Two-Stage Model Predictive Control for Voltage Collapse Prevention

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***Abstract*—The paper proposes a two-stage model predictive control (MPC) strategy for alleviating voltage collapse. The first stage uses a static load shedding algorithm to obtain stable MPC prediction simulations. The second stage uses the MPC simulations, and associated trajectory sensitivities, to establish a linear program (LP) that optimizes the control action. This LP builds on trajectory approximations that are generated from the sensitivities. The first stage prestabilizing process improves the accuracy of the trajectory approximations, with consequent improvement in the LP optimization results. The control action determined by the LP is subsequently applied to the actual system. The paper discusses the integration of this MPC algorithm into an energy management system (EMS) environment. A standard 10 bus voltage collapse case is used to illustrate the performance of the overall control strategy.**

I. INTRODUCTION

Voltage collapse can often be associated with large scale system blackouts, and is therefore one of the major security concerns for power system operations. It has been observed that, after some initial disturbances, many collapses involve a slow process of load restoration and generation reactive power limitation. The relatively slow pace of this process potentially provides sufficient time for implementation of operational decisions aimed at preventing the collapse.

During the past thirty years, a substantial effort has been devoted to designing on-line voltage stability controls. The extensive literature suggested that generation, transmission and load components can all provide effective means of alleviating voltage instability. Static approaches, which focus on power flow relationships under voltage collapse conditions, were well studied and formed the dominant trends in the early years [1]–[4].

Like transient angle and frequency stability problems, voltage instability is ultimately a dynamic phenomenon which needs to be more precisely handled by dynamic approaches. Along a voltage instability process, the nonlinear nature of power system dynamics, together with saturation and hybrid switching induced by local controls and protection devices, could cause significant deviations from normal operating conditions. Static methods provide online control strategies that are easily computed. However neglecting complicated switching and nonlinear dynamics may lead to control solutions that are either insufficient to stabilize the system or that have quite large social cost. Deficiencies in various static approaches were discussed in [5].

In recent years, industry has begun adopting online dynamic security assessment (DSA) tools, with explicit use of dynamic models and time domain simulation. These tools provide a good foundation for integrating online dynamic control applications. Based on the DSA framework, online controls can be sought, coordinated and justified for their longer term impact to the systems.

Power systems are large scale nonlinear systems with numerous physical limits (or constraints). Theoretically computing a closed loop optimal control strategy for these systems is impossible. Model predictive control (MPC), a mature control method that is widely accepted in the process and chemical industries, provides a practical way to address online optimal control problems for dynamical systems like power systems.

This paper extends earlier work that adopted trajectory sensitivities in MPC-based voltage stability controls [6]–[9]. A hierarchical MPC scheme is proposed within and energy management system (EMS) - dynamic security assessment (DSA) framework. With real-time EMS power flow data updated repeatedly, an internal hybrid system model is used to generate trajectory sensitivity information along prediction trajectories. Predictions based on trajectory sensitivities may become quite inaccurate along a unstable trajectories. The paper therefore develops a prestabilizing control strategy that provides stable prediction trajectories. Using the trajectory sensitivity information for the stable prediction trajectory, a single step linear programming (LP) problem is formed to obtain coordinated stability controls. Constraints are enforced to capture actual system limits. A benchmark voltage collapse case (10 bus system) is used to illustrate the performance of this control strategy.

The paper are organized as follows. Section II introduces power system dynamic modeling issues and trajectory sensitivity concepts. Section III focuses on the MPC strategy and algorithm design issues. A voltage collapse example using MPC control is provided in Section IV and conclusion are presented in Section V.

II. POWER SYSTEM DYNAMIC MODELING AND TRAJECTORY SENSITIVITIES

In response to large disturbances, power systems typically exhibit periods of smooth dynamic behaviour, interspersed with discrete events. Accordingly, it is common for dynamic models to consist of nonlinear differential-algebraic equations

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(DAEs), of the form

$$\dot{x} = f(x, y; \lambda), \quad x(0) = x_0 \tag{1}$$
$$0 = g(x, y; \lambda), \tag{2}$$

coupled with mechanisms for capturing the switching and impulsive effects that are introduced by discrete events [10]. In (1),(2), x describes the dynamic state variables, with initial values x_0, y are algebraic variables, and λ are parameters. In the power system context, x would describe quantities such as generator fluxes, and y would include bus voltage magnitudes. The details of such models are not important for this paper. The behaviour described by the model is important though, and can be represented by the flow

$$x(t) = \phi(x_0, t) \tag{3}$$
$$y(t) = \psi(x_0, t). \tag{4}$$

The flow is obtained by numerically integrating (1),(2), taking account of discrete events.

