Power System Parameter Estimation *

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SUMMARY *The nonlinear non-smooth nature of power system dynamics complicates the estimation of parameters from system measurements. The paper proposes a Gauss-Newton approach to computing a set of model parameters that give the best fit between measurements and the model response. This approach involves trajectory sensitivities, i.e., the sensitivity of the model trajectory to small changes in parameters. An overview of trajectory sensitivity analysis is provided. A small example, which exhibits both soft and hard nonlinearities, is used to illustrate the estimation algorithm.*

1 INTRODUCTION

System-wide measurements of power system disturbances are frequently used in post-mortem analysis to gain a better understanding of system behaviour [1,2]. In undertaking such studies, measurements are compared with the behaviour predicted by a model. Differences are used to tune the model, i.e., adjust parameters to obtain the best match between the model and the measurements.

An example of the model tuning procedure, and the importance of correct modelling, is provided in [2]. In that case, the power system lost stability following a large disturbance. A post-mortem analysis was undertaken to explore the nature of that instability. It was found that by using the 'standard' set of parameters, the model did not replicate the unstable behaviour. However an exhaustive investigation showed that correct behaviour could be predicted if a parameter in the load description was altered by a small amount.

This example illustrates the need for a systematic approach to estimating power system model parameters. The difficulty is that power system behaviour is nonlinear. Models must therefore also be nonlinear, and in fact may frequently contain hard nonlinearities, i.e., discontinuities. Parameter estimation techniques are quite well established for linear models [3]. However parameter estimation for nonlinear systems is a relatively new and unexplored field.

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This paper develops an iterative technique for determining parameter values which produce the best match between measured large disturbance system behaviour and the model response. The technique uses a Gauss-Newton approach, which in turn relies on trajectory sensitivity analysis.

Similar ideas have been used previously for estimating parameters of generators and AVR/exciters [4,5,6]. However the number of parameters that could be estimated using those earlier ideas was limited, because the trajectory sensitivities were generated numerically [5,7]. A more computationally efficient method of calculating trajectory sensitivities has recently been presented in [8]. This has enabled the extension to estimating many system-wide parameters.

A brief review of modelling and trajectory sensitivity concepts is given in Section 2. Those ideas are then used in Section 3 to develop the desired parameter estimation algorithm. Section 4 considers the application of this algorithm in a simple yet illustrative example. Conclusions are presented in Section 5.

2 MODEL AND TRAJECTORY SENSITIVITIES

2.1 Model

Power systems frequently exhibit a mix of continuous time dynamics, discrete-time and discrete-event dynamics, switching action and jump phenomena. It is shown in [8] that such systems, known generically as *hybrid systems*, can be modelled by a set of switched differential-algebraic equations, coupled with equations to describe state resetting, i.e.,

$$\dot{\underline{x}} = \underline{f}(\underline{x}, y) \tag{1}$$

$$0 = g^{(0)}(\underline{x}, y) \tag{2}$$

$$0 = \begin{cases} g^{(i-)}(\underline{x}, y) & y_{d,i} < 0 \\ g^{(i+)}(\underline{x}, y) & y_{d,i} > 0 \end{cases} \quad i = 1,\cdots,d \tag{3}$$

$$\underline{x}^{+} = \underline{h}_j\left(\underline{x}^{-}, y^{-}\right) \quad y_{e,j} = 0 \quad j \in \{1,\ldots,e\} \tag{4}$$

where $\underline{x} = [x^t z^t \lambda^t]^t$, and

- x are the continuous dynamic states, for example generator angles, velocities and fluxes,
- z are discrete dynamic states, such as transformer tap positions and protection relay logic states,
- y are algebraic states, e.g., load bus voltage magnitudes and angles,
- λ are parameters such as line reactances, controller gains and switching times.

In this model, the parameters λ form part of the state $\underline{x}$. This allows a convenient development of trajectory sensitivities. To ensure that parameters remain fixed at their initial values, the corresponding differential equations (1) are defined as $\dot{\lambda} = 0$.

