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Direct calculation of reactive power limit points

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A knowledge of points where sources of reactive power encounter limits is useful in determining the vulnerability of a power system to voltage collapse. This paper proposes a predictor/corrector technique which quickly and robustly finds such points. The predictor is based on sensitivity ideas, whilst the corrector is a slight modification of the standard power flow problem. It is shown that a knowledge of limit points can provide a fast estimate of the point of collapse of a power system, and that it can improve the usefulness of some common voltage collapse indices.

Keywords: network analysis, security assessment, loadflow

I. Introduction

Lack of adequate reactive power resources in a power system is a major contributing factor to the process of voltage collapse[1–3]. As loads in a power system increase, voltages across the network tend to decrease and reactive power losses increase. This increased reactive power demand would be supplied by voltage regulating devices such as generators or static var compensators, if possible. However, owing to physical limitations such devices cannot supply unlimited amounts of reactive power. Often sustained load growth will result in some source of reactive power, or perhaps a number of such sources, encountering limits, i.e. reaching a physical limitation in the amount of reactive power that they can supply[3].

Once a reactive power source has reached its maximum limit, it can no longer regulate voltage. Therefore sustained load growth results in accelerated voltage decay, and hence greater reactive power requirements. This may force other voltage regulating devices to their limits, with subsequent further acceleration in the rate of decline of voltages. This is illustrated in Figure 1. As loads increase from their initial value at point A, network voltages fall. At point B, the first reactive power source encounters its limit. Voltages begin to fall more rapidly. Sustained load increase results in a second reactive power source encountering its limit at point C. If load increased further, the point of collapse (PoC) would soon be reached. This behaviour is part of the process known as voltage collapse. Many factors other than those described in this overview influence the voltage collapse process[4,5]. However limits on reactive power resources are a major factor. This paper therefore focuses on a method for determining points where limits are encountered.

The encountering of limits by reactive power resources is a sign of vulnerability of a power system. It is therefore useful to have a method for determining limit points directly. In a planning environment, system planners could identify when and where reactive power resources became inadequate. Steps could be taken to reinforce those resources, or reduce the reactive power requirements. A knowledge of reactive power limit points could also be extremely useful in an operational environment. Knowing the level of load increase that would drive resources to their limits, operators could decide whether it was necessary to take steps to free-up reserves. For example, an operator may decide to reschedule generation, or to use some FACTS device to reduce the power flow through some part of the network.

The paper proposes a predictor/corrector technique which robustly finds all the reactive power limit points as the system is loaded according to a specified loading pattern. Referring to Figure 1, points B, C and D would be found. Note that the proposed technique is not a continuation method in the sense described in References 9–11. No attempt is made to obtain points along the path between the limit points. Consequently the proposed technique offers speed advantages over traditional continuation methods, but provides less information.

The predictor part of the algorithm was inspired by the sensitivity based ideas of References 6 and 7. Those methods rely on linearization of the power flow equations. They provide approximate limit points and point of collapse information without needing to perform any power flows apart from at the initial point. However because these techniques are based on linearization, their accuracy is very dependent on how close the initial point is to the points of interest, i.e. limit points and the point of collapse. If the initial point is some distance away, the

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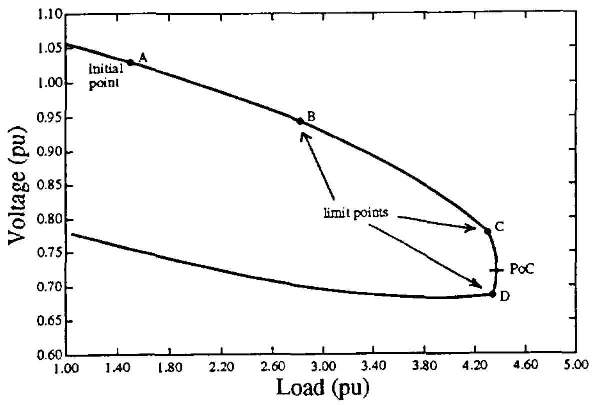

Figure 1. Typical nose curve

accuracy may not be so good. Further, it is not possible to determine *a priori* the accuracy of the results.

Other methods have been proposed, primarily for finding the PoC, but which take into account reactive power limits[8–10]. They provide complete information about the reactive power sources that are on limits at the PoC. But they do not explicitly provide information about the encountering of limits during the loading process. For example it is not possible to determine exactly the value of the loading parameter when a particular source encounters its limit.

