DIRECT COMPUTATION OF CRITICAL CLEARING TIME USING TRAJECTORY SENSITIVITIES

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Abstract: The transient energy function (TEF) and single machine equivalent (SIME) techniques have been used successfully over the years to compute the critical clearing time for faults in the system. Sensitivity of the energy margin has also been used to find critical generators for rescheduling of generation. This procedure involves using trajectory sensitivities explicitly and the computation of the critical energy V_{cr} depends on the system parameters. The need to compute V_{cr} makes it computationally intensive. In this paper we compute the sensitivity of the energy function to the fault clearing time t_{cl} directly. By computing this sensitivity for two values of t_{cl}, the results can be extrapolated to obtain a good estimate of t_{cr}. The method is illustrated for a structure-preserving model of a 3-machine, 9-bus system with nonlinear voltage dependent loads.

Keywords: Power Systems Stability, Trajectory Sensitivity, Transient Energy Function

I. INTRODUCTION

In the new restructuring scenario of power systems, it is very important to assess the dynamic stability of the operating point of the systems in the case of contingencies. The TEF technique is one of the powerful tools to achieve this information and has been the topic for research for the last few decades. Sensitivity approach in dynamic security assessment (DSA) and its analytical calculations were originally proposed in [1]. In Refs. [2] and [3], sensitivities of the normalized energy margin with respect to different system parameters were calculated for analyzing power system stability. The estimation of stability limits of power systems using sensitivity of the energy margin was carried out in [4]. In this paper, we use sensitivity of the energy function itself to the clearing time to estimate the critical clearing time for a particular fault. This sensitivity is computed using trajectory sensitivities. The big advantage in this method is that there is no need to calculate the critical value of the energy function. The sensitivity is computed for two values of t_{cl} and then extrapolated to obtain the estimate of the critical clearing time. This idea is similar to that of Ref. [5] where based on simulation the system is reduced to a single machine equivalent (SIME) and then t_{cr} is estimated.

Fairly restrictive modeling assumptions are required to rigorously establish energy functions. Accordingly, true Lyapunov stability arguments can only be made for systems that satisfy those assumptions. However the stability assessment approach proposed in this paper does not rely on Lyapunov concepts. Rather, the energy function is used purely as a metric, or measure, of the "distance" between the transient state (a point on the trajectory) and the stable equilibrium point. Therefore no restrictions need to be placed on system modeling. Additional computational tasks are involved in calculating the trajectory sensitivities. This results in extra differential-algebraic equations of a multimachine system. However, one can exploit the structural similarity in the Jacobian of both the system and sensitivity models.

The paper is organized as follows. System and sensitivity models for differential-algebraic equations (DAE) are discussed in section 2. Section 3 shows the method for estimation of t_{cr} for DAE models using TEF sensitivity. Section 4 gives an overview of the energy function and its sensitivities. Section 5 gives results for a test system using the method discussed in sections 3 and 4. The test system used here is a 3-machine, 9-bus power system.

II. SYSTEM SENSITIVITY MODELS

In simulating disturbances, switching actions take place at certain time instants. At these time instants, the algebraic equations change, resulting in discontinuities of the algebraic variables. Following [6], the equations can then be written as a set of differential-algebraic equations of the form

$$\dot{x} = f(x, y, \lambda) \tag{1}$$

$$0 = \begin{cases} g^{-}(x, y, \lambda) & s(x, y, \lambda) < 0 \\ g^{+}(x, y, \lambda) & s(x, y, \lambda) > 0 \end{cases} \tag{2}$$

and a switching occurs when $s(x, y, \lambda) = 0$.

In the above model, x are the dynamic state variables such as machine angles, velocities, etc.; y are the algebraic variables such as load bus voltage magnitudes and angles; and λ are the system parameters such as line reactances, generator mechanical input power, or fault clearing time. Note that the state variables x are continuous while the algebraic variables can undergo step changes at switching instants.

The initial conditions for (1)-(2) are given by

$$x(t_0) = x_0 \quad (3)$$

$$y(t_0) = y_0 \quad (4)$$

where y_0 satisfies the equation

$$g(x_0, y_0, \lambda) = 0 \quad (5)$$

For compactness of notation, the following definitions are used

$$\underline{x} = \begin{bmatrix} x \\ \lambda \end{bmatrix}$$

$$\underline{f} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

With these definitions, (1)-(2) can be written in a compact form as

$$\dot{\underline{x}} = \underline{f}(\underline{x}, y) \quad (6)$$

$$0 = \begin{cases} g^{-}(\underline{x}, y) & s(\underline{x}, y) < 0 \\ g^{+}(\underline{x}, y) & s(\underline{x}, y) > 0 \end{cases} \quad (7)$$

The initial conditions for (6)-(7) are

$$\underline{x}(t_0) = \underline{x}_0 \quad (8)$$

$$y(t_0) = y_0 \quad (9)$$

Trajectory sensitivity analysis studies the variations of the system variables with respect to the small variations in initial conditions x_0 and parameters λ (or equivalently $\underline{x}_0$).