Away from events, system dynamics evolve smoothly according to the familiar differential-algebraic model

$$\dot{\underline{x}} = \underline{f}(\underline{x}, y) \tag{5}$$

$$0 = g(\underline{x}, y). \tag{6}$$

At switching events (3), some component equations of g change. Algebraic variables y may undergo a corresponding step change. Reset events (4) force a discrete change in some z. Algebraic variables may again step to ensure g is always satisfied.

We shall define the flows of $\underline{x}$ and y respectively as

$$\underline{x}(t) = \phi_x(\underline{x}_o, t) \tag{7}$$

$$y(t) = \phi_y(\underline{x}_o, t) \tag{8}$$

where $\underline{x}(t)$ and $y(t)$ satisfy (1) - (4), along with initial conditions,

$$\phi_{\underline{x}}(\underline{x}_0, t_0) = \underline{x}_0 \tag{9}$$

$$g(\underline{x}_0, \phi_y(\underline{x}_0, t_0)) = 0. \tag{10}$$

2.2 Trajectory sensitivities

Trajectory sensitivities provide a way of quantifying the variation of a trajectory which results from changes to parameters and/or initial conditions [9]. Recent power system applications, apart from those relating to parameter estimation, can be found in [7,10,11]. The main concepts are summarized in this section and Appendix A. Further details can be found in [8].

To obtain the sensitivity of the flows $\phi_{\underline{x}}$ and ϕ_y to initial conditions, and hence to parameter variations, we form the Taylor series expansions of (7), (8). Neglecting higher order terms gives

$$\Delta\underline{x}(t) = \frac{\partial \underline{x}(t)}{\partial \underline{x}_0}\Delta\underline{x}_0 \equiv \underline{x}_{\underline{x}_0}(t)\Delta\underline{x}_0 \tag{11}$$

$$\Delta y(t) = \frac{\partial y(t)}{\partial \underline{x}_0}\Delta\underline{x}_0 \equiv y_{\underline{x}_0}(t)\Delta\underline{x}_0 . \tag{12}$$

It is important to keep in mind that $\underline{x}_0$ incorporates λ, so sensitivity to $\underline{x}_0$ includes sensitivity to λ. Equations (11), (12) provide the changes $\Delta\underline{x}(t)$ and $\Delta y(t)$ in a trajectory, at time t along the trajectory, for a given (small) change in initial conditions $\Delta\underline{x}_0 = [\Delta x_0^t \;\; \Delta z_0^t \;\; \Delta\lambda^t]^t$.

An overview of the computation of $x_{\underline{x}_0}$ and $y_{\underline{x}_0}$ is given in Appendix A.

3 PARAMETER ESTIMATION

3.1 Introduction

Measurements of power system dynamic behaviour are typically obtained using data acquisition systems (DASs) [1] which produce sequences m of samples $m_0, m_1, m_2, \ldots, m_q$ of system variables. The aim of parameter estimation is to find the set of model parameters which gives the best fit between the measurements and the model. It is assumed that the number of samples q is sufficiently large, as it is necessary to have at least $q+1 = p$ samples to estimate p unknowns (the parameters). This is normally the case.

Usually a DAS provides measurement sequences for many different quantities. However for clarity the parameter estimation algorithm will initially be developed assuming a single measurement sequence. This assumption will then be relaxed.

The development of the parameter estimation algorithm assumes that measurements correspond to algebraic states. This does not restrict the application of the algorithm though, as it is always possible to add extra algebraic constraints

$$g_i(\underline{x},y) = y_i - \rho(\underline{x},y) = 0$$

to generate an 'output' y_i which matches the measurement. These new functions augment the original algebraic constraints g given by (2) - (3).

3.2 Parameter estimation from a single measurement

The algebraic state corresponding to the measurement m will be denoted $\breve{y}$. The estimation process involves varying initial conditions x_0, z_0 and parameters λ to obtain the best match between the sequence m of measured samples and the flow $\breve{y}(t)$, provided by the model (8). A Gauss-Newton approach is proposed.

