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Sensitivity approaches for direct computation of critical parameters in a power system

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Abstract

In this paper we propose two techniques to estimate critical values of parameters of interest in a power system such as clearing time of circuit breakers, mechanical input power, etc. One is via the sensitivity of the transient energy function (TEF) and the other through computation of the norm of the trajectory sensitivities. Both these methods require some a-priori information about the range of the critical parameters. In real time operation, this information is generally available to the operator. A changing operating condition will result in new values of the critical parameters and the proposed technique can thus monitor them closely without the use of direct methods. The advantage of the first technique lies in not having to compute the unstable equilibrium point (uep) while in the second technique even the computation of stable equilibrium (sep) is not needed. However, there are additional computational costs involved in both the techniques, which can be addressed by faster algorithms for computing the sensitivities.

Keywords: Power systems stability; Trajectory sensitivity; Transient energy function; Critical clearing time

1. Introduction

In the new restructured scenario of power systems, it is very important to assess the stability of the operating point of the system for a set of contingencies on-line in a reliable manner. The transient energy function (TEF) technique is one of the powerful tools to achieve this information and has been the topic for research for the last few decades. Sensitivity approach in dynamic security assessment (DSA) is somewhat a recent technique and its analytical calculations were originally proposed in Ref. [1]. In Refs. [2,3], sensitivities of the normalized energy margin with respect to different system parameters were calculated for analyzing power system stability. Recent applications of trajectory sensitivity to power systems are to be found in Refs. [4–6]. In this paper, we use two approaches to compute the critical clearing time and mechanical input power directly based on computing the trajectory sensitivities: (i) the sensitivity of the energy function *itself* to the parameter of interest is computed to estimate the critical parameter value for a particular fault, (ii) instead of the energy function approach, we compute the norm of the trajectory sensitivity vector itself. Both methods avoid computing the uep. The second method also avoids constructing the energy function and computing the post-fault sep. The sensitivity is computed for two values of the parameter and then extrapolated to obtain an estimate of the critical value. While this may appear to be a drawback, in actual operation the operator has some knowledge of the critical values, and we need to do only two fast simulations. This idea is similar to that of Refs. [7,8] where, based on simulation and expected mode of instability, the system is reduced to a single machine equivalent (SIME) and then critical clearing time is estimated using extrapolation technique at two different values of clearing time.

Fairly restrictive modelling assumptions are required to rigorously establish the transient energy function for a power system model. Accordingly, true Lyapunov stability arguments can only be made for systems that satisfy those assumptions. However the stability assessment approach proposed in this paper does not rely on Lyapunov concepts. Rather, the energy function is used purely as a metric, or measure, of the 'distance' between the transient state (a point on the trajectory) and the post-fault stable equilibrium point. Therefore no restrictions need to be placed on system modelling. Another metric that can be used is the norm of the trajectory sensitivity vector itself. Additional computational tasks are involved in calculating the trajectory sensitivities. However, one can exploit the structural similarity in the Jacobian of both the system and sensitivity models [9].

The paper is organized as follows. System and sensitivity

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models for differential–algebraic equations (DAE) are discussed in Section 2. Section 3 discusses the method for estimation of t_{cr} for DAE models using the TEF sensitivity. Section 4 explains a direct approach using norm of the sensitivity vector as a metric instead of the energy function. Both approaches are model independent, so that they can be applied to systems with any complex level of system modelling. Section 5 gives results for two test systems applying the methods discussed in Sections 3 and 4. The test systems used here are the 3-machine, 9-bus system and the 10-machine, 39-bus system. Since the method outlined in Section 4 is independent of the energy function concept or system modelling details, we illustrate its application for the 3-machine and the 10-machine systems with detailed modelling as well. In this section we also compute critical value of mechanical power P_m for a given contingency. Section 6 gives conclusions and areas of future research.

2. System sensitivity models

In simulating disturbances, switching actions take place at certain time instants. At these time instants, the algebraic equations change, resulting in discontinuities of the algebraic variables. In general the power system can be cast in the form of a differential–algebraic discrete (DAD) model incorporating discrete events as in Ref. [6]. A special case is the model described by differential–algebraic equations of the form

$$\dot{x} = f(x, y, \lambda) \tag{1}$$

$$0 = \begin{cases} g^-(x, y, \lambda) & s(x, y, \lambda) < 0 \\ g^+(x, y, \lambda) & s(x, y, \lambda) > 0 \end{cases} \tag{2}$$

A switching occurs when the switching function $s(x, y, \lambda) = 0$.