Away from discontinuities, the differential-algebraic system can be written in the form

$$\dot{\underline{x}} = \underline{f}(\underline{x}, y) \quad (10)$$

$$0 = g(\underline{x}, y) \quad (11)$$

Differentiating (10) and (11) with respect to the initial conditions $\underline{x}_0$ yields

$$\dot{\underline{x}}_{\underline{x}_0} = \underline{f}_{\underline{x}}(t)\underline{x}_{\underline{x}_0} + \underline{f}_y(t) y_{\underline{x}_0} \quad (12)$$

$$0 = g_{\underline{x}}(t)\underline{x}_{\underline{x}_0} + g_y(t) y_{\underline{x}_0} \quad (13)$$

where $\underline{f}_{\underline{x}}$, $\underline{f}_y$, $g_{\underline{x}}$, and g_y are time varying matrices and are calculated along the system trajectories. $\underline{x}_{\underline{x}_0}(t)$ and $y_{\underline{x}_0}(t)$ are the trajectory sensitivities.

Initial conditions for $\underline{x}_{\underline{x}_0}$ are obtained by differentiating (8) with respect to $\underline{x}_0$ as

$$\underline{x}_{\underline{x}_0}(t_0) = I \quad (14)$$

where I is the identity matrix.

Using (14) and assuming that $g_y(t_0)$ is nonsingular along the trajectories, initial conditions for $y_{\underline{x}_0}$ can be calculated from (13) as

$$y_{\underline{x}_0}(t_0) = -[g_y(t_0)]^{-1} g_{\underline{x}}(t_0) \quad (15)$$

Therefore, the trajectory sensitivities can be obtained by solving (12) and (13) simultaneously with (10) and (11) using (8), (9), (14) and (15) as the initial conditions. At the discontinuity where $s(\underline{x}, y) = 0$, the jump condition in the sensitivity of $\underline{x}$ and y are computed as discussed in [7].

III. ESTIMATION OF CRITICAL CLEARING TIME USING TRAJECTORY SENSITIVITIES

In the literature, trajectory sensitivities have been used [8] to compute the energy margin sensitivity with respect to system parameters such as interface line flow, system loading. In these cases, v_{cr} depends on the parameters and hence computation of v_{cr} is necessary. This is computationally a difficult task. On the other hand, if the objective is to only get an estimate of t_{cr}, then we can avoid the computation of v_{cr}. Because the energy function $V(x)$ is used as a metric to monitor the system sensitivity for different t_{cl}, we can use it to estimate t_{cr} directly as follows. The sensitivity $S = \frac{\partial v}{\partial t_{cl}}$ is computed for two different values of t_{cl} which are chosen to be less than t_{cr}. Because the system under consideration is stable, the sensitivity S will display larger excursions for larger t_{cl}. Next, the reciprocal of η the maximum deviation of S is computed as $\eta = \frac{1}{\max(S) - \min(S)}$. A straight line is constructed from the two points (t_{cl1}, η_1) and (t_{cl2}, η_2). The estimated critical clearing time $t_{cr,est}$ is the intersection of the constructed straight line with the time-axis in the (t_{cl}, η)-plane as shown in Fig. 1

The energy function used in this paper is for the structure-preserving model with classical machine representation and nonlinear load representation which is discussed in the next section.

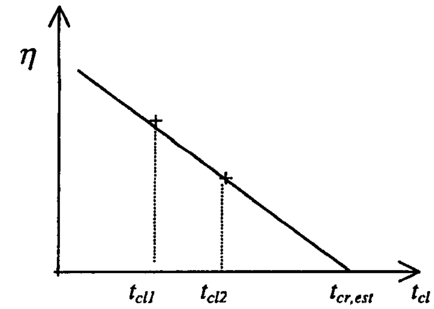

Fig. 1: Estimate of t_{cr}

IV. SYSTEM MODEL, ENERGY FUNCTION AND SENSITIVITY

In the rigorous justification of the structure-preserving energy function, all synchronous machines are assumed as classical models, i.e., they are represented by constant voltage in series with the transient reactance. Loads are assumed to have constant real power and voltage dependent reactive power. Furthermore, assume that reactive power load at the i-th bus can be expressed in the form

$$Q_{di}(V_i) = Q_{di}^s \left(\frac{V_i}{V_i^s} \right)^{\alpha} \tag{16}$$

where Q_i^s and V_i^s are the nominal steady state reactive power load and voltage magnitude at the i-th bus, and α is the reactive power load index.