The model produces the flow $\breve{y}(\underline{x}_0,t)$ for all $t \geq t_0$. But the samples in the sequence m are measured at certain time instants. Therefore for each time instant $t_k, k = 0,1,\cdots,q$, corresponding to each measurement sample, it is possible to create the model sample $\breve{y}_k(\underline{x}_0) = \breve{y}(\underline{x}_0,t_k)$, resulting in the sequence $\breve{y}_0(\underline{x}_0), \breve{y}_1(\underline{x}_0)\cdots, \breve{y}_q(\underline{x}_0)$. The aim of the parameter estimation process is to determine the value of $\underline{x}_0$, i.e., parameters and initial conditions, which makes the model response $\breve{y}_k(\underline{x}_0)$ closest to the measured samples m_k for all k.

Let the mismatch between the measured value and the model output at each sample time be

$$e_k(\underline{x}_0) = \breve{y}_k(\underline{x}_0) - m_k \qquad k = 0,1,\cdots,q$$

or in vector form

$$e(\underline{x}_0) = \breve{y}(\underline{x}_0) \tag{13}$$

where

$$e(\underline{x}_0) = [e_0(\underline{x}_0) \;\; e_1(\underline{x}_0)\cdots e_q(\underline{x}_0)]^t$$
$$\breve{y}(x_0) = [\breve{y}_0(\underline{x}_0) \;\; \breve{y}_1(\underline{x}_0)\cdots \breve{y}_q(\underline{x}_0)]^t$$
$$m = [m_0 \;\; m_1\cdots m_q]^t.$$

Then the best fit is obtained by the $\underline{x}_0$ which minimizes the least squares cost

$$J(\underline{x}_0) = \frac{1}{2}\sum_{k=0}^{q}\left|e_k(\underline{x}_0)\right|^2 = \frac{1}{2}\left\|e(\underline{x}_0)\right\|_2^2. \tag{14}$$

The problem has been reduced to a nonlinear least squares formulation which can be solved using the Gauss-Newton method [12]. This is an iterative approach which is based on linearizing $e(\underline{x}_0)$ around the point $\underline{x}_0^j$, i.e.,

$$\tilde{e}(\underline{x}_0,\underline{x}_0^j) = e(\underline{x}_0^j) + \frac{\partial e(\underline{x}_0^j)}{\partial \underline{x}_0}(\underline{x}_0 - \underline{x}_0^j). \tag{15}$$

From (13) it follows that

$$\frac{\partial e(\underline{x}_0^j)}{\partial \underline{x}_0} = \frac{\partial \breve{y}(\underline{x}_0^j)}{\partial \underline{x}_0} = \begin{bmatrix} \breve{y}_{\underline{x}_0}(\underline{x}_0^j,t_0) \\ \breve{y}_{\underline{x}_0}(\underline{x}_0^j,t_1) \\ \vdots \\ \breve{y}_{\underline{x}_0}(\underline{x}_0^j,t_q) \end{bmatrix} \equiv S(\underline{x}_0^j). \tag{16}$$

Notice that this matrix is composed of the trajectory sensitivities $\breve{y}_{\underline{x}_0}$ evaluated at the time steps $t_0,t_1,\cdots t_q$. Therefore $S(\underline{x}_0^j)$ shall be referred to as the *sensitivity matrix*. Substitution of $S(\underline{x}_0^j)$ into (15) gives

$$\tilde{e}(\underline{x}_0,\underline{x}_0^j) = e(\underline{x}_0^j) + S(\underline{x}_0^j)(\underline{x}_0 - \underline{x}_0^j).$$

The value of $\underline{x}_0$ obtained at the $(j+1)^{th}$ iteration is that value which minimizes $\frac{1}{2}\left\|\tilde{e}(\underline{x}_0,\underline{x}_0^j)\right\|_2^2$, i.e.,

$$\underline{x}_0^{j+1} = \underset{\underline{x}_0}{\arg\min}\left\{\frac{1}{2}\left\|\tilde{e}(\underline{x}_0,\underline{x}_0^j)\right\|_2^2\right\}.$$