The paper is structured as follows. The predictor/corrector algorithm is discussed in Section II. In Section III the details of the implementation of the corrector are provided. Examples are discussed in Section IV. Section V describes how a knowledge of the reactive power limit points can be used to provide a good fast estimate of the point of collapse. It also discusses the use of this knowledge for improving some voltage collapse indices.

II. A predictor/corrector algorithm

Consider the power flow equations:

$$f(x, \lambda(\tau)) = 0 \tag{1}$$

where x is the vector of state variables (voltages and angles at load buses, angles at generator buses), and $\lambda(\tau)$ is a vector of parameters (typically real and reactive power loads).

In the normal power flow problem, the parameters λ are fixed, and equation (1) is solved using an algorithm such as Newton–Raphson for the unknown state variables x. Having calculated the state variables all other system quantities such as line flows, system losses and/or reactive power generated at voltage controlled buses can be found.

Often we are interested in studying how solutions of equation (1) vary as parameters are changed. If one parameter is varied at a time, then a curve results. Figure 1 is an example of the type of curve that can result. If two parameters are allowed to change, then a surface is obtained. Variation of more parameters results in a higher dimensional hypersurface.

In the development of this algorithm, we shall restrict attention to the usual case where only one parameter is varied at a time. However we shall allow some generality in the loading pattern by defining parameter variation as

$$\lambda(\tau) = \lambda_0 + \tau\mu \tag{2}$$

where τ is the single loading parameter which will be varied, μ is a fixed vector of unit length which defines the direction in which the system will be loaded, and λ_0 gives the initial values of loads.

Many common loading patterns can be described by (2). For example, if we were interested in varying only the reactive power at one particular bus, then μ would be all zeros, except for a 1 in the location corresponding to the desired reactive power load. If we required real and reactive power load at some bus to be varied, but that its power factor remained constant, then μ would be all zeros, except for the two entries corresponding to the desired bus. The real power load entry would have a value $P_0/\sqrt{P_0^2 + Q_0^2}$ and the reactive power entry would be $Q_0/\sqrt{P_0^2 + Q_0^2}$, where P_0 and Q_0 were the initial load values.

As noted above, Figure 1 is an example of the type of curve that can result from allowing τ to vary in (1). Of particular interest are the points B, C and D. We would like to have a method for determining the values of x and τ (and hence λ) at those points. In this paper we are proposing a predictor/corrector technique that achieves that aim.

Flatabø *et al.*[6] have proposed a method, based on linearization of the power flow equations, for determining *approximately* the limit points. Such methods, which are based on sensitivity ideas, do not tell us exactly the limit points. But they do (generally) provide a good estimate of their location. Sensitivity ideas therefore form a good basis for the predictor part of the proposed algorithm. (Having determined the approximate location of a desired limit point, a corrector is then used to give the exact point. This is discussed later.)

To establish the predictor, we first find the total derivative of the power flow equations (1):

$$\begin{aligned} df = 0 &= f_x\, dx + f_\lambda \frac{d\lambda}{d\tau}\, d\tau \\ &= f_x\, dx + f_\lambda\, \mu\, d\tau \end{aligned} \tag{3}$$

So

$$\Delta x = -f_x^{-1} f_\lambda\, \mu \Delta\tau \tag{4}$$

or

$$\Delta x = S_{x\tau} \Delta\tau \tag{5}$$

where $S_{x\tau}$ gives the sensitivity of the state variables x to variations in the loading parameter τ. Define Q as the vector of reactive power productions at voltage regulated buses. The limit points, which are the points of interest to us, occur when any one of the Q entries reaches its maximum value, given by $Q_{\max}$. The Q are dependent variables:

$$Q = Q(x) \tag{6}$$

Therefore

$$\Delta Q = Q_x \Delta x$$

From (4):

$$\Delta Q = -Q_x f_x^{-1} f_\lambda\, \mu \Delta\tau \tag{7}$$

or

$$\Delta Q = Q - Q_0 = S_{q\tau} \Delta\tau \tag{8}$$

where $S_{q\tau}$ gives the sensitivity of the reactive power generation to variations in the loading parameter τ, and Q_0 is the initial value of Q.