Let the power system consists of n_0 buses, with generators attached to m of the buses. Hence there are $n_0 - m$ load buses with no generation. The power system is augmented by m fictitious buses representing the generator internal buses. The total number of buses in the augmented network is therefore $n = n_0 + m$.

The network is assumed to be lossless, so that all lines are modeled as series reactances. The bus admittance matrix Y is therefore purely imaginary, with elements $Y_{ik} = jB_{ik}$.

Let the complex voltage at the i-th bus be the phasor $V_i \angle \delta_i$ where δ_i is the bus phase angle with respect to a synchronously rotating reference frame.

The center of angle (COA) of a m-machine, n_0-bus system is defined as

$$\delta_0 = \frac{1}{M_T} \sum_{i=1}^{m} M_i \delta_{n_0+i} \tag{17}$$

where $M_T = \sum_{i=1}^{m} M_i$. It follows that

$$\omega_0 = \frac{1}{M_T} \sum_{i=1}^{m} M_i \omega_{g_i} \text{ and } \dot{\omega}_0 = \frac{1}{M_T} \sum_{i=1}^{m} M_i \dot{\omega}_{g_i}$$

In this paper the COA is chosen as reference. Therefore, the rotor angles and bus phase angles referred to the COA are

$$\theta_i = \delta_i - \delta_0 \qquad i=1,\ldots,n \tag{18}$$

By defining

$$\tilde{\omega}_{g_i} = \omega_{g_i} - \omega_0 \qquad i = 1,\ldots,m \tag{19}$$

The power system can be represented by the DAE model as

$$\dot{\theta}_{n_0+i} = \tilde{\omega}_{g_i} \quad i = 1,\ldots,m \tag{20}$$

$$M_i \dot{\tilde{\omega}}_{g_i} = P_{M_i} - \sum_{j=1}^{n} B_{n_0+i,j} V_{n_0+i} V_j \sin\left(\theta_{n_0+i} - \theta_j\right) - \frac{M_i}{M_T} P_{COA} \qquad i = 1,\ldots,m \tag{21}$$

$$P_{d_i} + \sum_{j=1}^{n} B_{ij} V_i V_j \sin(\theta_i - \theta_j) = 0 \quad i = 1,\ldots,n_0 \tag{22}$$

$$Q_{d_i}(V_i) - \sum_{j=1}^{n} B_{ij} V_i V_j \cos(\theta_i - \theta_j) = 0 \quad i = 1,\ldots,n_0 \tag{23}$$

where

$$P_{COA} = \sum_{i=1}^{m} \left(P_{M_i} - \sum_{j=1}^{n} B_{ij} V_i V_j \sin(\theta_i - \theta_j) \right)$$

The corresponding energy function is established as [9]

$$\begin{aligned} v(\tilde{\omega}_g, \theta, V) = {} & \frac{1}{2} \sum_{i=1}^{m} M_i \tilde{\omega}_{g_i}^2 \\ & - \sum_{i=1}^{m} P_{M_i} (\theta_{n_0+i} - \theta_{n_0+i}^s) + \sum_{i=1}^{n_0} P_{d_i} (\theta_i - \theta_i^s) \\ & - \frac{1}{2} \sum_{i=1}^{n_0} B_{ii} (V_i^2 - V_i^{s2}) + \sum_{i=1}^{n_0} \frac{Q_{d_i}^s}{\alpha V_i^{s\alpha}} (V_i^{\alpha} - V_i^{s\alpha}) \\ & - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} B_{ij} (V_i V_j \cos\theta_{ij} - V_i^s V_j^s \cos\theta_{ij}^s) \end{aligned} \tag{24}$$

where $\theta_{ij} = \theta_i - \theta_j$.

The sensitivity S of the energy function $v(x)$ with respect to clearing time ($\lambda = t_{cl}$) is obtained by taking partial derivatives of (24) with respect to t_{cl}

$$
\begin{aligned}
\frac{\partial v}{\partial t_{cl}} = & \sum_{i=1}^{m} M_i \tilde{\omega}_{g_i} \frac{\partial \tilde{\omega}_{g_i}}{\partial t_{cl}} - \sum_{i=1}^{m} P_{M_i} \frac{\partial \theta_{n_0+i}}{\partial t_{cl}} \\
& + \sum_{i=1}^{n_0} P_{d_i} \frac{\partial \theta_i}{\partial t_{cl}} - \sum_{i=1}^{n_0} B_{ii} V_i \frac{\partial V_i}{\partial t_{cl}} \\
& + \sum_{i=1}^{n_0} Q_{d_i}^{s} \frac{V_i^{\alpha-1}}{V_i^{s\alpha}} \frac{\partial V_i}{\partial t_{cl}} \\
& - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} B_{ij} (V_j \cos\theta_{ij} \frac{\partial V_i}{\partial t_{cl}} + V_i \cos\theta_{ij} \frac{\partial V_j}{\partial t_{cl}} \\
& - V_i V_j \sin\theta_{ij} \frac{\partial \theta_{ij}}{\partial t_{cl}})
\end{aligned} \qquad (25)
$$

The partial derivatives of $\tilde{\omega}_{g_i}$, θ and V with respect to t_{cl} are the sensitivities obtained from (12) and (13).