Assuming $S(\underline{x}_0^j)^t S(\underline{x}_0^j)$ is invertible, this minimization gives

$$\begin{aligned}\underline{x}_0^{j+1} &= \underline{x}_0^j - \left\{S(\underline{x}_0^j)^t S(\underline{x}_0^j)\right\}^{-1} S(\underline{x}_0^j)^t e(\underline{x}_0^j) \\ &= \underline{x}_0^j - \left\{S(\underline{x}_0^j)^t S(\underline{x}_0^j)\right\}^{-1} S(\underline{x}_0^j)^t (\breve{y}(\underline{x}_0^j) - m).\end{aligned} \tag{17}$$

An estimate of $\underline{x}_0$ which (locally) minimizes the cost function $J(\underline{x}_0)$ in (14) is obtained when $\Delta\underline{x}_0^j = \underline{x}_0^{j+1} - \underline{x}_0^j$ is close to zero. Note that this procedure will only give local minima though, as it is based on linearization. However if the initial guess for $\underline{x}_0$ is good, which is generally the case in power system studies, then a local minimum is sufficient.

3.3 Parameter estimation from more than one measurement

Normally a DAS (or a number of DASs scattered around a power system) will provide many measurements of a disturbance. Ideally all the available information should be used to give the best estimate of the initial conditions and parameters.

Assume there are l measurement sequences, $m^1, m^2, \cdots, m^l$ and the corresponding model flows are $\breve{y}^1, \breve{y}^2, \cdots, \breve{y}^l$. A sensitivity matrix S^i corresponding to each $\breve{y}^i$ can be defined as in (16). We now define

$$\breve{y}(\underline{x}_0) = \begin{bmatrix} \breve{y}^1 \\ \breve{y}^2 \\ \vdots \\ \breve{y}^l \end{bmatrix}, \quad m = \begin{bmatrix} m^1 \\ m^2 \\ \vdots \\ m^l \end{bmatrix}$$

and make corresponding changes in the definition of $e(\underline{x}_0)$; see (13). The sensitivity matrices are arranged as

$$S(\underline{x}_0) = \begin{bmatrix} S^1(\underline{x}_0) \\ S^2(\underline{x}_0) \\ \vdots \\ S^l(\underline{x}_0) \end{bmatrix}.$$

Then (17) can again be used to obtain the optimal value of $\underline{x}_0$. In this case, the optimal $\underline{x}_0$ provides the best fit of the model to all the measurements.

Note that if some measurements are known more accurately than others, then weighting factors can be used to weight the relative importance of the measurements.

4 EXAMPLES

The two machine infinite bus system shown in Fig. 1 will be used to illustrate the parameter estimation process. The dynamic behaviour of the system is governed by the swing equations of the two machines.

In order to introduce a hard nonlinearity (switching action) into this simple system, the mutual admittance between the generators has been modelled as

$$Y_{12} = \begin{cases} 1.5 \text{ when } (\delta_1 - \delta_2)^2 < 0.03^2 \\ 0.67 \text{ when } (\delta_1 - \delta_2)^2 < 0.03^2 \end{cases} \tag{18}$$

This could (crudely) represent voltage support devices running out of range as the angle across the line increased beyond a certain threshold. The modelling details are not so important; the main aim is to test

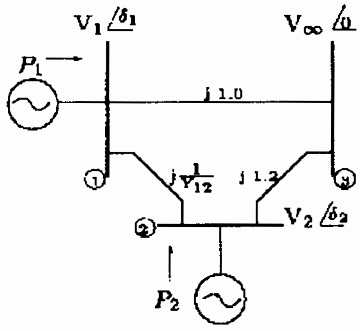

Figure 1: Two machine infinite bus system.

parameter estimation ideas in the presence of discontinuities.

For now it is assumed that the only available measurement is that of the real power flow from generator 1 to the infinite bus. Accordingly, the model is augmented by the algebraic equation

$$0 = \breve{y} - \sin\delta_1$$

where $\breve{y}$ is the measured power flow.