The reactive power limits, Q_{max}, are generally taken as fixed values. However it has been shown in Reference 12 that it is often more appropriate to model the limits as voltage dependent. In that case Q_{max} becomes a dependent variable:

$$Q_{max} = Q_{max}(x) \tag{9}$$

Therefore

$$\Delta Q_{max} = Q_{max,x}\Delta x$$

From (4) we obtain

$$\Delta Q_{max} = Q_{max} - Q_{max,0} = S_{m\tau}\Delta\tau \tag{10}$$

where $S_{m\tau}$ gives the sensitivity of the reactive power limit to variations in τ, and $Q_{max,0}$ is the initial value of Q_{max}. Note that for sources that have a fixed maximum limit, $S_{m\tau,j} = 0$.

We wish to find the change in τ which would cause each reactive power source to encounter its limit, i.e. what value of $\Delta\tau$ causes $Q_j = Q_{max,j}$ for each source j? To find this value, we set $Q_j = Q_{max,j}$ in (8):

$$Q_{max,j} - Q_{0,j} = S_{q\tau,j}\Delta\tau$$

From (10) we have:

$$Q_{max,j} = Q_{max,0j} + S_{m\tau,j}\Delta\tau$$

Substituting yields

$$Q_{max,0j} - Q_{0,j} = (S_{q\tau,j} - S_{m\tau,j})\Delta\tau \tag{11}$$

The minimum of those values of $\Delta\tau$ is therefore given by:

$$\Delta\tau_{min} = \min_j\left(\frac{Q_{max,0j} - Q_{0,j}}{S_{q\tau,j} - S_{m\tau,j}}\right) \tag{12}$$

An increase of $\Delta\tau_{min}$ in the parameter τ would cause the first limit to be encountered. In Figure 1, that would correspond to point *B*. So, from (12), it is possible to determine which source is the first to encounter a limit, and the approximate change in parameter τ which would force that source to its limit. Equation (5) would then provide the corresponding changes in the state variables x. We therefore have a prediction of the values of x and τ at the first limit point. The values are only estimates. The next step is to correct to the true point.

By definition, the limit points are points where a voltage regulating device first encounters its maximum reactive power limit, as the parameter increases. For lower values of the parameter, the voltage is constrained. For higher values, the reactive power is constrained. (For an SVC, the capacitive susceptance would instead be constrained.) But at the limit point, the voltage is at its setpoint value, and the reactive power is constrained to its limit value. The corrector makes use of this fact that both voltage and reactive power are fixed.

The equations which therefore define the limit point for the kth source are:

$$\left.\begin{aligned} f(x, \lambda(\tau)) &= 0 \\ Q_k(x) - Q_{max,k}(x) &= 0 \end{aligned}\right\} \tag{13}$$

i.e. the original power flow equations (1), together with the extra reactive power constraint. The unknowns in this set of equations are the state variables x together with the loading parameter τ. Notice that the number of equations and the number of unknowns have both increased by one.

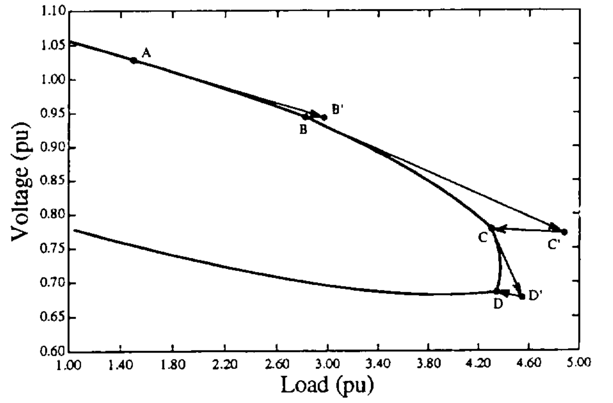

Figure 2. Predictor/corrector process

The predictor provides an initial guess for x, τ. Equations (13) can then be solved to within the desired tolerance using almost standard power flow techniques. Section III gives details of the modifications which are needed to standard power flows in order to solve this slightly different problem.

Figure 2 illustrates the predictor/corrector process, as described so far. Starting from the initial point A, linearization of the power flow equations yields a predicted limit power B′. The corrector uses B′ as an initial guess to solve for the actual limit point B. The predictor/corrector process can be repeated to obtain subsequent limit points. For example, to solve for limit point C, point B would be used as the initial point. Recall though that for parameter values greater than that at the limit point, the corresponding reactive power source switches from having regulated voltage to having fixed reactive power. So in solving for point C, this constraint change must be made. (In Figure 2 this constraint change shows as a discontinuity in the slope of the curve at the limit points. Notice that the predictor for point C is tangential to the curve to the right of point B, but not to the curve on the left.)