Trajectory sensitivities are motivated by forming the Taylor series expansion of the flow. In terms of the dynamic states, this gives

$$\begin{aligned}\Delta x(t) &= \phi(x_0 + \Delta x_0, t) - \phi(x_0, t)\\ &= \frac{\partial x(t)}{\partial x_0}\Delta x_0 + \text{ higher order terms} \quad (5)\\ &\approx \Phi(x_0, t)\Delta x_0. \quad (6)\end{aligned}$$

Likewise, the sensitivity of the $y(t)$ trajectory is given (to first order) by

$$\Delta y(t) \approx \Psi(x_0, t)\Delta x_0. \tag{7}$$

Trajectory sensitivities Φ, Ψ are well defined for non-smooth systems. Full details are provided in [11].

III. Model predictive control strategy for voltage stability enhancement

In this section, the MPC strategy designed for prevention of voltage instability will be discussed. The proposed MPC algorithm is based on trajectory sensitivity, and considers stability and optimality requirements.

A. MPC for prevention of voltage instability

Model predictive control (MPC) (also called receding horizon control or moving horizon control) is a discrete-time optimal control strategy. Figure 1 provides an illustration of the MPC process. Each control decision is obtained by first determining an estimate of the current system state. This provides the initial condition for prediction (simulation) of subsequent dynamic behavior. An internal dynamic model is used to predict system dynamic behavior, and control laws are computed based on that prediction [12]–[14]. At each MPC iteration, an open-loop optimal control sequence is computed to minimize (or maximize) a value function, with hard constraints being taken into account. Then the first control command of the sequence is issued to the system. This process is repeated at the next interval, with the updated real-time data.

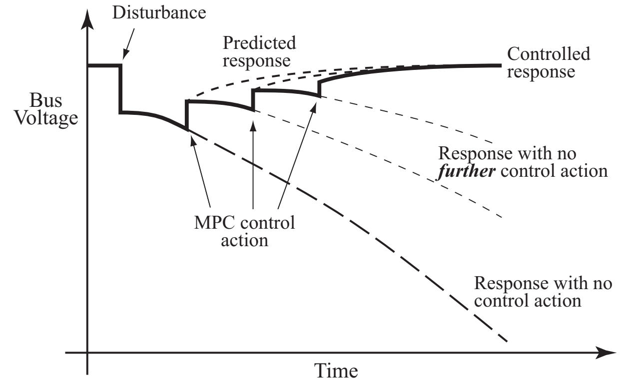

Fig. 1. MPC response.

The whole process of the control and measurement forms a closed loop control scheme.

Normally, open-loop MPC controls are calculated by solving an optimization problem based on the time-domain prediction of system behavior. For a finite horizon MPC problem with horizon N, starting from the initial state $x(k)$ with a future control sequence,

$$\mathbf{u}^k = \left\{u_k^k, u_{k+1}^k, \ldots, u_{k+N-1}^k\right\},$$

the value function along the controlled trajectory is defined as,

$$\mathcal{V}(x, \mathbf{u}, k) = \sum_{i=k}^{k+N-1} L(x(i), u_i^k) + F(x(k+N))$$

where $x(i) := x^u(i; x(k))$ denotes the predicted states at time instant i in the future. The stage cost $L(x(i), u_i^k)$ is usually the penalty term for state and control along the trajectory and satisfies:

$$L(x^*, u^*) = 0$$

(x^*, u^*) is the target equilibrium. $F(x(k+N))$ is defined as the terminal penalty for state deviation from the equilibrium. The constraint for the controls and states satisfies:

$$\left.\begin{aligned} u_i^k \in \mathbb{U}_i^k \\ x(i) \in \mathbb{X}_i \end{aligned}\right\} \quad i = k, \ldots, k+N-1$$

where $\mathbb{U}_i^k$ and $\mathbb{X}_k$ are convex and compact subsets with suitable dimensions.

