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# SVC BEHAVIOUR UNDER VOLTAGE COLLAPSE CONDITIONS

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Abstract - Voltage-susceptance diagrams are often used to identify system operating conditions which would lead to abnormal voltage behaviour. It is generally believed that system operation in the region where the slope of the voltage-susceptance characteristic is negative results in voltage instability/collapse. However this paper demonstrates that SVCs can extend stable operation into that region. It is shown that short term system stability is primarily governed by the interaction between the SVC control system and instantaneous voltage-susceptance characteristics. Given that SVCs are stable in the short term, then long term behaviour is dependent on transformer tapping. These effects are explored. The consequences of SVCs encountering limits are also considered.

Keywords - voltage collapse, SVCs, small disturbance stability, long term stability, load modelling, static bifurcations

## I INTRODUCTION

Numerous power systems throughout the world have suffered failures which have been associated with abnormal voltage behaviour. A mechanism for such failures has been described by Lachs [1] and Brownell and Clark [2]. The frequency and severity of these collapses has resulted in focusing a substantial research effort on the area of voltage instability/collapse. Even so, this subject is still not well understood.

It is recognized that one of the important factors contributing to long term voltage collapse is that of inadequate reactive power resources at critical locations within the network [1,3]. Therefore, to minimize the risk of voltage problems, it is becoming prudent to strategically locate voltage support devices such as static var compensators (SVCs) throughout power systems. An example of this type of reinforcement is provided by Allen et al [4]. The Queensland Electricity Commission (QEC) has also adopted this practice.

If this reinforcement philosophy is to be successful though, SVCs must exhibit stable behaviour under voltage collapse conditions. However little research has addressed this issue. Of particular concern is the fact that under stressed conditions, it may be necessary for a power system to operate at a point where a voltage-susceptance characteristic has a negative slope, i.e., where an increase in shunt capacitance at a bus ultimately causes a voltage decrease. The design of normal SVC control systems assumes that such a system characteristic is positive.

The QEC has encountered this negative slope situation in studies of future loading conditions. It was shown in those studies that if SVC behaviour was stable at points of negative slope, transmission line reinforcement could be deferred. Therefore a detailed investigation of SVC behaviour was undertaken. This paper provides details of that investigation.

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The paper is organized as follows. Section 2 provides a brief static bifurcation analysis of the power flow equations. This opens up questions regarding the short term transient behaviour of SVCs under voltage collapse conditions. Following the development of various system component models in Section 3, these questions are addressed in Section 4. Section 5 investigates aspects of the longer term behaviour of SVCs. Conclusions and recommendations for the secure operation of SVCs are given in Section 6.

## II BIFURCATION ANALYSIS

Investigations of voltage instability/collapse situations often involve static bifurcation analysis of the power flow equations, i.e., the system equilibrium point equations. This approach provides some useful information regarding SVC behaviour under collapse conditions.

Consider the highly stressed power system of Figure 1. This system is a simplified representation of the northern section of the QEC power system. If the common assumption that loads are modelled as constant power is made, then the familiar bifurcation diagram, of the form shown as curve a-a in Figure 2, results. (This assumption is based on the notion that load restoration devices, such as tap changing transformers,

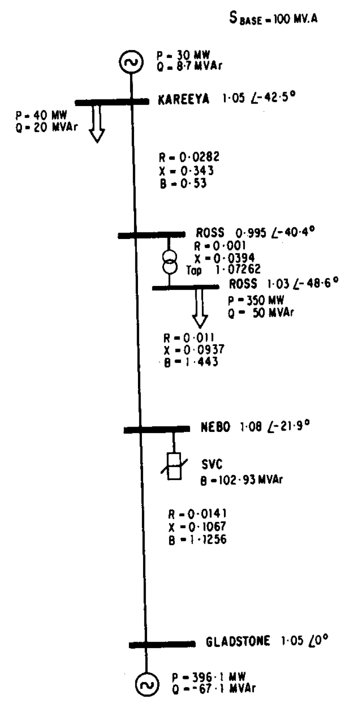


Figure 1: Example Power System

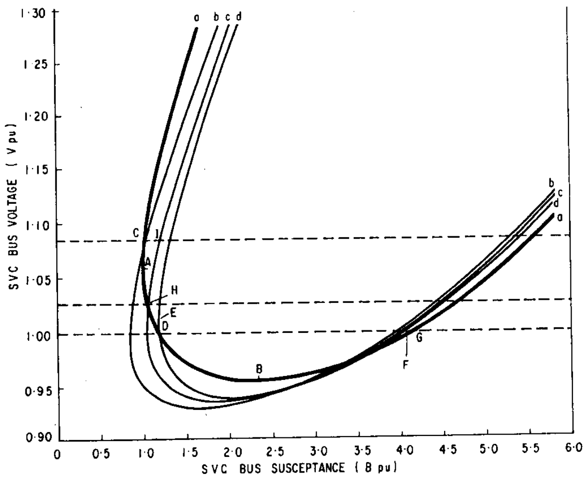


Figure 2: Voltage-Susceptance Characteristic Constant Power Loads

maintain loads at constant values.) Curve a-a shows the relationship between voltage and susceptance at the SVC bus. Normally such curves are drawn for load buses, where voltage varies depending on the susceptance. (This is equivalent to an SVC with zero gain.) Under those conditions, point A is a bifurcation point, because reduction of susceptance through A results in the merging, then disappearance, of solution points [5].

However if an SVC at the bus varied susceptance to maintain a constant voltage, then point A would no longer be a significant point. (In a power flow, the SVC bus would be modelled as having constant real power and voltage, i.e., a PV bus.) Variation of the voltage setpoint through A would not alter the number of solution points. However, if the setpoint voltage was reduced through point B , the number of solution points would indeed change. Two solution points would merge, then disappear. Therefore, if the SVC bus is modelled as a PV bus, point B becomes the bifurcation point.

Under what conditions though is it realistic to model an SVC bus type as PV? Can SVCs regulate voltage at points where the bifurcation diagram has a negative slope, i.e., that section of the curve a-a between points A and B ? These questions are addressed in Section 4. Before they can be answered however, various system component models must be established.

## III SYSTEM COMPONENT MODELS

### 3.1 SVC Control System Model

SVCs regulate voltage by varying shunt susceptance at a bus. Control system design is based on the notion that an increase in capacitive susceptance at a bus will cause its voltage to rise. Therefore, typically an SVC AVR can be modelled as a simple integrator, as shown in Figure 3. The gain of the $j^{th}$ SVC is $K_j$ . SVC susceptance $B_{sj}$ forms part of the self susceptance of the bus. Each SVC regulates bus voltage to its setpoint $|V^o_{sj}|$ .