Having found point C, the predictor/corrector process can again be used this time to find limit point D. Notice that the curve has much greater curvature near the point of collapse, so behaviour is much less linear. It has been found in such cases that occasionally there is a need to limit the length of the predictor vector, i.e. reduce the value of $\Delta\tau$ obtained from (12). The approach taken has been to ensure that no predicted voltages fall below a predetermined threshold, for example 0.6 p.u. The difference between the initial values of voltage and the threshold value can be used in (5) to determine the appropriate reduced value of $\Delta\tau$.

Comments

(1) Even when the corrector is solving for a limit point that is near the point of collapse, it does not suffer the convergence problems of normal power flows. In normal power flows, the Jacobian of the power flow equations is singular at the point of collapse. Solution techniques such as Newton–Raphson rely on inverting the Jacobian, so tend to exhibit poor convergence behaviour near the point of collapse. However the corrector solves the augmented problem (13), and so uses a different Jacobian. The corrector can successfully solve for limit points, even if they are right at the point of collapse.

(2) The computational effort involved in finding a limit point is quite small. The predictor involves some

manipulation of Jacobians, whilst the corrector is just a single power flow solution. Further, because the predictor provides a good starting point for that power flow solution, it generally only takes two or three iterations to solve to quite a tight tolerance. Therefore the critical points of a nose curve such as in Figure 1 can be found very cheaply. In Section V we discuss ideas for determining the point of collapse efficiently from the limit points.

(3) Because the predictor is based on linear approximations, it is possible that if two reactive power sources encounter limits at about the same value of the loading parameter, the predictor may choose the wrong one. (This has not been observed to date, but it is possible.) This can easily be detected following the corrector by checking whether 'extra' machines have encountered limits. If so, it is a simple matter to reformulate (13) to backtrack to the correct limit point.

(4) Notice in Figure 1 that limit point D is on the lower part of the nose curve. A possible stopping criterion for this predictor/corrector process is to stop if such a point is found. This can be determined by building and factorizing the Jacobian of the power flow equations (not the augmented corrector equations) at the limit point. If any of the diagonals of the factorized matrix change sign when compared with the previous point, then the limit point is on the lower section. (Note that at the point of collapse, the determinant of the power flow Jacobian changes sign. The product of these diagonals has the same sign as the determinant.)

(5) It is possible to find situations where the nose curve takes the form shown in Figure 3[13]. The predictor/corrector method has no problems finding such limit points. It is interesting to note that if the power flow Jacobian was formulated with voltage at the limited source constrained (but not reactive power), then a check of the diagonals would show the point to be on the upper section of the curve. If however reactive power was constrained (but voltage was free to vary), we would find that the point was on the lower section of the curve. This is one way to detect limit points which are the point of collapse.

(6) The purpose of the predictor is to provide an initial guess for the limit point, so that the corrector converges more robustly. In an operating environment where it may be desirable to find limit points every few minutes, it is possible that the predictor could be discarded in favour of using the old values of limit points as initial guesses for new values. Such tracking techniques are common in energy management system applications such as state estimation.

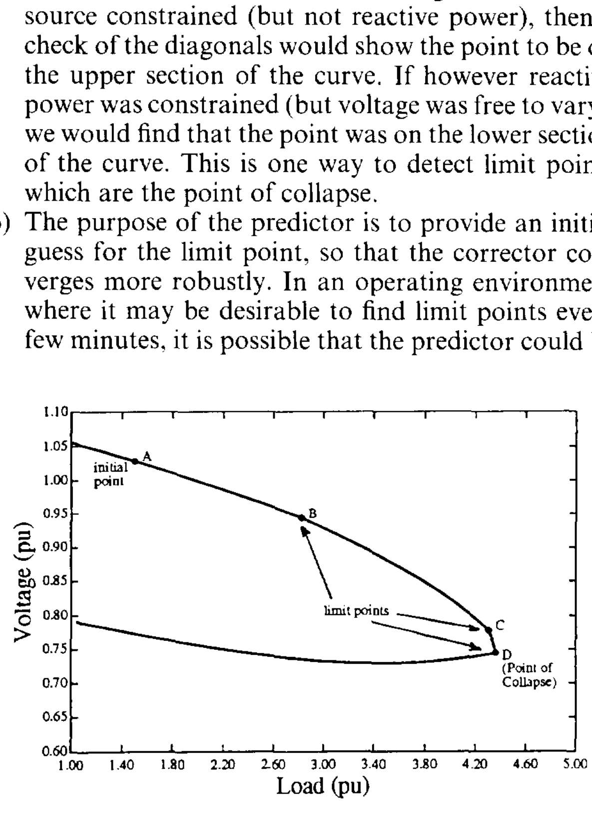

Figure 3. A limit point as the PoC

III. Implementation of the corrector

It has been found that benefits arise in the implementation of the power flow problem if it is structured so that each bus is described by two equations and contributes two variables[14,15]. In the base power flow formulation, bus voltage magnitude V_i and angle α_i are treated as variables for every bus, independent of bus type. The equations describing the different bus types are:

Slack bus

$$p_i(x) = \alpha_i - \alpha_i^0 = 0 \tag{14}$$

$$q_i(x) = V_i - V_i^0 = 0 \tag{15}$$

PV bus

$$p_i(x) = P_i(x) - P_i^0 = 0 \tag{16}$$

$$q_i(x) = V_i - V_i^0 = 0 \tag{17}$$

PQ bus

$$p_i(x) = P_i(x) - P_i^0 = 0 \tag{18}$$

$$q_i(x) = Q_i(x) - Q_i^0 = 0 \tag{19}$$

Note that P_i^0, Q_i^0 are the real and reactive power injected into bus i, so are positive for generation and negative for load. If the equations are ordered as:

$$f^t = [p_1\, q_1\, p_2\, q_2 \dots p_n\, q_n]^t$$

and the unknowns as:

$$x^t = [\alpha_1 V_1 \alpha_2 V_2 \dots \alpha_n V_n]^t$$

then the power flow Jacobian is composed totally of 2×2 blocks. Further, the Jacobian takes the same structure as the network admittance matrix, except that each admittance matrix element is replaced by a 2×2 block.

A number of advantages result from this structure:

(1) The Jacobian is simpler to build. No matter what the bus type, each bus contributes a 2×2 block to the diagonal. Links contribute two off-diagonal 2×2 blocks, independent of the end bus types.
(2) The similarity with the admittance matrix allows an extremely efficient factorization process. Each 2×2 block is treated as a single entity, with arithmetic operations replaced by simple matrix operations.
(3) Bus type changes do not affect the Jacobian structure. So type changes due to limits being met can be handled very efficiently.

Further details and discussion of this power flow implementation can be found in Reference 15.

One further benefit of this power flow structure is that it can be easily adapted to solve for limit points. Recall from (13) that when solving for limit points, the power flow problem is augmented by an extra equation, describing the reactive power balance at the source that is hitting its limit, and an extra variable, i.e. the loading parameter τ. Let the kth bus be the reactive power source corresponding to the limit point of interest. Before encountering its limit, this bus would be described in the standard power flow by (16), (17). To solve for the limit point, the equation $q_k(x) = 0$ given by (17) is replaced by

$$q_k(x) = Q_k(x) - Q_{\max,k}(x) = 0 \tag{20}$$

Also, the entry V_k in x is replaced by τ. These changes effectively add the extra equation and the extra variable that are required. Note also that the removal of V_k from x ensures that it remains constant.

We must also include the effect of the loading parameter τ on all the loads. This is done by modifying the power balance equations for all load buses (18), (19), as follows,

$$p_i(x) = P_i(x) - P_i^0 + \mu_{2i-1}\tau = 0 \tag{21}$$

$$q_i(x) = Q_i(x) - Q_i^0 + \mu_{2i}\tau = 0 \tag{22}$$

Often many μ_j will be zero.

In terms of the power flow Jacobian, the modifications needed for finding the limit points only affect the $2k$th row and the $2k$th column. The $2k$th row will consist of the partial derivatives of (20) with respect to x. The $2k$th column consists of the partial derivatives of all the equations with respect to τ. From (21) and (22) it can be seen that this is just μ. It is useful to illustrate this Jacobian structure using the three-bus system of Figure 4. In this system, we wish to find the values of x and τ which ensure that the generator at bus 3 is at a limit point. (For clarity of the illustration, we have not shown a slack bus.)