The behaviour of this system, for a particular disturbance, is shown in Fig. 2. The measured power flow $\breve{y}$ is plotted. The sensitivity of this trajectory to variations in the parameters M_1, M_2, D_1, D_2, the machine inertia and damping constants, is shown in Fig. 3.

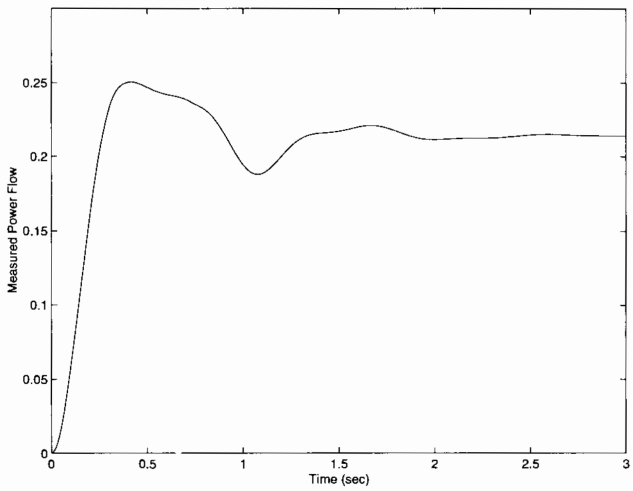

Figure 2: System trajectory.

To initially illustrate the parameter estimation process, the plot of Fig. 2 will be used as the measurement. The *actual* parameters in this case are

$$\lambda = \begin{bmatrix} M_1 \\ M_2 \\ D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.0138 \\ 0.0276 \\ 0.0570 \\ 0.1140 \end{bmatrix}. \tag{19}$$

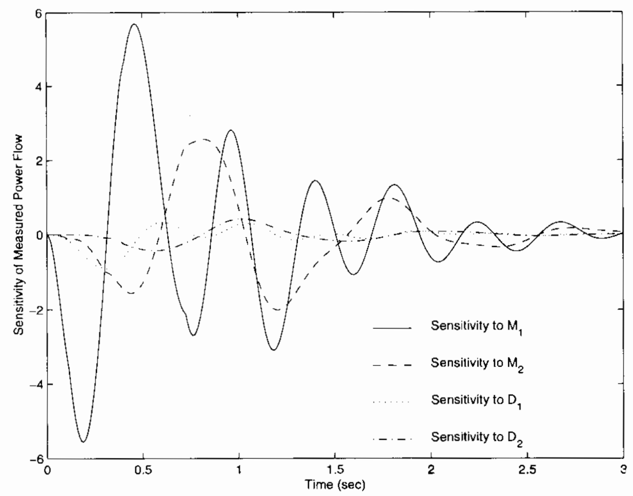

Figure 3: Sensitivity of $\breve{y}$ to parameters.

Those actual values are assumed to be unknown, so a guess of $\lambda = [0.012 \quad 0.020 \quad 0.05 \quad 0.05]^t$ was made. A measurement time interval of 0.1 seconds was used.

The parameter estimation process (17) converged from the initial guess to the actual parameter values in 5 iterations. The convergence process is illustrated in Fig. 4. The trajectories shown correspond to the initial (guessed) parameters and the subsequent 5 iterations. The measurement samples, (based on the actual parameter values in this case), are also shown. The changes in parameters at each iteration are given in Table 1. The parameters D_1, D_2 are the slowest to converge. This is consistent with Fig. 3, which shows that the trajectory is least sensitive to those parameters. Convergence in 5 iterations is quite acceptable, as the error in the initial guess of the parameter values was rather high.