The assumption that increased capacitive susceptance causes higher voltage, upon which the control system is based, is not necessarily true in voltage collapse situations. Therefore the interaction between the SVC control system and the system voltage-susceptance characteristic is vital to an understanding of the short term response of an SVC to a disturbance.

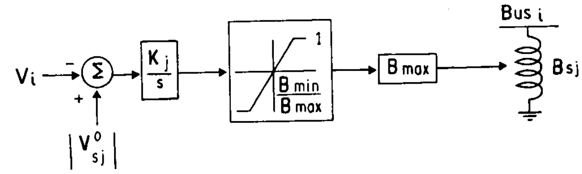


Figure 3: SVC Control System Schematic

### 3.2 Transformer Model

The AVR of a regulating transformer monitors the error between the controlled bus voltage and its setpoint value. If this error exceeds a preset deadband, it forces taps to change until the voltage error falls below that deadband, or a tap limit is encountered. In these investigations, this general behaviour has been represented by a simple transformer model whose taps are varied continuously (not in discrete steps) at a constant rate whenever the voltage error is greater than the deadband. This model enables useful qualitative information about the interaction between transformer tapping and SVCs over the long term to be determined.

### 3.3 Load Model

In the short term following system changes, load response is predominantly voltage dependent. A generally accepted model of this voltage dependence is given by

$$P_{di} = P^o_{di} |V_i|^{\zeta_i} \qquad (1a)$$

$$Q_{di} = Q^o_{di} |V_i|^{\eta_i} \qquad (1b)$$

where $\zeta_i, \eta_i$ are constant indices.

In the longer term, transformers tap to restore load bus voltages, and hence loads, to their predisturbance values. Also, in this longer time frame, load dynamics tend to cause load restoration. It has been found however that for qualitative investigations of SVC behaviour during voltage collapse, the effects of load dynamics can be neglected.

The assumption that transformer tapping is the only source of load restoration does not result in the loss of any significant modes of SVC behaviour.

## VI SHORT TERM BEHAVIOUR OF SVCS

### 4.1 Voltage-Susceptance Characteristics

The response of a power system to a disturbance, such as a feeder trip or a step change in load, occurs in three (reasonably distinct) phases. Instantaneously, network quantities such as bus voltages will adjust. In the short term, SVCs and generators will respond; SVCs and generator AVRs to voltage errors and generator angles to power mismatches. Note that these are dynamic processes, so cannot occur instantaneously. However, because SVCs have no inertia, they respond much more quickly than can machines. In the long term, system adjustments will occur due to devices with slower dynamics such as tap changing transformers. A different voltage-susceptance characteristic can be associated with each of these phases.

The *instantaneous characteristic* shows the initial relationship between voltage and susceptance at the SVC bus. To produce this characteristic, the network is modelled as normal, with loads given by (1), but generator angles and transformer taps are frozen at their initial values. Note that in this case, generators and transformers are not in equilibrium. This characteristic corresponds to the period before they have had time to respond to the disturbance. Because SVCs respond much more quickly than machines, SVC bus voltage, in response to susceptance changes, will initially follow this characteristic.

The *short term characteristic* shows the voltage-susceptance relationship after the generators have responded, but before taps have moved. This characteristic differs from the instantaneous one in that generator powers are constrained to their equilibrium values, rather than generator angles being constrained. Again the network is modelled as normal, with loads given by (1), and transformer taps are fixed at their initial values.

The *long term characteristic* gives the voltage-susceptance relationship after generators and transformers have responded. In this case, as in the previous case, generator powers are constrained to their equilibrium values, and the network is modelled as normal, again with loads given by (1). However, unlike the previous case, this time transformers are in equilibrium. This characteristic corresponds to normal power flow solutions.

Figure 2 can be used to illustrate the instantaneous and short term characteristics for the case where loads are modelled as constant power, i.e., $\zeta_i, \eta_i=0$. (Note that this is not a realistic load model for the short term period, but is convenient for demonstrating different possibilities for short term behaviour of SVCs. This is undertaken in Section 4.2.) In Figure 2, curve a-a is the short term characteristic. Curves b-b, c-c, d-d are instantaneous characteristics corresponding to various different initial conditions. Note that taps are fixed in this example.

Consider operating point C in Figure 2. The instantaneous characteristic is given by curve b-b. Therefore, as the SVC control system varies its susceptance following a disturbance, the voltage will respond according to curve b-b. Over time however, as generator angles move and the system settles to a new operating point, system quantities will change in accordance with curve a-a. Hence b-b dictates the initial SVC response, whilst a-a gives its response after the "short term" transient period.

## 4.2 Dependence of SVC Stability on Voltage-Susceptance Characteristics

As noted earlier, to produce the curves of Figure 2, loads were modelled as constant power. This is recognized as not being a realistic representation of load behaviour. However it does allow the various possibilities for short term behaviour of SVCs to be explored. Other (more realistic) load indices are considered later, in Section 4.3.

Because curve b-b of Figure 2 has a positive slope at the operating point C, the SVC control system will initially respond in the correct manner. (An increase in capacitive susceptance will cause an increase in voltage.) Over a longer time frame, the machine angles will move. But at each instant along that trajectory, SVCs will respond according to an instantaneous characteristic like b-b. As long as the slopes of those characteristics remain positive SVCs will behave in a desirable manner.

Now consider operating point D of Figure 2. At that point, the instantaneous characteristic d-d has a negative slope. Therefore the SVC control system will produce undesirable behaviour. If a disturbance caused the voltage seen by the SVC to rise, then the control system would force the SVC susceptance to a more inductive value. Because of the negative slope, the voltage would actually rise further, i.e., a positive feedback situation. Susceptance would quickly reach the bifurcation point E, at which time voltage would become infinitely sensitive to susceptance movement. System failure would occur.

If however a disturbance caused the SVC terminal voltage to fall, the control system would force SVC susceptance to become more capacitive, resulting in a further fall in voltage. Susceptance would continue to go more capacitive, with the voltage varying according to curve d-d. Notice though that voltage would quickly pass through a minimum, encountering a section of d-d where the slope was positive. The control system would begin to function properly again, resulting in the SVC settling to operating point F. Ultimately, as generators responded, the system would settle back to curve a-a at point G.

Interestingly, if the SVC gain was zero at operating point D, the system would be small disturbance stable. Analysis leading to that conclusion is given in Appendix A. For positive gain though, the system linearized around point D contains a positive eigenvalue, indicating instability. The positive eigenvalue changes to negative if the SVC gain is altered to a negative value. This confirms the need for consistency between the gain of the SVC and the slope of the system's instantaneous characteristic, in this case d-d.