In the above model, x are the dynamic state variables such as machine angles, velocities, etc.; y are the algebraic variables such as load bus voltage magnitudes and angles; and λ are the system parameters such as line reactances, generator mechanical input power, or fault clearing time. Note that the state variables x are continuous while the algebraic variables can undergo step changes at switching instants.

The initial conditions for Eqs (1)and (2) are given by

$$x(t_0) = x_0, \qquad y(t_0) = y_0 \tag{3}$$

where y_0 satisfies the equation

$$g(x_0, y_0, \lambda) = 0 \tag{4}$$

For compactness of notation, the following definitions are used:

$$\underline{x} = \begin{bmatrix} x \\ \lambda \end{bmatrix}, \qquad \underline{f} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

With these definitions, Eqs. (1) and (2) can be written in a compact form as

$$\dot{\underline{x}} = \underline{f}(\underline{x}, y) \tag{5}$$

$$0 = \begin{cases} g^-(\underline{x}, y) & s(\underline{x}, y) < 0 \\ g^+(\underline{x}, y) & s(\underline{x}, y) > 0 \end{cases} \tag{6}$$

The initial conditions for Eqs. (5) and (6) are

$$\underline{x}(t_0) = \underline{x}_0, \qquad y(t_0) = y_0 \tag{7}$$

Trajectory sensitivity analysis studies the variations of the system variables with respect to the small variations in initial conditions x_0 and parameters λ (or equivalently $\underline{x}_0$).

Away from discontinuities, the differential–algebraic system can be written in the form

$$\dot{\underline{x}} = \underline{f}(\underline{x}, y) \tag{8}$$

$$0 = g(\underline{x}, y) \tag{9}$$

Differentiating Eqs. (8) and (9) with respect to the initial conditions $\underline{x}_0$ yields

$$\dot{\underline{x}}_{\underline{x}_0} = \underline{f}_{\underline{x}}(t)\underline{x}_{\underline{x}_0} + \underline{f}_y(t)y_{\underline{x}_0} \tag{10}$$

$$0 = g_{\underline{x}}(t)\underline{x}_{\underline{x}_0} + g_y(t)y_{\underline{x}_0} \tag{11}$$

where $\underline{f}_{\underline{x}}$, $\underline{f}_y$, $g_{\underline{x}}$, and g_y are time varying matrices that are calculated along the system trajectories, and $\underline{x}_{\underline{x}_0}(t)$ $\dot{x}$ and $y_{\underline{x}_0}(t)$ are the trajectory sensitivities.

Initial conditions for $\underline{x}_{\underline{x}_0}$ are obtained by differentiating Eq. (7) with respect to $\underline{x}_0$ as

$$\underline{x}_{\underline{x}_0}(t_0) = I \tag{12}$$

where I is the identity matrix.

Using Eq. (12) and assuming that $g_y(t_0)$ is nonsingular along the trajectories, initial conditions for $y_{\underline{x}_0}$ can be calculated from Eq. (11) as

$$y_{\underline{x}_0}(t_0) = -[g_y(t_0)]^{-1} g_{\underline{x}}(t_0) \tag{13}$$

Therefore, the trajectory sensitivities can be obtained by solving Eqs. (10) and (11) simultaneously with Eqs. (8) and (9) using Eqs. (7), (12), and (13) as the initial conditions. At the discontinuity where $s(\underline{x}, y) = 0$, the trajectory sensitivities $\underline{x}_{\underline{x}_0}$, $y_{\underline{x}_0}$ typically undergo a jump. Derivation of these jump conditions is provided in Ref. [6].