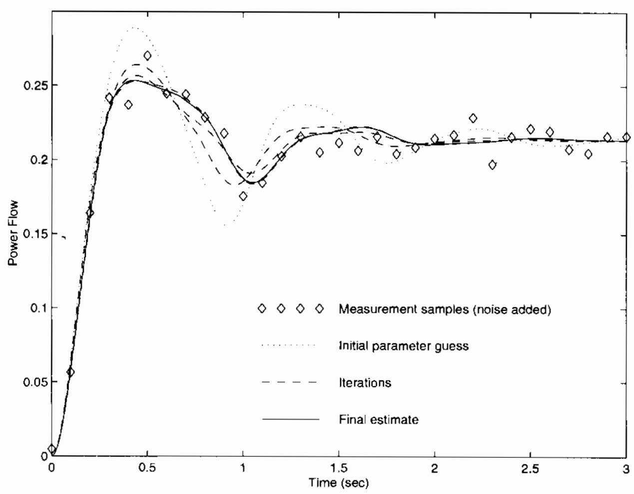

Figure 4: Convergence process. No noise in measurement samples.

Table 1
Parameter changes at each iteration.

Iter.	ΔM_1	ΔM_2	ΔD_1	ΔD_2
1	0.0004	-0.0007	-0.0108	-0.0380
2	-0.0022	-0.0026	-0.0012	-0.0352
3	-0.0002	-0.0050	0.0039	0.0086
4	0.0001	0.0007	0.0010	0.0004
5	0.0000	0.0000	0.0000	0.0001

To further explore the estimation process, noise was added to the 'measurement'. The noise was normally distributed, with a mean of 0 and a standard deviation of 0.01. Again the estimation process converged in 5 iterations from the same initial guess as before. The parameter changes at each iteration were similar to those given in Table 1. The convergence process is illustrated in Fig. 5. Notice that the trajectory converged to a filtered version of the noisy measurement. The estimated parameters were $\lambda = [0.0133 \quad 0.0264 \quad 0.0579 \quad 0.1019]^t$. These values are good estimates of the actual values given in (19). This is especially so, considering the level of noise added to the measurement, and the small number of measurement samples.

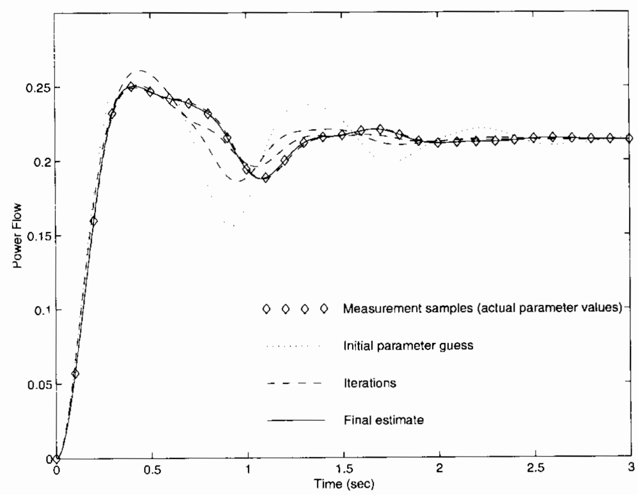

Figure 5: Convergence process. Noisy measurement samples.

The sensitivity of the estimation process to the noise level was investigated by increasing the noise standard deviation four-fold, to 0.04. With a maximum parameter change tolerance of 0.005, the estimation procedure converged in 5 iterations. The estimated parameters were $\lambda = [0.0134 \quad 0.0242 \quad 0.0530 \quad 0.0772]^t$. Results are illustrated in Fig. 6. This figure provides a feel for the corruption caused by the noise. It can be seen that the actual system trajectory is extremely well disguised in the noise. Yet the estimated parameters are close to the actual values, and fairly accurately