Operating point H in Figure 2 is particularly interesting because the instantaneous characteristic c-c has positive slope while the long term characteristic, curve a-a, has negative slope. It is shown in Appendix A that if the SVC gain was zero at this point, the point would be small disturbance unstable. Therefore, because of the continuity of eigenvalues with parameter changes, the point is unstable for small values of gain. However, the characteristic c-c is positive, so we know that for sufficiently high gain, the SVC will respond in a stable manner. For example, if the SVC setpoint was suddenly changed from its value at H to 1.08pu, the SVC would quickly respond by moving along curve c-c to the point I. SVC susceptance would become more capacitive. Notice though that over time, as generator angles moved, the system would settle back to point C. Ultimately SVC susceptance would be more inductive than it was before the change in setpoint.

Comments:

1. At points where the instantaneous characteristic is steep, such as H, too high a gain in the SVC will cause oscillatory instability of its control system. It occurs because very small changes in susceptance (caused by the control system) result in large voltage fluctuations, and hence voltage overshoot.
2. Switched shunt controls generally respond slowly to voltage variations, so they can be thought of as having low gain. Therefore such devices would provide no assistance in maintaining stable operation at points such as H.
3. It is apparent from the cases considered that provided the instantaneous characteristic has a positive slope, a non-negative gain can be chosen to ensure that the SVC regulates voltage correctly. Note therefore that the instantaneous characterist-

ics have a far greater influence on SVC stability than does the short term characteristic, curve a-a .

## 4.3 The Influence of Load Indices

It was noted in Section 3.3 that when a power system is subjected to a disturbance, load powers initially vary with voltage. (This voltage dependence is modelled by (1).) Therefore it could be expected that load indices significantly influence the form of the instantaneous characteristics, and hence short term SVC behaviour. Figure 2 showed these characteristics for the case where loads were modelled as constant power. However that type of load behaviour is not typical of real power systems. Therefore more realistic load indices shall now be considered.

Results obtained in [6] enable at least a partial understanding of the effects of load indices. It was conjectured that for lossless systems, if

$$\zeta_i \geq 1 \quad \text{and} \quad \eta_i = 2 \qquad (2a)$$

or

$$\zeta_i = 2 \quad \text{and} \quad \eta_i \geq 1 \qquad (2b)$$

then instantaneous characteristics could not have bifurcation points, i.e., each characteristic would express a unique relationship between voltage and susceptance. Thus, if all loads were modelled according to (2), then such characteristics must always have a positive slope. Consequently SVC control systems would always interact correctly with the power system.

In [6] a robustness argument (based on perturbation of eigenvalues) allowed the set of load indices (2) to be expanded to include 'nearby' indices. A similar robustness argument can be used to show that the indices of (2) are applicable to lossy systems, provided the losses are small.

Figure 4 shows cases which are similar to those of Figure 2, but with loads modelled as voltage dependent. Uniform load indices of $\zeta_i = 1$ , $\eta_i = 3$ were used. (Tests on the QEC system have indicated that those indices provide a useful model of actual system load-voltage dependence.) In this case, the long term characteristic a-a includes the effects of transformer tapping, and hence load restoration. Notice in Figure 4 that the instantaneous characteristics have positive slope everywhere. Therefore, the SVC control system will respond correctly, in the short term, at all points. However, to determine the ultimate stability of the system, the behaviour of the slowly responding transformers must be considered. This long term behaviour is investigated in the next section.

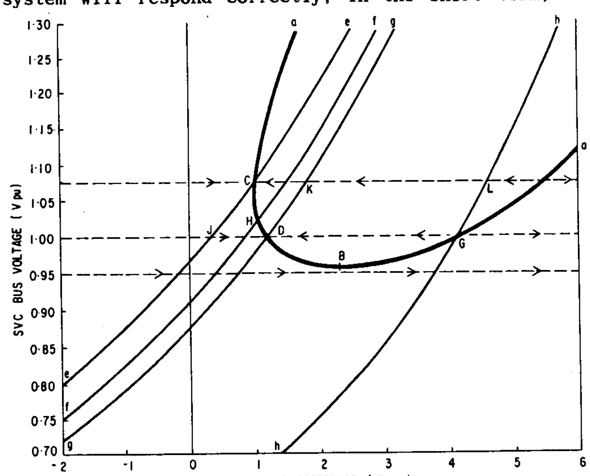


Figure 4: Voltage-Susceptance Characteristics Voltage Dependent Loads

## V LONG TERM BEHAVIOUR OF SVCS

It is clear from [1] that many factors, such as tap changing, load dynamics and generator reactive power limiting, contribute to long term voltage collapse. However, the significant aspects of long term SVC behaviour can be highlighted through the use of the simplified system component models described in Section 3. Generator reactive power limits have not been considered. Whilst such reactive power limiting does tend to accelerate voltage collapse, it does not directly influence the way in which SVCs respond during such a collapse.

### 5.1 System-SVC Interaction

In Section 4.3 it was shown that if loads could be modelled as voltage dependent with indices close to those of (2) then, in the short term, an SVC would allow stable system operation at all points on curve a-a . The following simple examples allow long term system behaviour to be investigated. These examples are once again based on the system of Figure 1.

Consider the situation depicted in Figure 4, with the system initially operating at point C . How would the system behave if the SVC setpoint was suddenly altered to 1.00pu? SVCs respond much more quickly than generators or transformers, so susceptance would vary along curve e-e and initially approach point J . However generator angles and transformer taps would then begin to move. Because the SVC setpoint voltage was lowered from 1.08pu to 1.00pu, network voltages would be correspondingly depressed. Loads would reduce due to their voltage dependence. Transformers would therefore tap so as to restore their controlled bus voltages, and loads would increase accordingly. Because of the increasing system load, the SVC would require more capacitive susceptance to maintain its setpoint voltage. Therefore the system would move along the line between points J and D , before finally settling to point D . This response is shown in Figure 5. Notice that there is initially a rapid decrease in the capacitive susceptance of the SVC. This is followed by a period of oscillations where generator angles respond to the setpoint change. Transformer taps vary to restore load bus voltage. (Recall that in this simplified model, tapping is

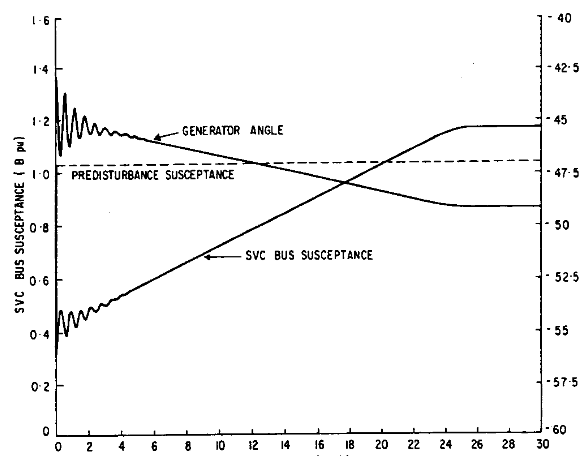


Figure 5: Response to an SVC Setpoint Change

continuous, not discrete. Taps vary at a constant rate.) The effect of this tapping can be seen in the comparatively slow ramping of SVC susceptance and generator angle. As predicted from Figure 4, SVC susceptance at the final operating point is more capacitive than it was before the reduction in the setpoint.