3. Estimation of critical clearing time using trajectory sensitivities

In the literature, trajectory sensitivities have been used [10] to compute the energy margin sensitivity with respect to system parameters such as interface line flow, system loading, etc. In these cases, the critical energy ν_{cr} is the energy at the controlling uep, and hence depends on the parameters. Therefore, computation of $\partial \nu_{cr}/\partial t_{cl}$ is necessary. This is computationally a difficult task. On the other hand,

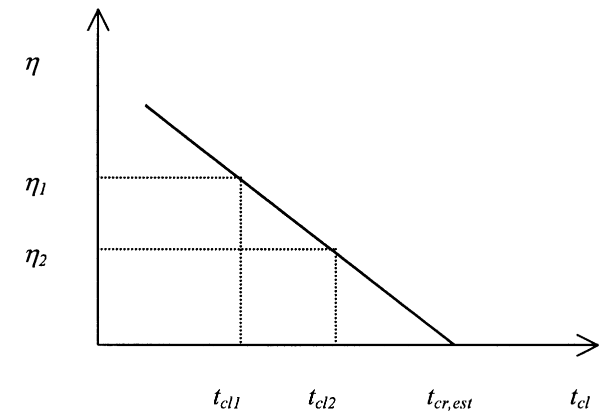

Fig. 1. Estimate of $t_{\rm cr}$.

because the energy function $\nu(x)$ is used here only as a metric to monitor the system sensitivity for different $t_{\rm cl}$, we can avoid the computation of $\nu_{\rm cr}$.

The process of estimating critical values of parameters will be illustrated using the clearing time $t_{\rm cl}$. However, the process is appropriate for any parameter that can induce instability. A later example considers mechanical power $P_{\rm m}$. We can use the sensitivity $\partial\nu/\partial t_{\rm cl}$ to estimate $t_{\rm cr}$ directly. With classical model for machines, the energy function $\nu(x)$ for a structure-preserving model is provided in Appendix A. The sensitivity S of the energy function $\nu(x)$ with respect to clearing time ($\lambda = t_{\rm cl}$) is obtained by taking partial derivatives of Eq. (A6) with respect to $t_{\rm cl}$ as

$$S = \frac{\partial \nu}{\partial t_{\rm cl}} = \sum_{i=1}^{m} M_i \tilde{\omega}_{g_i} \frac{\partial \tilde{\omega}_{g_i}}{\partial t_{\rm cl}} - \sum_{i=1}^{m} P_{M_i} \frac{\partial \theta_{n_0+i}}{\partial t_{\rm cl}} + \sum_{i=1}^{n_0} P_{d_i} \frac{\partial \theta_i}{\partial t_{\rm cl}} - \sum_{i=1}^{n_0} B_{ii} V_i \frac{\partial V_i}{\partial t_{\rm cl}} + \sum_{i=1}^{n_0} Q_{d_i}^{\rm s} \frac{V_i^{\alpha-1}}{V_i^{s\alpha}} \frac{\partial V_i}{\partial t_{\rm cl}} - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} B_{ij} \left(V_j \cos\theta_{ij} \frac{\partial V_i}{\partial t_{\rm cl}} + V_i \cos\theta_{ij} \frac{\partial V_j}{\partial t_{\rm cl}} - V_i V_j \sin\theta_{ij} \frac{\partial \theta_{ij}}{\partial t_{\rm cl}} \right) \quad (14)$$

The partial derivatives of $\tilde{\omega}_{g_i}$, θ and V with respect to $t_{\rm cl}$ are the sensitivities obtained from Eqs. (10) and (11) and jump conditions.

The sensitivity $S = \partial\nu/\partial t_{\rm cl}$ is computed for two different values of $t_{\rm cl}$ which are chosen to be less than $t_{\rm cr}$. Since we are computing only first order trajectory sensitivities, the two values of $t_{\rm cl}$ must be less than $t_{\rm cr}$ by at the most 20%. This might appear to be a limitation of the method. However, extensive experience with the system generally will give us a good estimate of $t_{\rm cr}$. Because the system under consideration is stable, the sensitivity S will display larger excursions for larger $t_{\rm cl}$ [6]. Since sensitivities generally increase rapidly with increase in $t_{\rm cl}$, we plot the reciprocal of the maximum deviation of S over the post-fault period as $\eta = 1/(\max(S) - \min(S))$. A straight line is then constructed through the two points ($t_{\rm cl1}, \eta_1$) and ($t_{\rm cl2}, \eta_2$). The estimated critical clearing time $t_{\rm cr,est}$ is the intersection of the constructed straight line with the time-axis in the ($t_{\rm cl}$, η)-plane as shown in Fig. 1.

