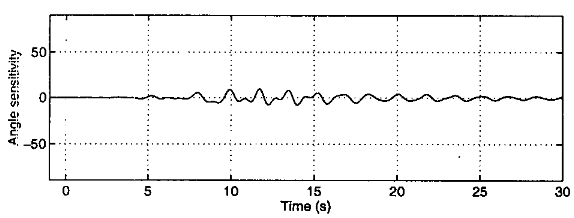

Fig. 13. Angle sensitivity for generator Ringhals34.

D. Sensitivity to switching time

Parameter sensitivity analysis can also be used to investigate the importance of the timing of switching events. In this case, it has been used to study the timing of two events: shunt reactor connection, and the tripping of the nuclear units at Ringhals12. Again the angle deviation $\delta_{10,7}$ between Denmark and Norway was considered. The sensitivities of $\delta_{10,7}$ to the two switching times are given in Figures 14 and 15.

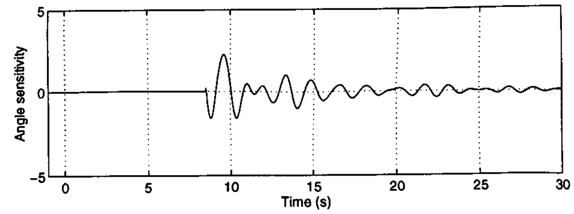

Fig. 14. Sensitivity of $\delta_{10,7}$ to the reactor switching time.

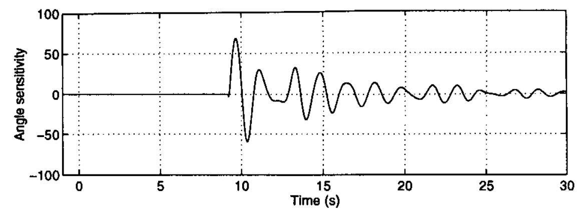

Fig. 15. Sensitivity of $\delta_{10,7}$ to Ringhals12 tripping time.

Based on the results of Figure 14, it may be concluded that the reactor switching time played a minor role in the overall disturbance. However it is interesting to consider the effect on stability of earlier or later switching. Figure 16 helps in answering that question. In that figure, the deviation of the angle $\delta_{10,7}$ from its steady-state value is plotted against its sensitivity, for the first few swings following switching. It can be seen that the angle and its sensitivity are largely in phase, i.e., the angle and the sensitivity increase together and decrease together. Now based on (9), if the parameter (the reactor switching time) was increased, a component of the trajectory sensitivity would be added to the original trajectory. Hence in this case the angle deviation would be larger for an increased reactor switching time, i.e, for delayed switching.

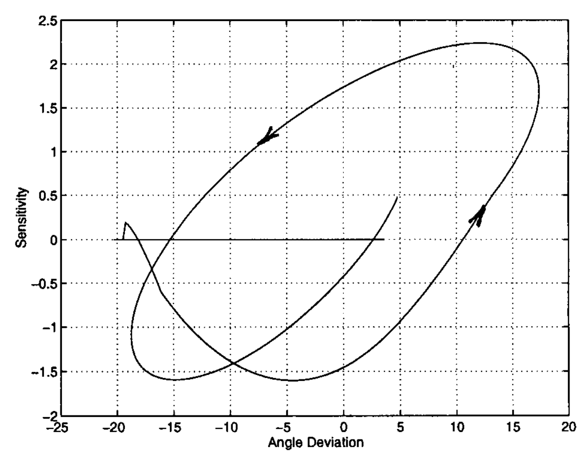

Fig. 16. Correlation between $\delta_{10,7}$ and its sensitivity for reactor switching.

In contrast to the reactor switching time, Figure 15 indicates that the tripping time of Ringhals12 was very important. The sensitivity is very high, with a small change in switching resulting in a large change in the angle deviation. Figure 17 shows that the angle and its sensitivity were again in phase. Hence delayed tripping would have led to increased angle deviation, and decreased security. This result is to be expected, as it was the tripping of these units that ultimately saved the system.