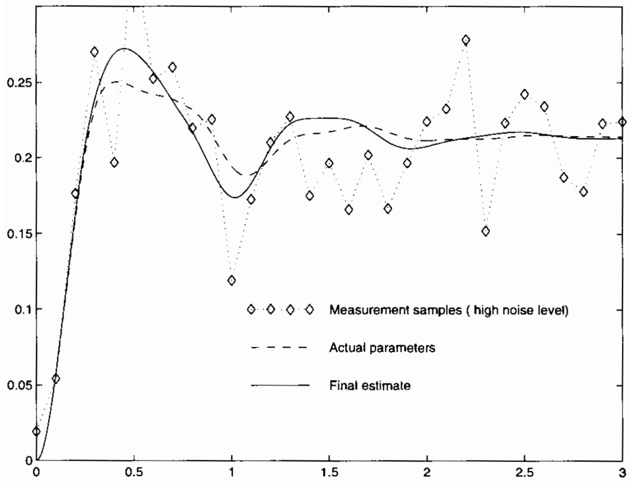

Figure 6: Convergence process. High noise in measurement samples.

reproduce the actual system behaviour. The main error lies in the estimate of D_2. Reasons for that include:

- The estimation algorithm sees the noise as poorly damped oscillations. This is because of the small number of samples and high level of noise.
- Fig.3 shows that the measured power flow is quite insensitive to variations in D_2.

The effect of multiple measurements on the estimation algorithm was examined by adding an extra measurement; that of the real power flow over the line from generator 2 to the infinite bus. All previous cases were repeated. In all cases, the extra measurement led to faster convergence. Table 2 gives the parameter changes at each iteration for the zero noise case. A comparison with Table 1 shows the faster convergence rate. For this case, the final estimated parameter values were exactly the same as earlier. For the other cases, the noise in the extra measurement let to slightly different final values. The estimated parameter values were consistent with those found earlier though.

Table 2
Parameter changes at each iteration. Multiple measurements.

Iter.	ΔM_1	ΔM_2	ΔD_1	ΔD_2
1	0.0002	-0.0016	-0.0004	-0.0472
2	-0.0016	-0.0041	-0.0100	-0.0233
3	-0.0005	-0.0021	0.0036	0.0061
4	0.0000	0.0001	-0.0001	0.0003

5 CONCLUSIONS

A parameter estimation procedure has been proposed for finding power system parameters from measurements of system disturbances. The procedure is based on solving a nonlinear least squares problem using a Gauss-Newton approach. This requires trajectory sensitivities, which effectively provide gradient information at each iteration. The algorithm works reliably, even when measurements are corrupted by significant noise.

Whilst the estimation algorithm has been proposed for obtaining parameters which best fit measurements, it can easily be adapted for model reduction purposes[6]. In that case, the 'measurements' are provided by the full model. The estimation process provides parameter values for the reduced model, such that a best fit between trajectories of the reduced and full models is achieved for scenarios of interest.

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APPENDIX A

TRAJECTORY SENSITIVITY COMPUTATION

Away from events, where system dynamics evolve smoothly, the sensitivities $\underline{x}_{\underline{x}_0}$ and $y_{\underline{x}_0}$ are obtained by differentiating (5), (6) with respect to $\underline{x}_0$. This gives

$$\dot{\underline{x}}_{\underline{x}_0} = \underline{f}_{\underline{x}}(t)\underline{x}_{\underline{x}_0} + \underline{f}_y(t) y_{\underline{x}_0} \tag{20}$$

$$0 = g_{\underline{x}}(t)\underline{x}_{\underline{x}_0} + g_y(t) y_{\underline{x}_0} \tag{21}$$

where $\underline{f}_{\underline{x}} \equiv \dfrac{\partial \underline{f}}{\partial \underline{x}}$, and likewise for the other Jacobian matrices. Note that $\underline{f}_{\underline{x}}, \underline{f}_y, g_{\underline{x}}, g_y$ are evaluated along the trajectory, and hence are time varying matrices. It is shown in [8] that the solution of this (potentially high order) DA system can be obtained as a by-product of solving the original DA system (5), (6).