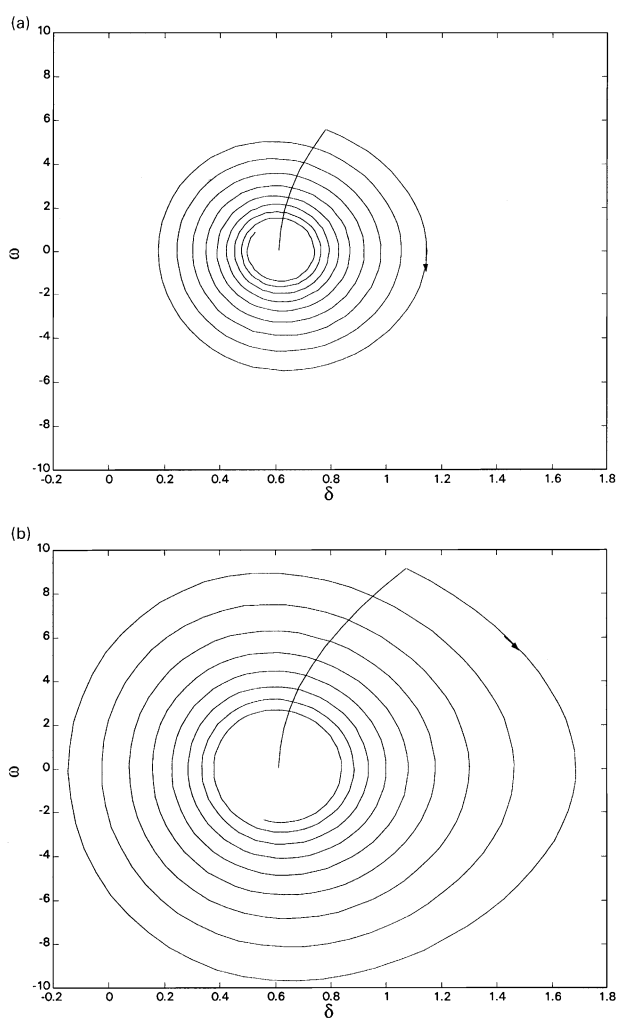

Fig. 2. (a): Phase plane behavior for small $t_{\rm cl}$ ($\approx 50\%$ of $t_{\rm cr}$). (b): Phase plane behavior for $t_{\rm cl}$ close to $t_{\rm cr}$ ($\approx 80\%$ of $t_{\rm cr}$).

4. Direct use of trajectory sensitivities to compute critical clearing time

In this section we outline an approach using trajectory sensitivity information *directly* instead of via the energy function to estimate the critical clearing time. To motivate this approach let us consider a single machine infinite bus system described by

$$\begin{aligned} M\ddot{\delta} + D\dot{\delta} &= P_{\rm m} \quad 0 < t \le t_{\rm cl}, \\ M\ddot{\delta} + D\dot{\delta} &= P_{\rm m} - P_{\rm em}\sin\delta \quad t > t_{\rm cl} \end{aligned} \quad (15)$$