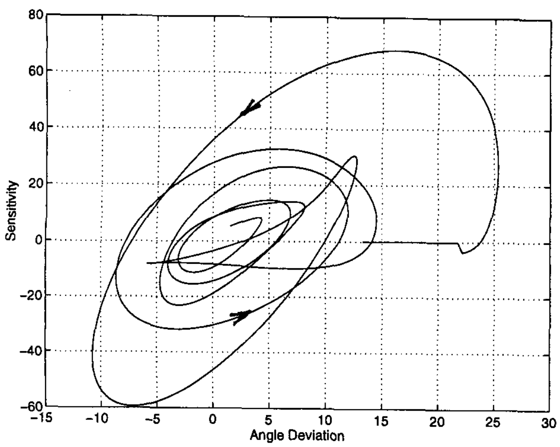

Fig. 17. Correlation between $\delta_{10,7}$ and its sensitivity for Ringhals12 tripping.

V. Conclusions

Limited tools are available for systematically investigating the large disturbance behaviour of power systems. The results presented in the paper show that trajectory sensitivity analysis provides a valuable addition to current techniques.

Trajectory sensitivities can be used to explore the factors which influence large disturbance behaviour of a system. Such factors may range from line parameters to switching/tripping times of critical devices. Trajectory sensitivities provide a way of judging the relative importance of those various factors.

Because trajectory sensitivities provide a first order approximation to the change in the trajectory corresponding to changes in parameters, they open up many analysis possibilities.

Acknowledgements

This work was supported by research grants from Sydkraft, Sweden and the Centre for Integrated Dynamics and Control at the University of Newcastle. The ideas were stimulated by discussions with Prof. M.A. Pai.

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Biographies

Ian A. Hiskens (S'77,M'80,SM'96) received the B.Eng. (Elec.) degree and the B.App.Sc.(Math.) degree from the Capricornia Institute of Advanced Education, Rockhampton, Australia in 1980 and 1983 respectively. He received the Ph.D. degree from the University of Newcastle, Australia in 1990. He worked in the Queensland Electricity Supply Industry from 1980 to 1992. Dr. Hiskens is currently a Senior Lecturer in the Department of Electrical and Computer Engineering at the University of Newcastle. His major research interests lie in the area of power system analysis, in particular system dynamics and control, security, and numerical techniques. Other research interests include nonlinear systems and control.

Magnus Akke (M'92) was born in Lund, Sweden in 1961. He received his M.E.E. degree, Licentiate degree and Ph.D. from Lund Institute of Technology, Sweden, in 1986, 1989 and 1997 respectively. He also has a Bachelors degree in Business Administration from Lund University, Sweden. In 1990 he started at Sydkraft – a Swedish power utility – where his work tasks are power system analysis and relay protection. He has been a visiting scientist at the University of Newcastle, Australia, and at Cornell University, USA.

Discussion

M. A. Pai (Power and Energy Systems Area, University of Illinois) This paper is a first realistic application of trajectory sensitivity theory to the analysis of a disturbance. It is a nonlinear analysis and helps to find the critical parameters involved in the disturbance. The major contribution in terms of theory in the paper is to extend the trajectory sensitivity analysis to the entire time frame as opposed to only the post-fault system as in [2,4]. Since the technique is model independent it provides an alternative to TEF based DSA techniques. The computational overhead is substantial though the fact that the Jacobians of (11), (12) and (1) and (2) will be the same if one were to use simultaneous implicit method. Coupled with recent work in iterative solver techniques for dynamic simulation [A,B], the technique may prove to be a viable one in the future.

The authors may like to clarify the following points.

1. The jump condition (17) in the sensitivity of x is not reflected in Figs. 8-10, 11-13 and 14-15. In the first two cases the parameters were impedance and line respectively it may not appear. However, in Figs. 14-15, it will be interesting to know what were the switching times.
2. Have the authors considered lessening the computational burden by not integrating the entire set of sensitivity equations (11) but only those whose sensitivities are of interest.
3. Can the results of Section D be extended to DSA work for computing sensitivities to t_{cl}?
4. Have the authors investigated sensitivities with respect to initial conditions (operating point) of (1) and (2).