Initial conditions for $\underline{x}_{\underline{x}_0}$ are obtained from (9) as

$$\underline{x}_{\underline{x}_0}(t_0) = I$$

where I is the identity matrix, and for $y_{\underline{x}_0}$ from (21),

$$0 = g_{\underline{x}}(t_0) + g_y(t_0) y_{\underline{x}_0}(t_0).$$

Equations (20), (21) describe the evolution of the sensitivities $\underline{x}_{\underline{x}_0}$ and $y_{\underline{x}_0}$ between events. However at an event, the sensitivities are generally not continuous. It is necessary to calculate *jump conditions* describing the step change in $\underline{x}_{\underline{x}_0}$ and $y_{\underline{x}_0}$. For clarity, consider a single switching/reset event, so the model (1)-(4) reduces to the form

$$\dot{\underline{x}} = \underline{f}(\underline{x}, y) \tag{22}$$

$$0 = \begin{cases} g^-(\underline{x}, y) & s(\underline{x}, y) < 0 \\ g^+(\underline{x}, y) & s(\underline{x}, y) > 0 \end{cases} \tag{23}$$

$$\underline{x}^+ = \underline{h}\left(\underline{x}^-, y^-\right) \text{ when } s(\underline{x}, y) = 0. \tag{24}$$

Let $(\underline{x}(\tau), y(\tau))$ be the point where the trajectory encounters the hypersurface $s(\underline{x}, y) = 0$, i.e., the point where an event is triggered. This point is called the *junction point* and τ is the *junction time.*

Just prior to event triggering, at time τ^-, we have

$$\underline{x}^- = \underline{x}\left(\tau^-\right) = \phi_{\underline{x}}\left(\underline{x}_0, \tau^-\right)$$

$$y^- = y^-\left(\tau^-\right) = \phi_y\left(\underline{x}_0, \tau^-\right)$$

where

$$0 = g^-\left(\underline{x}^-, y^-\right).$$

Similarly, $\underline{x}^+, y^+$ are defined for time τ^+, just after the event has occurred. It is shown in [8] that the jump conditions for the sensitivities $\underline{x}_{\underline{x}_0}$ are given by

$$\underline{x}_{\underline{x}_0}\left(\tau^+\right) = \underline{h}^*_{\underline{x}} \underline{x}_{\underline{x}_0}\left(\tau^-\right) - \left(\underline{f}^+ - \underline{h}^*_{\underline{x}} \underline{f}^-\right) \tau_{\underline{x}_0} \tag{25}$$

where

$$\underline{h}^*_{\underline{x}} = \left(\underline{h}_{\underline{x}} - \underline{h}_y \left(g_y^-\right)^{-1} g_{\underline{x}}^-\right)\Big|_{\tau^-}$$

$$\tau_{\underline{x}_0} = -\frac{\left(s_{\underline{x}} - s_y \left(g_y^-\right)^{-1} g_{\underline{x}}^-\right)\Big|_{\tau^-} \underline{x}_{\underline{x}_0}\left(\tau^-\right)}{\left(s_{\underline{x}} - s_y \left(g_y^-\right)^{-1} g_{\underline{x}}^-\right)\Big|_{\tau^-} \underline{f}^-}$$

$$\underline{f}^- = \underline{f}\left(\underline{x}\left(\tau^-\right), y^-\left(\tau^-\right)\right)$$

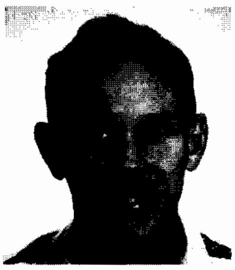
$$\underline{f}^+ = \underline{f}\left(\underline{x}\left(\tau^+\right), y^+\left(\tau^+\right)\right).$$

The sensitivities $y_{\underline{x}_0}$ immediately after the event are given by

$$y_{\underline{x}_0}\left(\tau^+\right) = -\left(g_y^+\left(\tau^+\right)\right)^{-1} g_{\underline{x}}^+\left(\tau^+\right) \underline{x}_{\underline{x}_0}\left(\tau^+\right).$$

Following the event, i.e., for $t > \tau^+$, calculation of the sensitivities proceeds according to (20), (21), until the next event. The jump conditions provide the initial conditions for the post-event calculations.

Real power systems involve many discrete events. The more general case follows naturally though, and is presented in [8].



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