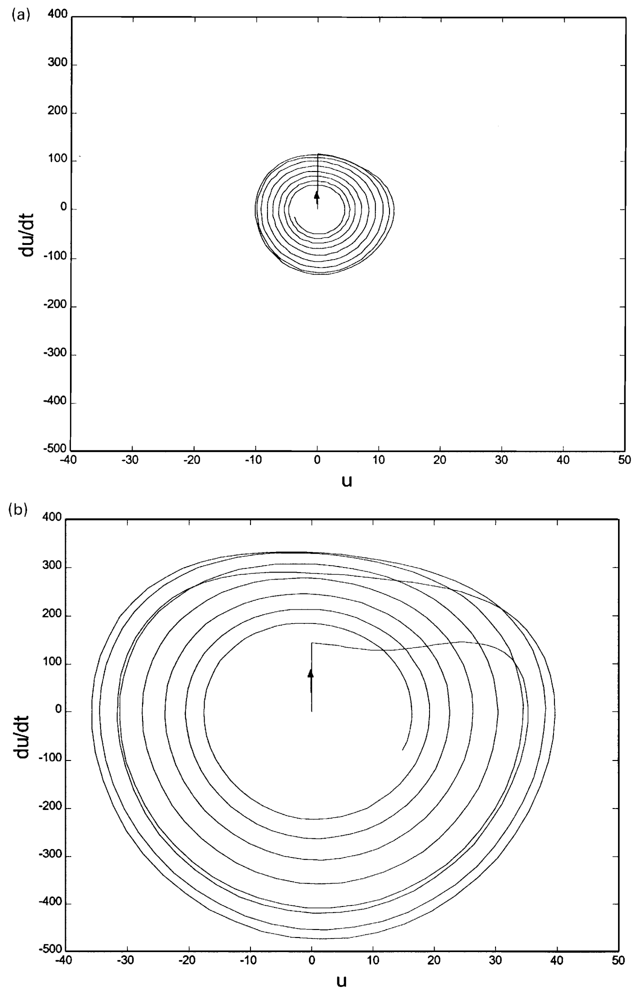

Fig. 3. (a) Sensitivity plane behavior for small t_{cl} ($\approx 50\%$ of t_{cr}). (b) Sensitivity plane behavior for t_{cl} close to t_{cr} ($\approx 80\%$ of t_{cr}).

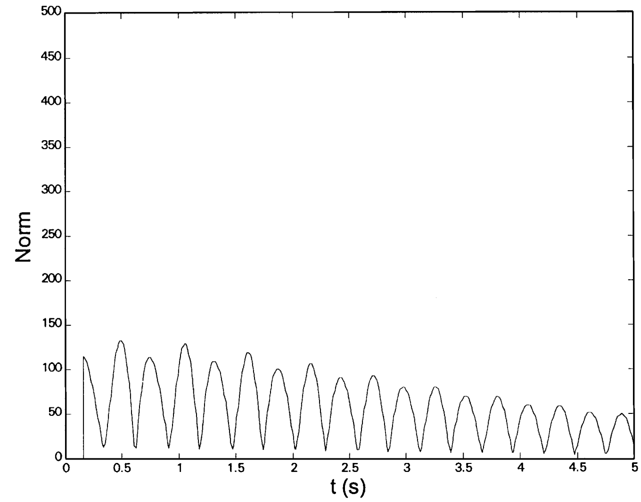

Fig. 5. Sensitivity norm for small t_{cl} ($\approx 50\%$ of t_{cr}).

The corresponding sensitivity equations, using the same approach as in Section 2, are

$$M\ddot{u} + D\dot{u} = 0 \quad 0 < t \le t_{cl},$$
$$M\ddot{u} + D\dot{u} = (-P_{em} \cos \delta)u \quad t > t_{cl} \tag{16}$$

where $u = \partial\delta/\partial t_{cl}$.

The phase plane portrait of the system for two values of t_{cl}, one small and the other close to t_{cr} are shown in Fig. 2(a) and (b). The corresponding behaviors of sensitivities in the $(u, \dot{u})$-plane are shown in Fig. 3(a) and (b). From this it is seen that the sensitivity magnitudes increase much more rapidly as t_{cl} approaches t_{cr}. Also, the trajectories in the $(u, \dot{u})$-plane can cross each other since the system (16) is time varying while that is not the case for the system (15) which is an autonomous system. Qualitatively, both trajectories in the (δ, ω)-plane and the $(u, \dot{u})$-plane give the same information about the stability of the system, but the

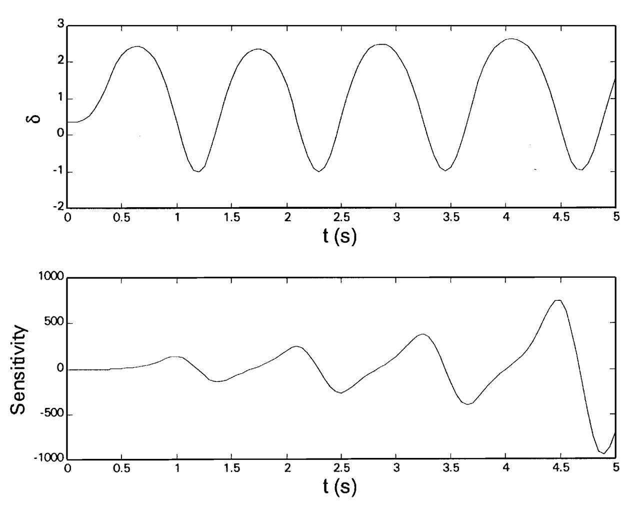

Fig. 4. Rotor angle and its sensitivity for t_{cl} very close to t_{cr} for 3-machine system.