If the setpoint was now returned to 1.08pu, the system would follow curve g-g from D to K . This setpoint change would cause higher network voltages, and hence higher loads. This time transformers would tap to reduce controlled bus voltages, and loads, so SVC susceptance would become less capacitive. System response would follow the line from K to C .

Long term system behaviour is stable in the previous cases. However stable behaviour is not guaranteed at all points on a-a . Consider point G . A setpoint change from G would cause movement along curve h-h . If the setpoint was raised slightly, network voltages and loads would increase. Transformers would tap to reduce controlled bus voltages, and hence loads, resulting in a reduction in capacitive SVC susceptance. System response would follow the line from G to D . Point G is therefore unstable in the sense that the system will not remain in a neighbourhood of that point when subjected to small disturbances. As a further example, if the setpoint was raised to 1.08pu, the system would initially move to point L . Then, following the previous argument, the system would traverse the line from L to C , ultimately settling at C .

Again referring to point G , if the setpoint was lowered below 1.00pu, system collapse would probably result. Taps would increase in response to the depressed voltages, so SVC susceptance would go more capacitive. The system would therefore move away from G to the right. But no solution point could be encountered. (All solution points fall on curve a-a .) As transformers tapped up, network voltages would fall. Unless tap limits were encountered, this process would continue until network voltages fell to a level where synchronizing power could not be maintained. Angle separation between machines would then occur.

(Recall that point G of Figure 2 was stable. However the system corresponding to that figure did not contain any tap changing transformers. It is the dynamics of these transformers which cause unstable behaviour at point G of Figure 4.)

Referring to Figure 4, it can be seen that in the region outside of curve a-a , system trajectories move in the direction of increasing capacitive susceptance. Inside (above) a-a , trajectories move in the direction of reducing capacitive susceptance. Therefore trajectories move away from points on a-a which are to the right of B , and toward points on a-a which are to the left of B . Hence point B forms the boundary between stable points, i.e., those points to the left of B , and unstable points, i.e., those to the right of B . Point B is therefore a bifurcation point of the system. This is consistent with the ideas of Section 2.

## 5.2 SVC Limits

The consequences of an SVC encountering its capacitive susceptance limit are of interest in the study of voltage collapse. When the capacitive limit is reached, voltage support ceases. Therefore, if a system change caused a fall in the SVC bus voltage, the SVC would not be able to go more capacitive to support the voltage.

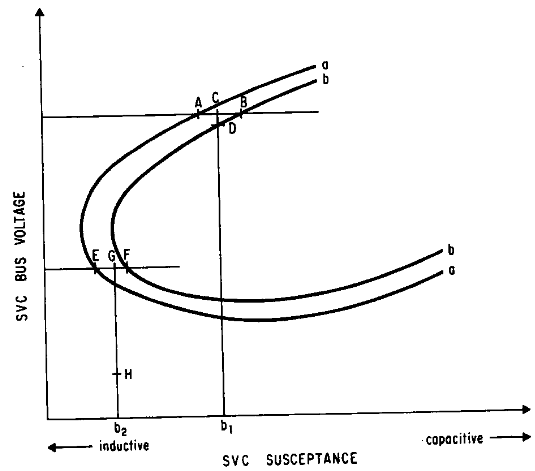


Figure 6: SVC Limit Behaviour

Under normal system conditions, when the SVC is operating on the positive slope of the long term characteristic, a bounded drop in SVC bus voltage will result if the capacitive limit is encountered. Figure 6 illustrates this behaviour. The effect of a system change which moved the long term characteristic from a-a to b-b is shown. If the SVC was initially operating at point A , and if no limit was encountered, then in response to machine angle adjustments and transformer tapping, the SVC would ultimately settle to point B . If however the capacitive limit value was $b_1$ , then the limit would be encountered at point C . The SVC bus voltage would fall, so the system would proceed from C to ultimately settle at point D .

Now consider the same system change, but with the SVC operating on the negative slope at point E . If the capacitive limit was not encountered, the system would ultimately settle to point F . However, if the capacitive limit value was $b_2$ , then the system would proceed from E to G , where the limit would be encountered. Voltage would fall, so the system would follow the line from G toward H . Nowhere would the long term characteristic b-b be intersected, so voltages would continue to decline, leading to a voltage collapse situation.

The first example of Section 5.1, i.e., where the system was initially operating at point C of Figure 4, can be further used to demonstrate this unstable behaviour. For this case, the capacitive limit of the SVC was set to 1.10pu. Recall from the earlier example that when the setpoint fell from 1.08pu to 1.00pu, capacitive susceptance decreased. This was followed by a period of increasing capacitive susceptance as transformers tapped to restore load bus voltages. This time however, before all transformer setpoint voltages were reached, the SVC limit was encountered. Voltage regulation ceased. The time response of the voltages at the SVC bus and the ROSS load bus are shown in Figure 7. After the SVC ceased regulating, transformers continued tapping up because some load bus voltages were still below their setpoints. This only succeeded in depressing voltages at the load buses and the transformer high voltage buses. Further tapping caused a continual decline in network voltages, until the point was reached where the depressed voltages did not allow sufficient synchronizing power between machines to be maintained. Voltages fell rapidly and system separation occured.

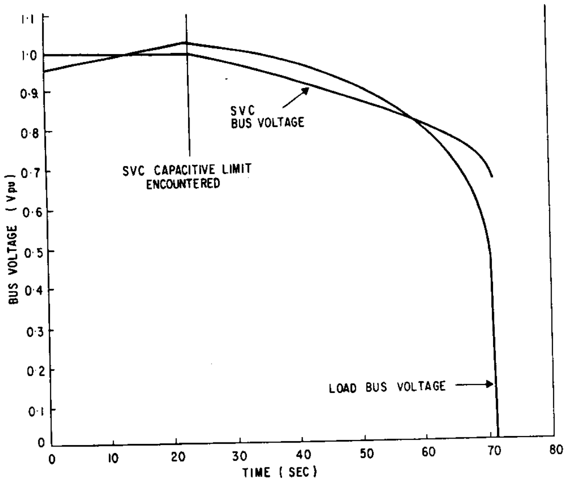


Figure 7: Voltage Behaviour, SVC Limit Encountered

This example illustrates one voltage collapse scenario. Others are possible. The load model has a significant effect on the ultimate mode of failure of a power system [7,8]. Research is continuing in this area.