Denoting the equality constraint sets as $0 = E(x, \mathbf{u}, k)$, and inequality constraints as $0 \geq I(x, \mathbf{u}, k)$, solving the optimization problem,

$$\begin{gathered}\min_{u \in \mathbb{U}, x \in \mathbb{X}} \mathcal{V}(x, \mathbf{u}, k)\\ 0 = E(x, \mathbf{u}, k)\\ 0 \geq I(x, \mathbf{u}, k)\end{gathered}$$

will generate the optimizing control sequence,

$$\mathbf{u}^{*k} = \left\{u_k^{*k}, \ldots, u_k^{*k+N-1}\right\}.$$

The first control u_k^{*k} will be issued to the system and this process will be repeated when the next system state $x(k+1)$ is available.

Compared to the theoretical optimal control approaches, MPC has some advantages for industrial applications. MPC allows the inclusion of hard constraints in a nonlinear optimal control problem. By solving a relatively simple optimization problem with an objective function similar to a value function of the stabilized system, both the stability and optimality properties of the controls can be considered, and the control computation can be carried out online.

In power systems, MPC applications have started to attract researchers' attention in the last 5 years. Voltage stability problems appear to be the most explored power system applications for MPC schemes. The first application of MPC for emergency voltage control problems appeared in 2002 [15]. Reference [16] was the first one that adopted a hybrid model to deal with a voltage control problem in an MPC framework. To reduce the computational burden for hybrid system controls, trajectory sensitivity concepts were introduced to MPC-based voltage control problems [6], [7]. In [7], [9], closed loop control performance with trajectory sensitivity was first investigated. To reduce the computational burden of a centralized MPC scheme, distributed approaches to MPC have become another area of interest in power systems [17], [18].

B. EMS-DSA framework

Dynamic models and real-time data are two essential components for MPC applications. To best utilize the available resources from existing control tools, a three level hierarchical structure is proposed in this paper: the lower substation level with local control loops, the EMS-DSA level for normal operations and system-level online dynamic assessment, and the higher MPC decision level for emergency voltage control. Figure 2 provides a block diagram that illustrates this structure.

Under normal conditions, real-time power flow data, topology information and events (disturbance and outages) are provided to the EMS for state estimation, topology and bad data processing. When a disturbance happens, real-time data will be delivered to the DSA blocks. A dynamic case preparation block will set up the models ready for simulation, and based on the time domain simulation, the assessment tools will feed back the results/suggestions to the EMS for further operational decisions.

When a large disturbance is detected and a potential voltage collapse is predicted, DSA will activate the MPC block by sending real time power flow data and simulation results to it. System dynamics as well as trajectory sensitivity information will be sampled to establish a dynamic embedded optimization problem, which in turn will provide the optimal open loop control. This optimization will also require information on control availability and costs/penalties to formulate the objective functions and constraints. This information can be obtained directly from the EMS.

C. MPC algorithm

For a voltage stability problem, security requirements are always given the first priority. A well designed MPC strategy should be able to find a control sequence that can successfully steer the system from a potential collapse situation to a stable operating point (normally with a certain stability margin). Minimum disruption to consumers is also extremely important. For example, load control (shedding) is an effective countermeasure but quite disruptive to consumers. Such disruptiveness can be incorporated into the cost function. Having satisfied the security requirements, the aim should be to minimize the disruptiveness of the control action. Coordination among the various different forms of control should take account of their longer term effects on consumers.

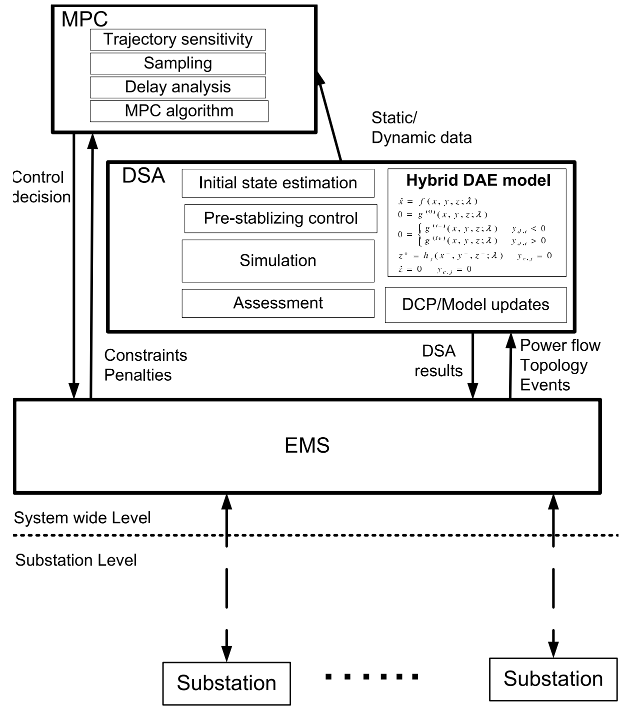

Fig. 2. MPC structure in EMS-DSA framework.