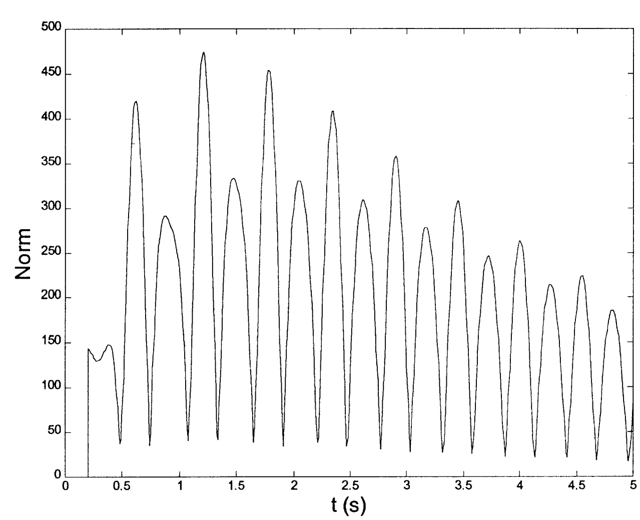

Fig. 6. Sensitivity norm for t_{cl} close to t_{cr} ($\approx 80\%$ of t_{cr}).

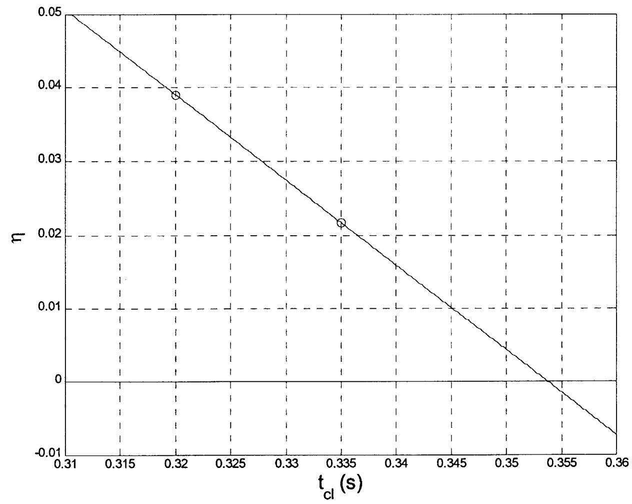

Fig. 7. Estimate t_{cr} for fault at bus 5 using energy function sensitivity.

sensitivities seem to be stronger indicators because of their rapid changes in magnitude as t_{cl} increases. Hence, we can associate sensitivity information with the stability level of the system for a particular clearing time. When the system is very close to instability, the sensitivity reflects this situation much more quickly as seen in Fig. 4 for a 3-machine test system. This qualitative relationship was discussed for the general nonlinear dynamic systems by Tomovic [11]. One possible measure of proximity to instability may be through some norm of the sensitivity vector. The Euclidean norm is one such possibility.

For the single machine system, if we plot the norm $\sqrt{u^2 + \dot{u}^2}$ as a function of time for different values of t_{cl}, one can get a quick idea about the system stability as shown in Figs. 5 and 6. For a stable system, although the sensitivity norm tends to zero eventually, it transiently assumes a very high value when t_{cl} is close to t_{cr}.

Thus, we associate with each value of t_{cl} the maximum value of the sensitivity norm. The procedure to calculate the estimated value of t_{cr} is the same as described in Section 3 but using the sensitivity norm instead of the energy function sensitivity. Here, the sensitivity norm for an m-machine system is defined as

$$\mathbb{S} = \sqrt{\sum_{i=1}^{m} \left(\left(\frac{\partial \delta_i}{\partial t_{cl}} - \frac{\partial \delta_j}{\partial t_{cl}} \right)^2 + \left(\frac{\partial \omega_i}{\partial t_{cl}} \right)^2 \right)}$$

where the jth-machine is chosen as the reference machine.

Table 1
Estimate t_{cr} using TEF sensitivity and sensitivity norm for 3-machine system

Faulted bus	TEF sensitivity $t_{cr,est}$ (s)	Sensitivity norm $t_{cr,est}$ (s)	Actual t_{cr} (s)
5	0.354	0.352	0.352
8	0.333	0.333	0.334

The norm is calculated for two values of $t_{cl} < t_{cr}$. For each t_{cl}, the reciprocal η of the maximum of the norm is calculated. A line through these two values of η is then extrapolated to obtain the estimated value of t_{cr}. If mechanical input power is chosen as the parameter instead, the technique will give an estimate of critical value of P_m for the machine.