## VI CONCLUSIONS

The paper has investigated the behaviour of SVCs at operating points where the slope of the system's long term voltage-susceptance characteristic is negative. This investigation has shown that SVC stability is influenced greatly by instantaneous characteristics obtained by constraining generator angles rather than powers. When these characteristics have positive slope, the SVC control system interacts properly with the power system.

Conditions which ensure that the instantaneous characteristics always have positive slope have been considered. Modelling of the loads was found to be particularly important. If loads are modelled as voltage dependent, then it appears that for a large (and realistic) class of indices, the instantaneous characteristics satisfy the positive slope requirement. Therefore, in systems where loads have appropriate voltage dependence, SVCs can successfully operate at points where the slope of the long term characteristic is negative. It was concluded that under these load modelling conditions, the absolute stability limit is given by the bifurcation point B of Figure 4.

However it has been shown that if an SVC was to encounter a limit while operating at a point where the long term characteristic had negative slope, system collapse would almost certainly occur. This is because the voltage could no longer be regulated at the SVC bus. Therefore, even though operation on a negative slope is practical (when load voltage indices are close to those of (2)), care must be taken to ensure that SVCs operating in that region do not hit limits. System operators would need to be made aware of this special operational requirement.

Having established that it is practical for SVCs to operate on the negative slope of the long term characteristic, power system planners and operators may be able to extend the power transfer capabilities of their systems. This may lead to a greater utilization of existing transmission plant, deferment of transmission reinforcement and consequent savings in capital expenditure.

## VII ACKNOWLEDGEMENTS

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## VIII REFERENCES

1. W.R. Lachs, "Voltage Collapse in EHV Power Systems", IEEE PES Winter Meeting, Paper No. A 78 057-2, New York, January/February 1978.
2. G. Brownell and H.K. Clark, "Analysis and Solutions for Bulk System Voltage Instability", *IEEE Comp. App. in Power*, July 1989, pp 31-35.
3. D.J. Hill, P-A. Lof and G. Andersson, "Analysis of Long-Term Voltage Stability", *10th Power Systems Comp. Conf.*, Graz, Austria, August 1990.
4. G.N. Allen, V.E. Henner and C.T. Popple, "Optimization of Static Var Compensators and Switched Shunt Capacitors in a Long Distance Interconnection", CIGRE Paper 38.07, 1988 Session, August/September 1988.
5. J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer-Verlag, New York, 1983.
6. I.A. Hiskens, "Energy Functions, Transient Stability and Voltage Behaviour in Power Systems", Ph.D. Thesis, *Department of Electrical Engineering and Computer Science, University of Newcastle*, Australia, March 1990.
7. I. Dobson et. al., "A Model of Voltage Collapse in Electric Power Systems", *Proc. 27th Conf. on Decision and Control*, Austin, Texas, December 1988.
8. K. Walve, "Modelling of Power System Components at Severe Disturbances", CIGRE Paper No. 38-18, *International Conf. on Large High Voltage Electric Systems*, August 1986.
9. S. Torseng, "Shunt-connected Reactors and Capacitors Controlled by Thyristors", *IEE Proc., Part C*, Vol. 128, No. 6, November 1981
10. I.A. Hiskens and D.J. Hill, "Energy Functions, Transient Stability and Voltage Behaviour in Power Systems with Nonlinear Loads", *IEEE Trans. on Power Systems*, Vol. PWRS-4, No. 4, November 1989.
11. B.C. Kuo, *Automatic Control Systems*, Prentice-Hall, Englewood Cliffs, New Jersey, Second Edition, 1967.
12. R.H. Craven and C.B. McLean, "The Impact of Voltage Stability Limitations on the Operation of the Queensland Electricity Commission's Power System", *7th CEPSI*, Paper No. 4-45, Brisbane, Australia, October 1988.

## APPENDIX A

### A.1 System Model

In this analysis, we are interested only in cases where SVC gains are zero, and taps are fixed. It is shown in [10] that under these conditions, the power system can be simply represented as

$$\dot{\underline{\omega}}_g = -\underline{M}_g^{-1}\underline{D}_g\underline{\omega}_g - \underline{M}_g^{-1}\underline{T}_g^t\underline{f}_g(\underline{\alpha}_g, \underline{\alpha}_\ell, |\underline{V}|) \tag{3a}$$

$$\dot{\underline{\alpha}}_g = \underline{T}_g\underline{\omega}_g \tag{3b}$$

$$\underline{0} = \underline{f}_\ell(\underline{\alpha}_g, \underline{\alpha}_\ell, |\underline{V}|) \tag{4a}$$

$$\underline{0} = \underline{g}(\underline{\alpha}_g, \underline{\alpha}_\ell, |\underline{V}|) \tag{4b}$$

where

$\underline{M}_g, \underline{D}_g$ - diagonal matrices of inertia, damping constants

| | |
|---|---|
| $\underline{T}_g$ | - specially structured matrix of ±1 entries |
| $\underline{\omega}_g$ | - generator frequency deviations |
| $\underline{\alpha}_g$ | - generator phase angles |
| $\underline{\alpha}_\ell$ | - load bus phase angles |
| $\lvert\underline{V}\rvert$ | - load bus voltage magnitudes |
| $-\underline{f}_g$ | - accelerating power at generators |
| $\underline{f}_\ell$ | - real power balance at load buses |
| $\underline{g}$ | - reactive power balance at load buses |

Note that in this representation, generator dynamics are described by the classical machine model. This is sufficiently accurate for the present investigation, as we are predominantly interested in machine/network interactions, rather than detailed behaviour of the machines themselves.

## A.2 Small Disturbance Stability Analysis

Small disturbance stability analysis involves the investigation of the eigenvalues of the system $\underline{A}$ matrix formed by linearizing the model (3),(4) about a particular operating point. An operating point is small disturbance stable if and only if all eigenvalues of $\underline{A}$ lie in the open left half complex plane [11], i.e., they all have negative real parts.