We now illustrate this technique using a 3-machine test system.

V. NUMERICAL RESULTS

A 3-machine, 9-bus power system is used to illustrate the technique [10]. For this system, a self-clearing fault is simulated at bus 5 and cleared at two different values of t_{cl}. The corresponding values of η are computed, and the results are shown in Fig. 2. The reactive power load index α is chosen as 2.

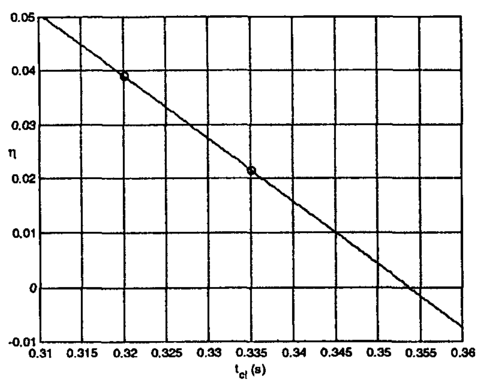

Fig. 2. Estimate t_{cr} for fault at bus 5

By using the technique described in sections 3 and 4, the estimated critical clearing time is $t_{cr,est}$ = 0.354 s. The actual critical clearing time obtained by successive simulations is t_{cr} = 0.352 s. Sensitivities of the energy function corresponding to the two values of t_{cl} are shown in Fig. 3 and Fig. 4.

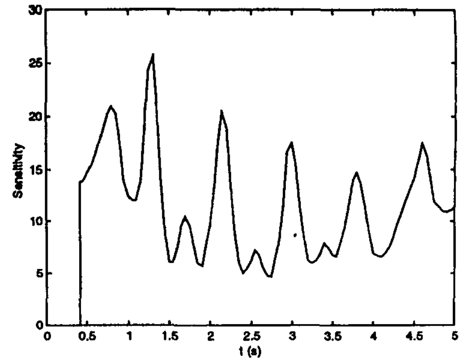

Fig. 3: Sensitivity of the energy function for t_{cl} = 0.32 s

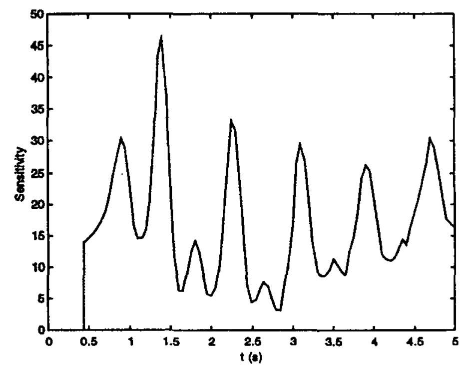

Fig. 4: Sensitivity of the energy function for t_{cl} = 0.335 s

The procedure is repeated for the same system with the fault at bus 8. The results are shown in Fig. 5. The estimated critical clearing time is $t_{cr,est}$ = 0.333 s. By successive simulations the critical clearing time is found to be t_{cr} = 0.334 s in this case.

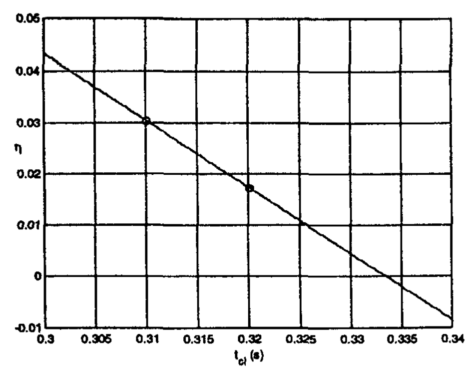

Fig. 5. Estimate t_{cr} for fault at bus 8

These examples show that the technique given in sections 3 and 4 gives a good way to estimate the critical clearing time of faults. A similar process can be used to estimate the critucal value of any parameter.

VI. CONCLUSIONS

In this paper, a direct technique to compute the critical clearing time for faults in power systems is proposed. The numerical results on a 3-machine, 9-bus system have shown that the technique gives fairly accurate results. We have extended the technique to larger systems and the results are encouraging. The procedure presented in this paper can be adapted to provide the critical value of any parameter.

VII. ACKNOWLEDGEMENTS

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IX. BIOGRAPHIES

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