Since this technique does not require computation of the energy function, it can be applied to power systems without any restriction on system modelling. This is a major advantage of this technique.

5. Numerical results

5.1. Classical model representation

A 3-machine, 9-bus [12] and a 10-machine, 39-bus power systems [13] are used to illustrate the technique. For the 3-machine system, a self-clearing fault is simulated at bus 5 and cleared at two different values of t_{cl} less than t_{cr}. The corresponding values of η discussed in Section 3 (TEF sensitivity) are computed, and the results are shown in Fig. 7. The reactive power load index α is chosen as 2.

The procedure is repeated for the same system with the fault at bus 8. The estimated critical clearing time and the actual value obtained for both the cases are shown in Table 1.

Next, the technique discussed in Section 4 (sensitivity norm) is applied to the same system. The estimated results are shown in Table 1 for self-clearing faults at bus 5 and bus 8. The simulation result using the sensitivity norm technique for fault at bus 5 is shown in Fig. 8. Hence, the values of t_{cr} obtained from both techniques are very close to the actual value.

For the 10-machine system [13], the estimated value of clearing time for a self clearing fault at bus 17 using the TEF sensitivity technique is obtained as 0.276 s, and the value obtained by sensitivity norm technique is 0.278 s. They agree very well with the actual value of $t_{cr} = 0.277$ s.

5.2. Detailed model

The sensitivity norm technique was used to assess stability of the detailed model of the 3-machine, 9-bus system. In this case, all machines are represented as a 2-axis model with turbine, governor, and exciter [12]. A fault is simulated at bus 7 and cleared by tripping the line 5–7 of the system. By applying the sensitivity norm technique, the estimated critical clearing time is $t_{cr,est} = 0.113$ s. The actual value is $t_{cr} = 0.115$ s. Again, the technique gives an estimated value that is very close to the actual value.

For the 10-machine, 39-bus system, the sensitivity norm technique is applied to estimate the critical clearing time for faults at various locations in the system. The results are summarized in Table 2.

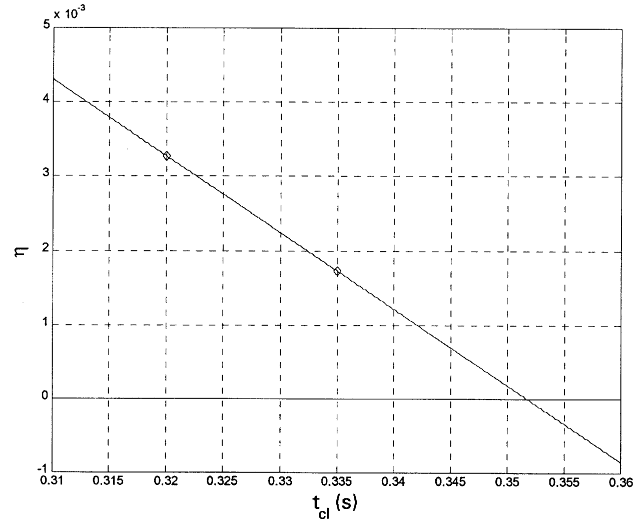

Fig. 8. Estimate t_{cr} for fault at bus 5 using norm sensitivity.

5.3. Computation of critical loading of generator

Next, the sensitivity norm technique is used to estimate the critical value of generator loading, or equivalently, the mechanical input power P_m. A fault is simulated in the system at bus 21 and cleared at $t_{cl} = 0.1$ s by tripping the line 21–22. Two simulations for two values of P_m are carried out. The change from normal operating values in P_m is distributed uniformly among all loads in the system, so that the loading of the rest of the generators is unchanged. The sensitivity norm is calculated for the two specified values of P_m and then extrapolated to obtain the estimated value of the critical P_m for the chosen generator. The estimated results for a few generators are shown in Table 3.

To validate the results it was verified that with the critical value of P_m the system goes unstable.

These examples show that the techniques described in Sections 3 and 4 give a good way to estimate the critical clearing time of faults and the critical value of mechanical input power for a generator. A similar process can be used to estimate the critical value of any other parameter.