It is easily shown that linearizing (3),(4) gives

$$\begin{bmatrix} \Delta\dot{\underline{\alpha}}_g \\ \Delta\dot{\underline{\omega}}_g \end{bmatrix} = \begin{bmatrix} \underline{0} & \vdots & \underline{T}_g \\ \cdots & & \cdots \\ -\underline{M}_g^{-1}\underline{T}_g^t\underline{F} & \vdots & -\underline{M}_g^{-1}\underline{D}_g \end{bmatrix} \begin{bmatrix} \Delta\underline{\alpha}_g \\ \Delta\underline{\omega}_g \end{bmatrix} = \underline{A} \begin{bmatrix} \Delta\underline{\alpha}_g \\ \Delta\underline{\omega}_g \end{bmatrix} \tag{5}$$

where $\underline{F}$ is obtained from the power flow Jacobian

$$\mathbb{F} = \begin{bmatrix} \frac{\partial \underline{f}_g}{\partial \underline{\alpha}_g} & \vdots & \frac{\partial \underline{f}_g}{\partial \underline{\alpha}_\ell} & \frac{\partial \underline{f}_g}{\partial \lvert\underline{V}\rvert} \\ \cdots & & \cdots & \cdots \\ \frac{\partial \underline{f}_\ell}{\partial \underline{\alpha}_g} & \vdots & \frac{\partial \underline{f}_\ell}{\partial \underline{\alpha}_\ell} & \frac{\partial \underline{f}_\ell}{\partial \lvert\underline{V}\rvert} \\ \frac{\partial \underline{g}}{\partial \underline{\alpha}_g} & \vdots & \frac{\partial \underline{g}}{\partial \underline{\alpha}_\ell} & \frac{\partial \underline{g}}{\partial \lvert\underline{V}\rvert} \end{bmatrix} = \begin{bmatrix} \underline{J}_{gg} & \vdots & \underline{J}_{g\ell} \\ \cdots & & \cdots \\ \underline{J}_{\ell g} & \vdots & \underline{J}_{\ell\ell} \end{bmatrix} \tag{6}$$

as

$$\underline{F} = \underline{J}_{gg} - \underline{J}_{g\ell}\underline{J}_{\ell\ell}^{-1}\underline{J}_{\ell g} \tag{7}$$

It is shown in [6] that due to the structure of $\underline{A}$, the number of positive eigenvalues of $\underline{A}$ is typically equal to the number of negative eigenvalues of $\underline{F}$. Therefore the small disturbance stability of the system can be determined from the signs of the eigenvalues of $\underline{F}$.

We now proceed to show that those eigenvalue signs can be determined from the slopes of the voltage-susceptance characteristics. This analysis is dependent upon the application of Schur's formula to (6), i.e., if $\underline{J}_{\ell\ell}$ is nonsingular then

$$\begin{aligned} \det \mathbb{F} &= \det \underline{J}_{\ell\ell} \det [\underline{J}_{gg} - \underline{J}_{g\ell}\underline{J}_{\ell\ell}^{-1}\underline{J}_{\ell g}] \\ &= \det \underline{J}_{\ell\ell} \det \underline{F} \end{aligned} \tag{8}$$

Because SVC gain is zero, voltage is not being regulated, so point A of Figure 2 is a bifurcation point of the power flow equations. (Recall also that taps are fixed.) Therefore $\mathbb{F}$ is singular at A, i.e., $\det \mathbb{F} = 0$. Above A, in the 'normal' operating region, $\det \mathbb{F} > 0$. Typically the sign of $\det \mathbb{F}$ changes at A, so that at points below A, e.g., points D and H, $\det \mathbb{F} < 0$. The sign of $\det \mathbb{F}$ and the sign of the slope of the short term characteristic a-a are therefore in agreement [12].

Now consider the instantaneous characteristics, e.g., curves b-b,c-c,d-d of Figure 2. Each such characteristic corresponds to generator angles $\underline{\alpha}_g$ being constrained, rather than to generator powers being enforced. Therefore these characteristics are determined from the algebraic equations (4) only, with $\underline{\alpha}_\ell$ and $\lvert\underline{V}\rvert$ being the only free variables. The corresponding Jacobian is therefore composed of the partial derivatives of $\underline{f}_\ell$ and $\underline{g}$ with respect to $\underline{\alpha}_\ell$ and $\lvert\underline{V}\rvert$. From (6), that Jacobian is $\underline{J}_{\ell\ell}$, (a submatrix of $\mathbb{F}$). It follows from bifurcation theory [5] that bifurcation points on the instantaneous characteristics, e.g., point E, correspond to $\det \underline{J}_{\ell\ell} = 0$. Therefore using the same argument as for the long term characteristic, it may be concluded that the sign of the slope of the instantaneous characteristic and that of $\det \underline{J}_{\ell\ell}$ are typically in agreement.

Based on the above reasoning, it can be deduced that at point D, $\det \mathbb{F} < 0$ and $\det \underline{J}_{\ell\ell} < 0$. Therefore by (8), $\det \underline{F} > 0$. Such a condition may be due to an even number of negative eigenvalues. However, it is argued in [6] that in general $\underline{F}$ will have no negative eigenvalues at such points. Therefore, under normal conditions, point D is small disturbance stable.

At point H, it is apparent that $\det \mathbb{F} < 0$ whilst $\det \underline{J}_{\ell\ell} > 0$. Hence by (8), $\det \underline{F} < 0$. $\underline{F}$ must have at least one negative eigenvalue, so the point is unstable.

* * *

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## Discussion

**M. K. Pal** (Public Service Electric and Gas Company, Newark, N.J.): This paper attempts to demonstrate that SVCs can extend stable operation into the region that is considered voltage unstable in the absence of such controls, and that the operation must be well within limits. These facts are well known, although the actual mechanism of extending the stability limit is not fully understood by many. The authors' treatment of the subject, especially in the first half of the paper, unfortunately, does not improve the situation.

Throughout the paper the authors seek to explain the behavior of SVCs with the help of the voltage-susceptance curves. They use the concepts of long term, short term and instantaneous voltage-susceptance characteristics, and note that a necessary condition for stable operation of an SVC is that at the operating point the instantaneous characteristic must have positive slope. There are, however, inconsistencies in their explanation of how these instantaneous and other characteristics were developed. For example, one of the curves (short term) was apparently drawn with loads given by equation (1) of the paper, before transformer taps had moved and with generator powers constrained to their equilibrium values. It is not clear how a power balance would be obtained.

It is, also, not clear how the voltage-susceptance curves can look like those shown in the paper. The stable SVC behavior for constant power load is explained with the help of the instantaneous voltage-susceptance characteristic. However, since, presumably, this was drawn using a generator internal voltage (constant flux linkage), it should appear to the left of the steady-state curve a-a of Figure 2 of the paper, in the normal operating range. In other words, the slope on the instantaneous curve would be less favorable than that on the steady-state curve. Obviously, the explanation of stable operation of SVCs for constant power load as provided by the authors cannot be valid.