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Ian A. Hiskens and Magnus Akke: We wish to thank Prof Pai for his interest in the ideas presented in the paper. Trajectory sensitivities complement numerical integration by quantifying the influence of parameters on large disturbance behaviour. This extra information often provides valuable insights into the system response. An example of the usefulness of sensitivities is provided by this paper.

As indicated by Prof Pai, trajectory sensitivities can be generated for any nonlinear, non-smooth system model. The ideas are therefore applicable to systems which include detailed representations of complicated switching devices, for example transformer tap changer AVRs and protection relays. Sensitivities are proving to be very useful in the analysis of voltage collapse.

If an implicit numerical integration technique is used to generate the system trajectory, then at each time step matrices that are closely related to the Jacobians of (1),(2) are built and factorized. These same matrices are required for numerically integrating the sensitivity equations (11),(12). Because they are already built and factorized, computation of the sensitivities only involves forward and backward substitution. Iterative solver techniques offer an exciting possibility for further computational improvements.

We will address Prof Pai's questions in order:

1. It can be seen from (17) that the jump in sensitivities from $x^-_{\lambda_i}(\tau)$ to $x^+_{\lambda_i}(\tau)$ is dependent upon the difference $\{f^{(2)} - f^{(1)}\}$. If the jth component of $\{f^{(2)} - f^{(1)}\}$ is zero, then the jth components of $x^-_{\lambda_i}(\tau)$ and $x^+_{\lambda_i}(\tau)$ will be equal. Figures 8 to 15 reflect that behaviour. All those figures plot the sensitivity of generator angle difference to various parameters. But recall from the swing equation that generator angle variations are given by

$$\dot{\delta} = \omega.$$

From (3) it can be seen that the function f_j governing a generator angle state is therefore $f_j = \omega$. Now because ω is a state, it is always continuous, even at switching events. Therefore at such an event $f_j^{(2)} = \omega^+ = \omega^- = f_j^{(1)}$, and hence $\delta^+_\lambda = \delta^-_\lambda$. The second order dynamics of angles (as described by the swing equation) ensure that angle sensitivities never display jumps. Other state sensitivities do generally display jumps though.

2. In solving (11),(12), it is necessary to keep track of the sensitivities of all states to a particular parameter, i.e., for each parameter λ_i, one must retain the complete sensitivity vector x_{λ_i}. (The sensitivity vector y_{λ_i} can be obtained from x_{λ_i} via (12).) Intuitively, this is saying that a parameter change may (over time) influence all states. The changes in states in turn influence other states. Discarding part of x_{λ_i} would upset that behaviour.
In practice, it has been found that parameters may sometimes only influence a subset of the states. This behaviour seems to offer a way of exploring system coherency in a large disturbance setting. Work is progressing on this topic.

3. Fault clearing time t_{cl} can be treated like any other parameter. It follows from [2,4] that high sensitivity indicates the system is close to instability whilst low sensitivity indicates relative security. If the fault clearing time is exactly equal to the critical clearing time, then sensitivity will be infinite. Mathemetically this occurs because the system runs to a UEP, where f_x is unstable. From (11), the trajectory sensitivities must be unstable. It follows intuitively also. An infinitesimal decrease in clearing time will return the system to the region of stability, whereas an increase will drive the system to instability. Therefore the trajectory is infinitely sensitive to the parameter change.

4. In later work, parameter sensitivities x_λ and y_λ are treated as a special case of the more general initial condition sensitivities x_{x_0} and y_{x_0}. This is achieved by representing the parameters of interest λ as states, through the introduction of extra state equations $\dot{\lambda} = 0$. These equations ensure that λ remains constant at the nominal (initial) values along the trajectory. However sensitivity to the initial conditions of this equation is exactly parameter sensitivity.

We again thank Prof Pai for his interest in the paper, and his encouragement of this line of research.