1) Conditions for stability: System stability is a property of the underling dynamic state x. On the other hand, algebraic states y such as voltages V and power injections P and Q are much easier to measure and update from the real world through the EMS. For this reason, almost all power system applications of MPC establish stability conditions in terms of constraints on algebraic quantities.

In this paper, the constraints used as pseudo-stability conditions are

$$\begin{aligned} V^{min} &\leq V \quad\ \leq V^{max} \\ I_{fd}^{min} &\leq I_{fd} \quad \leq I_{fd}^{max} \\ I_{stat}^{min} &\leq I_{stat} \leq I_{stat}^{max} \\ I_{line}^{min} &\leq I_{line} \leq I_{line}^{max} \end{aligned}$$

where V are bus voltages, I_{fd} and I_{stat} are generator field currents and stator currents respectively, and I_{line} are line currents. Let $y_s = [V, I_{fd}, I_{stat}, P_{line}]^T$ be a vector of selected algebraic states. These constraints can be expressed in a compact form as,

$$y_s^L \leq y_s \leq y_s^H.$$

It will be assumed that operating points lying within the polyhedral set $\mathbb{Y}_s^* = [y_s^L, y_s^H]$ are secure.

The constraints on generator field and stator currents are used as an alternative to reactive power limits, as they provide a more physically-based assessment of generator reactive capability.

Considering the security requirements, the objective function $\mathcal{V}(y,u,k)$ may include the penalty terms,

$$\sum_{i=k+1}^{k+N} C_i^y \rho(y_i, \mathbb{Y}_s^*)$$

as stage costs. The term $\rho(b,A)$ is a Hausdorff distance [19] between a point b and a set A, and can be defined,

$$\rho(b,A) = \min_{a\in A} \|a-b\|.$$

When $b \in A$, $\rho(b,A)=0$. As an example, consider the polyhedral set $A=[a_L, a_H]$, where a_L and a_H are vectors and $\|\cdot\|$ is a 1-norm,

$$\rho(b,A) = \frac{1}{2}\left(\|b-a_H\|_1 + \|b-a_L\|_1 - \|a_H-a_L\|_1\right).$$

When such distance terms are used in the objective function of an MPC optimization algorithm, the constant term $\|a_H - a_L\|_1$ and scaling $\frac{1}{2}$ can be removed without affecting the final optimization results.

2) Objectives for optimality: Load shedding is disruptive to consumers, and is therefore less desirable than generation rescheduling and generator terminal voltage set-point changes. The ultimate goal of the voltage stability control problem is to restore the system to a secure state (with the implication that this ensures stability), while minimizing the disruption caused by load shedding. Considering that optimality requirement, the objective function $\mathcal{V}(y,u,k)$ may include the following penalty terms as the stage costs,

$$\sum_{i=k}^{k+N-1} C_i^\lambda \lambda_i^k \tag{8}$$

where λ_i^k is the vector of participating load shedding percentage at the i_{th} prediction step. It satisfies the entry-wise inequality constraints

$$0 \le \lambda_i(j) \le \bar{\lambda}_i(j), j = 1 \dots n$$

with 0 for no load shedding and $\bar{\lambda}_i(j)$ for all available load being shed. When $\bar{\lambda}_i(j)$ is set to 1, it indicates that all load at bus j can be shed. Even though the other controls such as generation rescheduling may also induce some cost by increasing the network losses, their costs are negligible compared to load shedding.

Let the control at time $k-1$ be u^{k-1}, which is a known value at time k. By defining the control changes as,

$$\Delta u_i^k = u_i^k - u^{k-1}, \quad i = k, \dots, k+N-1$$

and for load shedding control

$$\Delta\lambda_i^k \quad = \quad \lambda_i^k - \lambda^{k-1}$$

then (8) can be rewritten as,

$$\sum_{i=k}^{k+N-1} C_i^\lambda \lambda_i^k = \sum_{i=k}^{k+N-1} C_i^\lambda \lambda^{k-1} + \sum_{i=k}^{k+N-1} C_i^\lambda \Delta\lambda_i^k.$$

The first term on the right is known for the computation at time k, and so can be removed from the objective function.