Actually, the problem in the authors' explanation has its root in the general misconceptions in the field of voltage stability. The misconceptions resulted from an attempt to predict system performance, which is basically dynamic, using a static system formulation. An increase in shunt capacitance at a bus cannot cause the voltage to decrease under any operating condition. It is true that under certain control actions, such as under the control of an SVC, while operating in the lower portion of the P-V curve, a lowering of the voltage set-point would be accompanied by an eventual increase in the reactive injection from the SVC. However, this is not a natural system response. The "negative slope," as the authors call it, should not be taken too literally. It simply reflects the low voltage steady-state solution based on constant power static load model.

The constant power static load model led to the conclusion in section 4 of the paper that point D in Figure 2 is small disturbance unstable with SVC but stable without. A constant power load is not a static load. This means it cannot jump instantaneously from one demand level to another as the demand changes. Similarly, following a sudden system change, the load will change momentarily. It will then be restored to the constant power level by the restoring mechanism. A definite time lag is involved in the process. The instantaneous characteristic of all loads is static, i.e. it is predominantly voltage dependent. In this sense the explanation provided in section 5 of the paper is conceptually correct. As long as the SVC has enough margin, stable operation can be extended until the angle stability limit is reached.

A truly satisfactory explanation of stable operation in the lower portion of the P-V curve for constant MVA load (self-restoring load) would require simultaneous consideration of the relevant dynamics of the load and the SVC control mechanism. Note that, for static load, such as constant impedance, SVC is not necessary for maintaining voltage stability, although it will certainly help in precise voltage control.

We provide below a simple but rigorous explanation of the mechanism of the extension of voltage stability limit due to SVC. Consider the system shown in Figure 1, supplying a unity power factor load whose voltage is being controlled by an SVC. We will use an SVC model similar to that shown in Figure 3 of the paper, and described by

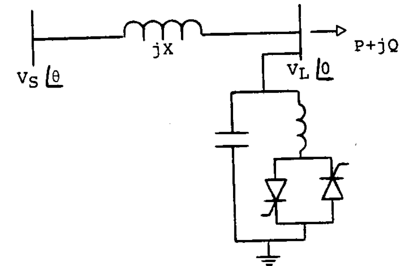


Fig.1 A power system with constant sending-end voltage supplying a load whose voltage is maintained constant by an SVC.

$$T_Q \frac{d}{dt} B = V_{ref} - V_L \qquad (1)$$

For our purpose, the dynamic behavior of the constant power load may be represented by a simple first order delay model, whose dynamics may be represented by

$$T_L \frac{d}{dt} G = P_o - V_L^2 G \qquad (2)$$

where

$V_L$ is the load voltage
$P_o$ is the power set point
$G$ is the load conductance which is adjusted to maintain constant power
$T_L$ is the load time constant

The load model given by equation (2), describes the basic dynamics, pertinent to voltage stability , of a wide variety of loads. With proper interpretation, it can also approximately represent the basic dynamics of induction motors. (Note that other load models may be used.) The power balance equations are:

$$P = V_L^2 G = \frac{V_S V_L}{X} \sin\theta \qquad (3)$$

$$Q = 0 = \frac{V_S V_L}{X} \cos\theta - \frac{V_L^2}{X} + V_L^2 B \qquad (4)$$

After linearizing equations (1) - (4), and eliminating the non-state variables, we obtain the state-space model $\dot{x} = Ax$, where

$$x = \begin{bmatrix} \Delta B \\ \Delta G \end{bmatrix}$$

The elements of A are:

$$a_{11} = -\frac{V_L}{2T_Q G} \sin 2\theta , \quad a_{12} = \frac{V_L}{T_Q G} \sin^2\theta$$

$$a_{21} = -\frac{V_L^2}{T_L} \sin 2\theta , \quad a_{22} = -\frac{V_L^2}{T_L} \cos 2\theta$$

The characteristic equation is

$$\lambda^2 + a\lambda + b = 0$$

where

$$a = -(a_{11} + a_{22}) = \frac{V_L}{2T_Q G} \sin 2\theta + \frac{V_L^2}{T_L} \cos 2\theta$$

$$b = a_{11}a_{22} - a_{12}a_{21} = \frac{V_L^3}{2T_Q T_L G} \sin 2\theta$$

For stability, $a > 0$, $b > 0$. After some algebraic manipulation and noting from equations (3) and (4) that $\tan\theta = GX/(1-BX)$, we arrive at the following stability condition

$$1 - \tan^2\theta > -\frac{T_L}{T_Q V_L}\frac{X}{1-BX} \tag{5}$$

Equation (5) shows that, when $T_Q << T_L$, which would be true for most load types, voltage stability limit can extend to $\theta$ approaching 90°, i.e. well into the lower portion of the P-V curve. (Note that for unity power factor load, $\theta$ = 45° corresponds to the maximum power point on the P-V curve; $\theta$ > 45° corresponds to operation on the lower portion of the curve. Also, beyond $\theta$ = 45°, the reactive requirement increases rapidly.) On the other hand if $T_L = 0$, i.e. if the load adjusts to constant power instantaneously, the stability limit occurs at $\theta$ = 45°, implying that no improvement in voltage stability limit over that obtainable from static shunt reactive support is possible. It may be of interest to note that if, in the above analysis, a constant power static load model is used, the result will correspond to that for the dynamic load case with $T_L = 0$.

In the voltage-susceptance characteristic shown (curve a-a in Figure 2 of the paper), point A corresponds to $\theta$ = 45° in the simple example of Figure 1. Point B corresponds to $\theta$ = 90°, at which point angle stability limit would be encountered for constant sending- and receiving-end voltage. It is not clear why the analysis was extended beyond this point. Point G is unstable from the angle stability point of view. A system cannot be operated stably at this point, unless the voltages are adjusted to maintain stability.

It is not clear why the appendix was included. The objective seems to be to relate voltage stability limit without SVCs, to the singularity of the power flow and other Jacobians. The literature is replete with such analyses, all of which are based on flawed mathematics. The problem with these analyses is clearly demonstrated in [A]. As noted earlier, the instantaneous characteristics, if drawn correctly, would be oriented differently. In that case, the authors' analysis would show that all points between A and B, i.e. both D and H, are stable, and only a region immediately above A is unstable. This is, of course, absurd, and is a consequence of the flawed model.

[A] M. K. Pal, Discussion of "An Investigation of Voltage Instability Problems," by N. Yorino, H. Sasaki, Y. Masuda, Y. Tamura, M. Kitagawa and A. Oshimo, 91 WM 202-2 PWRS, IEEE/PES winter meeting February, 1991.

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**I.A. HISKENS and C.B. McLEAN:** We thank Dr. Pal for the opportunity to explain more fully some of the concepts developed in our paper. We will answer Dr. Pal's criticisms in the order in which they appear in his discussion.