6. Conclusions

Technique based on sensitivity information at two values of clearing time and using linear extrapolation is employed to estimate the value of critical clearing time. The difference between the two techniques is that they use a different metric to measure the sensitivity of the system. The numerical results on the 3-machine, 9-bus and 10-machine, 39-bus systems have shown that the techniques give accurate results both for classical model and the detailed model. The procedure using sensitivity norm has also been used to compute critical loading of the generator. Thus the technique is quite general and can be adapted to compute the critical value of any parameter in the system. Two potential drawbacks of the technique are the need to have some a priori idea about the critical value and the need to compute the trajectory sensitivities. The former can be addressed by the fact that in on-line operation, this information is usually available. As for the latter, improved computational techniques exploiting the similarity of the Jacobian and use of Krylov subspace technique will help in reducing the computation time [9]. Detailed models can be handled with no difficulty. The TEF method using uep concepts is known to fail in certain cases, and hence the proposed techniques can be considered as an alternative for computing t_{cr}. The knowledge of the critical loading of generator will assist in preventive rescheduling [15]. Further research in improving the speed of simulation is being pursued.

Table 2
Estimated value of the critical clearing time vs the actual value

Faulted bus	Line tripped	$t_{cr,est}$ (s)	t_{cr} (s)
4	4–5	0.210	0.212
15	15–16	0.204	0.206
17	17–18	0.169	0.168
21	21–22	0.122	0.125

Table 3
Estimated value of critical input power P_m vs the actual value

Generator	$P_{mcr,est}$ (pu)	$P_{m,cr}$ (pu)
3	10.7	10.4
5	6.3	6.4
8	12.4	12.2

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Appendix A. Energy function for the structure preserving model

The post fault power system can be represented by the DAE model in the center of angle reference frame as [13]

$$\dot{\theta}_{n_0+i} = \tilde{\omega}_{g_i} \quad i = 1, \ldots, m \tag{A1}$$

$$M_i \dot{\tilde{\omega}}_{g_i} = P_{M_i} - \sum_{j=1}^{n} B_{n_0+i,j} V_{n_0+i} V_j \sin(\theta_{n_0+i} - \theta_j) - \frac{M_i}{M_T} P_{COA} \tag{A2}$$

$i = 1, \ldots, m$

$$P_{d_i} + \sum_{j=1}^{n} B_{ij} V_i V_j \sin(\theta_i - \theta_j) = 0 \;\; i = 1, \ldots, n_0 \tag{A3}$$

$$Q_{d_i}(V_i) - \sum_{j=1}^{n} B_{ij} V_i V_j \cos(\theta_i - \theta_j) = 0 \;\; i = 1, \ldots, n_0 \tag{A4}$$

where m is the number of machines, n_0 is the number of buses in the systems, and

$$P_{\mathrm{COA}} = \sum_{i=1}^{m} \left(P_{M_i} - \sum_{j=1}^{n} B_{ij} V_i V_j \sin(\theta_i - \theta_j) \right)$$

We assume constant real power loads and voltage dependent reactive power load of the form

$$Q_{d_i}(V_i) = Q_{d_i}^{\mathrm{s}} \left(\frac{V_i}{V_i^{\mathrm{s}}} \right)^{\alpha} \tag{A5}$$

where Q_i^{s} and V_i^{s} are the nominal steady state reactive power load and voltage magnitude at the ith bus, and α is the reactive power load index.

The corresponding energy function is established as [14]

$$\begin{aligned} \nu(\tilde{\omega}_g, \theta, V) = {} & 1/2 \sum_{i=1}^{m} M_i \tilde{\omega}_{g_i}^2 - \sum_{i=1}^{m} P_{M_i}(\theta_{n_0+i} - \theta_{n_0+i}^{\mathrm{s}}) \\ & + \sum_{i=1}^{n_0} P_{d_i}(\theta_i - \theta_i^{\mathrm{s}}) - 1/2 \sum_{i=1}^{n_0} B_{ii}(V_i^2 - V_i^{\mathrm{s}2}) \\ & + \sum_{i=1}^{n_0} \frac{Q_{d_i}^{\mathrm{s}}}{\alpha V_i^{\mathrm{s}\alpha}} (V_i^{\alpha} - V_i^{\mathrm{s}\alpha}) \\ & - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} B_{ij}(V_i V_j \cos\theta_{ij} - V_i^{\mathrm{s}} V_j^{\mathrm{s}} \cos\theta_{ij}^{\mathrm{s}}) \end{aligned} \tag{A6}$$

where $\theta_{ij} = \theta_i - \theta_j$.

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