3) Single step MPC algorithm: Consider a single step MPC algorithm with $N=1$. Based on the definition of trajectory sensitivity (6),(7) perturbations to the algebraic states y caused by controls can be approximated by

$$\Delta y_{k+1}^k = \bar{y}_{k+1}^k - y_{k+1}^k = \Psi_u(x_k^k, u_k^k, T)\Delta u_k^k$$

where the superscript k indicates that MPC is predicting ahead from current time kT, subscripts k and $k+1$ denote the sampling time along the predicted trajectory, $\bar{y}$ is the approximate post-control trajectory and y is the nominal (pre-control) predicted trajectory.

With this approximation and the penalty terms defined in the Sections III-C1 and III-C2, a single step MPC algorithm can be formulated as the linear program (LP),

$$\begin{aligned} \min\ & C_{k+1}^y(\|y_{k+1}^k - y_s^H + \Psi_u(x_k^k, u_k^k, T)\Delta u_k^k\|_1 \\ & \quad + \|y_{k+1}^k - y_s^L + \Psi_u(x_k^k, u_k^k, 1)\Delta u_k^k\|_1) + C_k^\lambda \Delta\lambda_k^k \\ \text{s.t. } & \Delta u_k^k \in \Delta\mathbb{U}_k. \end{aligned} \tag{9}$$

As mentioned earlier, when a system is evolving along a voltage instability process, the first priority is to save the system from future collapse. Therefore, it is natural to choose $C_{k+1}^y \gg C_k^\lambda$ when the predicted algebraic states y_{k+1}^k lie outside the target set $\mathbb{Y}_s^*$. On the other hand, if it has entered $\mathbb{Y}_s^*$, the distance penalty term $\|y_{k+1}^k - y_s^H + \Psi_u(x_k^k, u_k^k, T)\Delta u^k\|_1 + \|y_{k+1}^k - y_s^L + \Psi_u(x_k^k, u_k^k, T)\Delta u^k\|_1$ will be constant. This penalty can be converted to hard constraints to ensure that future control changes do not steer the system out of the target set. This yields another LP form of MPC algorithm:

$$\begin{aligned} \min\ & C_k^\lambda \Delta\lambda_k^k \\ \text{s.t. } & y_s^L \le y_{k+1}^k + \Psi_u(x_k^k, u_k^k, T)\Delta u_k^k \le y_s^H \\ & \Delta u_k^k \in \Delta\mathbb{U}_k. \end{aligned}$$

4) Prestabilizing control: Using the MPC algorithm (9), the control results obtained from the optimization are dependent upon the approximation provided by the trajectory sensitivities. Since trajectory sensitivities are defined by neglecting the higher order terms in the perturbed trajectories (5), it is important to understand the impact of this approximation on the accuracy of the predicted trajectories. In [20], it is shown that for a variety of hybrid dynamical systems, this first order approximation provides a good prediction of perturbed system dynamics.

On the other hand, when the system is unstable, the trajectory sensitivity evaluated along the unstable trajectory may provide a poor indicator for the post-control trajectories. This can be illustrated by a simple example. Consider the first order system,

$$\dot{x} = x - \sqrt{x}$$

which has a closed-form solution,

$$x(x_0, t) = 1 + 2e^{\frac{t}{2}}(\sqrt{x_0} - 1) + e^t(\sqrt{x_0} - 1)^2.$$

The closed-form solution of the trajectory sensitivity Φ can be obtained by evaluating $\frac{\partial x(x_0,t)}{\partial x_0}$.

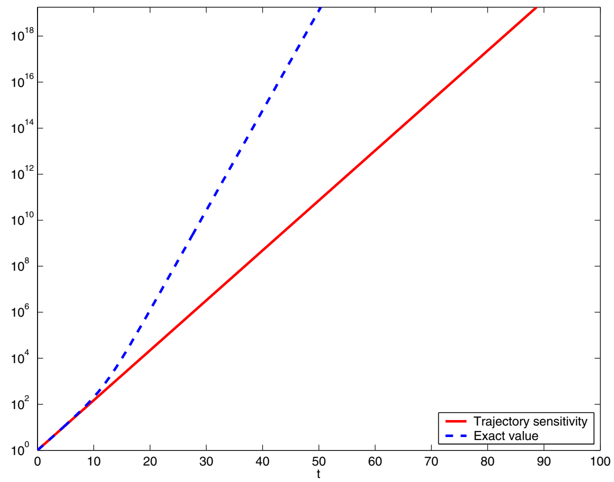

Fig. 3. Trajectory sensitivity vs. exact sensitivity values.