Dr. Pal is concerned about the equilibrium values of generator powers used to obtain the short term characteristics. In the short term, before governor action, any load-generation imbalance is picked up by generators in proportion to their damping constants (or inertia constants in the case of zero damping) [B]. Equilibrium values of generator powers are therefore given by their predisturbance values adjusted by the appropriate proportion of the power imbalance.

Dr. Pal is of the opinion that in the normal operating region the instantaneous characteristic should always be steeper than the short term characteristic. However Figures 2 and 4 provide counterexamples to that claim, as they are actual characteristics for the system of Figure 1. In fact, it is not clear to us that any general statement can be made regarding the relative slopes of the instantaneous and short term characteristics at an arbitrary operating point. The relative slopes would appear to be system and operating point dependent. This however does not affect the validity of our analysis at all. As explained in the paper, provided the instantaneous characteristic has positive slope, network response will be consistent with SVC control and stable operation will occur. Only when instantaneous characteristics have negative slope will the SVC respond in an incorrect manner. But it follows from the analysis of the Appendix that this negative slope condition cannot occur in the normal operating region. In that region, det $\underline{\mathbb{F}}$ and det $\underline{F}$ are both positive, so from (8), det $\underline{J}_{\ell\ell}$ must also be positive. Therefore the slope of the instantaneous characteristic must be positive. (Note that this is not meant to be a rigorous argument, but is true for normal power systems.)

Dr. Pal is under the misconception that our analysis is based on a constant power static load model. However it is emphasised in the paper that this is not the case. In Sections 4.1 and 4.2 it was stated that a constant power static load model was used solely for the purpose of exploring the various possibilities for short term behaviour of SVCs. Having established these possibilities, Section 4.3 then considered more realistic load models. Dr. Pal agrees with us that in determining the instantaneous characteristics, loads should be modelled as voltage dependent. However he seems to be unaware of the influence that load indices can have on the shape of these characteristics. For example, for real power load indices $\zeta_i < 1$, it cannot be guaranteed that an increase in shunt capacitance at a bus will not result in a voltage decrease. This type of behaviour is theoretically possible. Further details can be found in [6,C].

Dr. Pal's discussion of the system shown in his Figure 1 is interesting, but limited by his use of a constant admittance model for the instantaneous voltage dependence of the load, see (2). Further, it is not valid to draw conclusions about the constant load power case by setting $T_L$=0 as this is a singular perturbation of the original system. A more informative investigation is possible if (2) and (3) are replaced by

$$T_L \dot{P}_L = P_o - V_L^{\zeta} P_L \tag{2'}$$

$$P = V_L^{\zeta} P_L = \frac{V_S V_L}{X}\sin\theta \tag{3'}$$

where $P_L$ is a base load which is adjusted to maintain constant power. With this formulation we can determine the effect of the load index on stability. Again, to investigate stability the system is linearized. In general $T_Q << T_L$, so we shall assume that in the short term $\Delta P_L \approx 0$. SVC stability (in the short term) is then given by

$$\Delta\dot{B} = -\frac{1}{T_Q}\frac{\Delta V_L}{\Delta B}\Delta B$$

where

$$\frac{\Delta V_L}{\Delta B} = \frac{1}{det}\frac{V_S V_L^3}{X}\cos\theta$$

$$det = \frac{V_S^2 V_L}{X^2}\left[\cos^2\theta + (\zeta - 1)\sin^2\theta\right]$$

$det$ is the determinant of the Jacobian of partial derivatives of algebraic equations (3'),(4) with respect to the algebraic variables $V_L, \theta$. This Jacobian has the same significance in this system as

$J_{\ell\ell}$ has in the Appendix. When $\frac{\Delta V_L}{\Delta B}$ is positive, the SVC is stable. Negative $\frac{\Delta V_L}{\Delta B}$ results in instability.

If $\zeta>1$, $det$ is always positive. Stability is then governed by the $\cos\theta$ term in $\frac{\Delta V_L}{\Delta B}$. The system is stable for $\theta$ up to $90^\circ$. This is consistent with Dr. Pal's findings, as he used $\zeta=2$. However if $\zeta<1$, the sign of $det$ depends on the value of $\theta$. Stability of the operating point therefore also depends on $\theta$. (This loss of structural stability at $\zeta=1$ is consistent with the discussion of Section 4.3.) Consider the constant power case, i.e., $\zeta=0$. $det$ is positive for $\theta<45^\circ$ and negative for $\theta>45^\circ$. The stability limit therefore occurs at $\theta=45^\circ$. Notice though that as this stability limit is approached, the eigenvalue does not pass through zero, but instead approaches $-\infty$ then jumps to $+\infty$. This is a consequence of the singularity of the algebraic equations, i.e., $det=0$. Notice also that at this point, the $V_L$-B characteristic becomes vertical, i.e., a small change in B causes an infinite change in $V_L$. This is consistent with the findings of Section 4.2.

Dr. Pal concludes that there is correspondance between points A,B of Figure 2 and $\theta=45^\circ,90^\circ$ respectively for his example system. This is incorrect. Dr. Pal's example is based on the load responding statically as constant admittance. In terms of this static load response, his example is equivalent to the situation depicted in Figure 4 not Figure 2. There is correspondance between the equivalent points of Figure 4 and the $\theta=45^\circ,90^\circ$ cases, but only in the sense that they indicate similar types of limiting conditions. Note though that Figure 4 is based on an interconnected system (Figure 1 of the paper), whereas Dr. Pal's system is radial. Dr. Pal's suggestion that point G would be unstable is correct when referring to Figure 4. However it is not correct for Figure 2. This difference is discussed in the paper.

The Appendix provides the mathematical background for interpreting the stability of points on voltage-susceptance characteristics from the slopes of these characteristics. This mathematics enables the connection between load indices and the slope of instantaneous characteristics, and hence SVC stability, to be established. This is valuable information for guiding power system planners and operators when deciding whether it is acceptable for SVCs to operate on the negative slope portion of long term characteristics.

Dr. Pal claims that the Appendix contains flawed mathematics, but provides no details. He refers to [A] in which he discusses two small systems; one which is irrelevant and one which is a special degenerate case of the general systems considered in the Appendix. We remain confident that our analysis is correct.

We hope that in answering Dr. Pal's criticisms we have been able to demonstrate that our analysis is valid and informative.

[B] D.J. Hill, "On the equilibria of power systems with nonlinear loads", IEEE Trans. on Circuits and Systems, Vol. 36, No. 11, November 1989, pp 1458-1463.

[C] I.A. Hiskens and D.J. Hill, "Failure modes of a collapsing power system", Proceedings of the Workshop on Bulk Power System Voltage Phenomena, Deep Creek Lake, MD, August 1991, to appear.

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