Any perturbation to the initial state value $x_0 = 1$ will drive the state trajectory unstable. Figure 3 shows the exact scaled difference $\bar{\Phi}_x = \frac{\phi_x(x_0+\Delta x_0,t)-\phi_x(x_0,t)}{\Delta x_0}$ (with $\Delta x_0 = 0.01$) and the trajectory sensitivity Φ, on a logarithm scale. For this case, the trajectory sensitivity diverges from the exact difference within a short period.

This divergence is a consequence of the unstable system dynamics. In such cases, the error introduced by neglecting the higher order terms may be significant. It follows that the first order approximation obtained using $\Phi_x(x_0, t)$ will provide a poor indication of how control changes Δu will affect the unstable nominal trajectory.

Conversely, when the system dynamics are asymptotically stable, trajectory sensitivities evaluated along a stable trajectory usually provide an adequate estimate of trajectory perturbations.

To resolve the problem caused by inaccurate trajectory approximations, a stable post-disturbance simulation is highly desirable. This can be achieved by determining an initial estimate of the control action required to stabilize the system, and using that estimate within the MPC simulation. By stabilizing the MPC simulation, the corresponding trajectory sensitivities are well behaved and can be used in the MPC optimization (9). Note that this initial prestabilizing control is not applied to the actual system, but is only used within the MPC model. Hence, there is no need for this initial control guess to be optimal, just stabilizing. This prestabilized system behavior forms the basis for the optimization process (9). That process subsequently determines adjustments to the prestabilizing control decisions that give the optimal stabilizing strategy. Only that final optimal control decision is applied to the actual system.

Earlier research efforts on static-based voltage stability controls have been adopted for determining the prestabilizing control. One encouraging approach has been to view the prestabilizing control as moving the operating point from the outside of the power flow solvability region to inside. A static voltage stability control method proposed in [2] considered a similar situation, and so has been used. To summarize this approach, starting from a point A outside the power flow solution space, the method will iteratively apply an optimal multiplier method [21] to locate boundary points on the solution space. The process is terminated when the boundary point B that has a minimal distance to point A is obtained. This iterative process is based on the fact that the vector that is normal to the boundary hypersurface will be aligned with the left eigenvector associated with the zero eigenvalue of the singular power flow Jacobian. Sensitivity of the distance with respect to the controls [22] can then be used to determine the changes that should be made to bring the operating point to the solution space boundary.

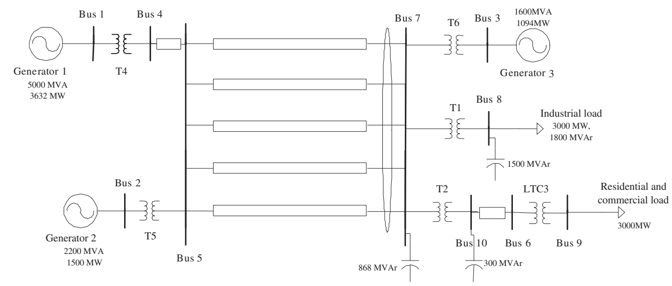

Fig. 4. 10 bus voltage collapse test system.

IV. Example

The simple 10 bus system shown in Figure 4 is well established as a benchmark for exploring voltage stability issues [7], [23], [24]. The system has 3 generators and 2 loads. LTC3 is a load tap changer (LTC) that automatically adjusts its tap ratio according to the voltage magnitude at load bus 9.

A. Voltage collapse

An outage of any one of the feeders between buses 5 and 7 will lead to voltage collapse. This is illustrated in Figure 5 for a line outage at 10 seconds. Voltages at buses 3, 6 and 9 are shown. Bus 9 supplies the residential load, and is regulated by the LTC. Following the initial drop caused by the fault, the voltage starts to recover as a consequence of LTC tap ratio adjustments. When viewed from bus 6 though, this LTC adjustment results in a load recovery process, which leads to the voltage falling at that bus. All the other buses follow the same trend of decreasing voltage, except generator terminal buses, such as bus 3, which are regulated by automatic voltage regulators (AVRs). At 35 seconds, increasing field current causes the reactive power limit at generator 3 to be reached. The over-excitation limiter (OXL) at generator 3 reduces the internal field voltage. A sudden drop in the bus voltages can be observed. After that, further LTC tap ratio adjustments cause voltages across the system to drop.

B. Prestabilizing control

Using the prestabilizing concept discussed in Section III-C4, the initial estimate of load shedding suggests a need to shed 17.44% of the load at bus 8. Bus voltages with this control applied are shown in Figure 6. With this control, the bus voltage on bus 9 is raised instantaneously to 1.11 pu. Since

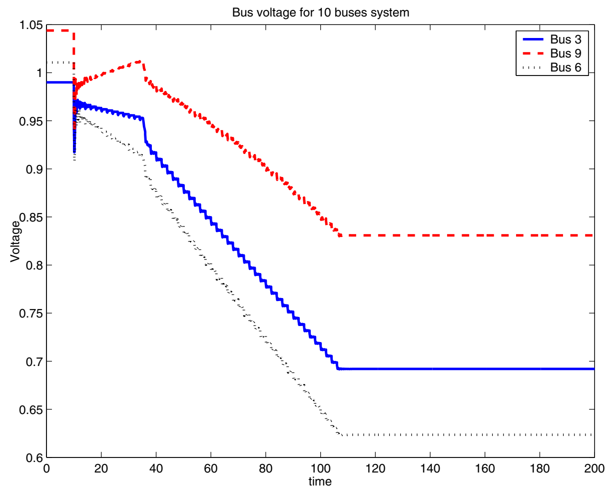

Fig. 5. Bus voltage behaviour with no MPC control action.

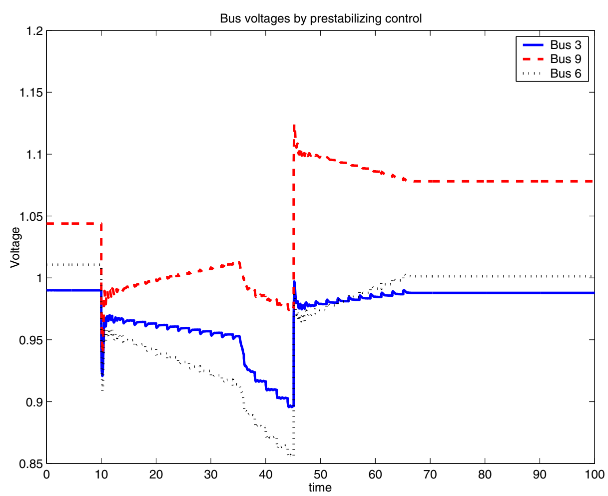

Fig. 6. Bus voltage response to the prestabilizing load shedding estimate.

this value lies outside the deadband for LTC3, it will decrease its tap ratio until the bus voltage drops to 1.08 p.u, the upper threshold of the deadband.

C. MPC

When there is no prestabilizing control to generate stable simulation trajectories, trajectory sensitivities are evaluated along an unstable trajectory. The blue curve in Figure 7 shows the load control signals computed by this MPC algorithm. At the first step, the control sheds 11.83% of the load at bus 8. After subsequent MPC steps, the load shedding amount was reduced to 10.08%. Figure 7 also shows the control action when the prestabilizing control was used to generate stable trajectories for MPC computations. It can be seen that final control amounts are not dependent upon the use of the prestabilizing process. However, the MPC solution calculated using prestabilized trajectories is less aggressive.

In addition to load shedding, rescheduling generator active power and terminal voltage set-points provides other countermeasures for overcoming voltage stability problems. By

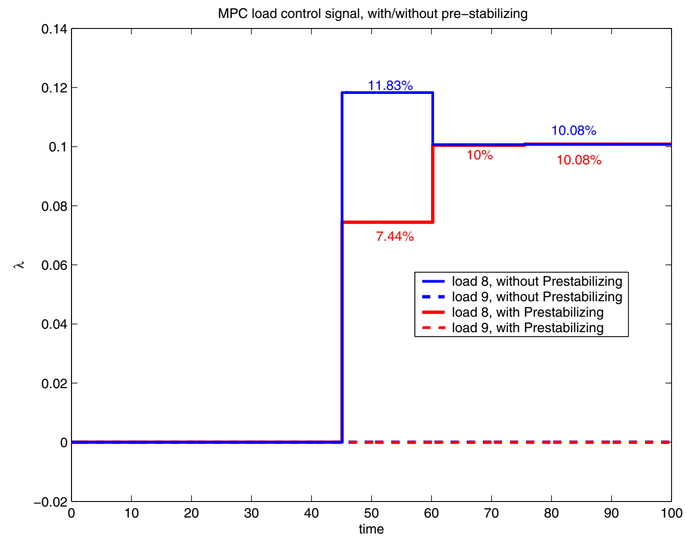

Fig. 7. MPC control of load shedding with/without prestabilizing.

TABLE I
MPC RESULTS FOR DIFFERENT CONTROLS

Case Number	λ_8	λ_9	T^2_{mech}	T^3_{mech}	V^2_{set}	V^3_{set}
1	10.08%	0	-	-	-	-
2	7.54%	0	9	14	-	-
3	8.59%	0	-	-	0.985	0.983
4	6.17%	0	9	14	0.985	0.983

combining these controls, the required load shedding amount can be reduced. Table I lists the final control decisions that result from the use of various different combinations of controls. The symbol '-' indicates that that particular control was not used. From the table it can be seen that when more controls are available, the required load shedding amount will be reduced.

Besides regulating bus voltage and field current, MPC can also be used to avoid line overloads. Figure 8 shows the active power flow between buses 5 and 7 when MPC controls are used to regulate bus voltages and field currents. The active power flow stabilizes around 1145 MW, which exceeds the feeder thermal limit of 1100 MW. To avoid such overloading, a line flow limit was added to the constraints. The resulting regulated line flow is shown in Figure 9.

V. CONCLUSION

The paper proposes an MPC strategy for alleviating voltage collapse. This control strategy can be integrated into EMS-DSA frameworks for online voltage stability control. Through the use of trajectory sensitivities, the MPC algorithm can be formulated as a linear programming problem that can be efficiently solved for large scale systems. To improve the accuracy of this approach, a static prestabilizing strategy can be used to generate stable prediction trajectories. The proposed controller has been tested on a standard 10 bus system. Results suggest that the proposed MPC strategy can effectively and efficiently prevent voltage collapse.

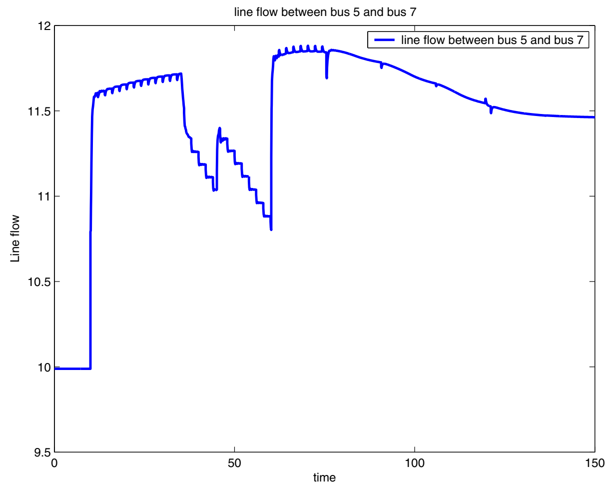

Fig. 8. Line flow on a feeder between buses 5 and 7.

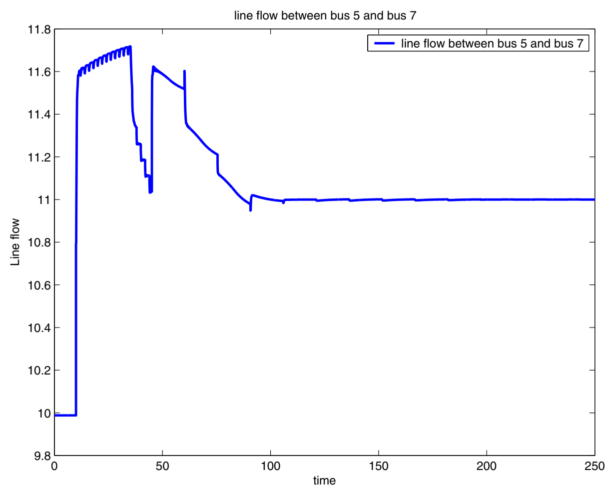

Fig. 9. Regulated line flow on a feeder between buses 5 and 7.

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