Define

$$H_{ij} = \frac{\partial p_i}{\partial \alpha_j}, \quad N_{ij} = \frac{\partial p_i}{\partial V_j}, \quad J_{ij} = \frac{\partial q_i}{\partial \alpha_j}, \quad L_{ij} = \frac{\partial q_i}{\partial V_j} \tag{23}$$

Then the Jacobian has the form:

$$\begin{bmatrix} dp_1 \\ dq_1 \\ dp_2 \\ dq_2 \\ dp_3 \\ dq_3 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} H_{11} & N_{11} \\ J_{11} & L_{11} \end{bmatrix} & \begin{bmatrix} H_{12} & N_{12} \\ J_{12} & L_{12} \end{bmatrix} & \begin{bmatrix} H_{13} & 0 \\ J_{13} & 0 \end{bmatrix} \\ \begin{bmatrix} H_{21} & N_{21} \\ J_{21} & L_{21} \end{bmatrix} & \begin{bmatrix} H_{22} & N_{22} \\ J_{22} & L_{22} \end{bmatrix} & \begin{bmatrix} 0 & \mu_3 \\ 0 & \mu_4 \end{bmatrix} \\ \begin{bmatrix} H_{31} & N_{31} \\ J_{31} & L_{31} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} H_{33} & 0 \\ J_{33} & 0 \end{bmatrix} \end{bmatrix} \times \begin{bmatrix} d\alpha_1 \\ dV_1 \\ d\alpha_2 \\ dV_2 \\ d\alpha_3 \\ d\tau \end{bmatrix} \tag{24}$$

This simple example illustrates a number of facts about the modified Jacobian. The first is that nonzero values of τ_i introduce new off-diagonal elements in the Jacobian.

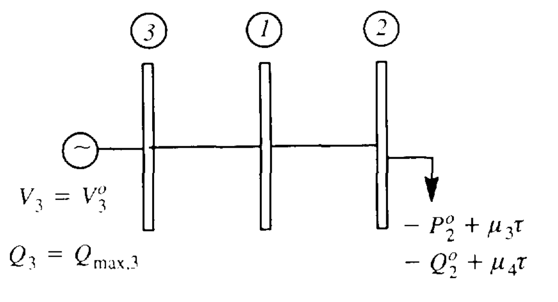

Figure 4. Three-bus system

Because of the 2×2 block structure, if either μ_3 or μ_4 were nonzero in the example, then a new nonzero off-diagonal block would be created in the matrix. If the Jacobian is treated as having an admittance matrix structure, then this is equivalent (in this example) to a new connection between buses 2 and 3. It is easy to see that, in general, a new connection will be formed between the reactive power source which is being forced to its limit, and any bus i that has nonzero μ_{2i-1} or μ_{2i}. This is significant for implementation. These new off-diagonal terms must be taken into account in sparse storage schemes. However it has been found that it is sufficient to treat these new connections the same as if a physical connection existed between the buses. It is also important that they be taken into account when determining the optimal ordering of buses for factorization. Just like physical connections, these fictitious links can cause fill-in. Again, it appears appropriate to treat the new connections the same as physical connections. One consequence is that if many load buses are participating in the loading pattern, i.e. have nonzero μ_i, then the reactive power source that is being forced to its limit will have many connections. It will therefore be ordered near the end of the optimal ordered bus list.

A second observation from the three bus example is that a zero appears on the diagonal. In general, if the source at the kth bus is being forced to its limit, then the $2k$th diagonal will be zero. This is also significant from an implementation point of view, because the factorization process divides by the diagonal elements. In the example, if bus 3 were ordered first, the power flow would halt, with an error indicating that the Jacobian was singular. However with bus 3 ordered third, there is no problem. During the factorization process, as bus 1 then bus 2 are eliminated, the $2k$th diagonal becomes nonzero as a result of fill-in. This can be seen from inspection of (24).

In general, before the limiting bus, i.e. the bus that is being forced to its limit, can be eliminated, at least one load bus with a nonzero μ_i must be eliminated, together with all the buses along a path between that load bus and the limiting bus. This condition is necessary for the diagonal to be filled in. But it is not sufficient. For example, consider again the system of Figure 4, and assume that bus 1 was a PV bus, that the line between buses 1 and 2 was lossless, and that $\mu_3 = 0$, $\mu_4 \neq 0$. Then the zero diagonal would never get filled in, so the system would always be singular. To see why this is so, consider variation of τ, which effectively varies the reactive power load at bus 2. Then the reactive power flow from bus 1 to bus 2 would vary accordingly, but because that feeder was lossless, the real power flow would not change at all. Any change in reactive power flow would be supplied by bus 1 as it is a PV bus. So variation of τ would have no effect on real or reactive power flow from bus 3 to bus 1. Reactive power at bus 3 would not vary at all, and so could not be driven to its limit value.

However, if the feeder between buses 1 and 2 was not lossless, the zero diagonal would get filled in by a small value. In this case, variation of τ would cause a variation in the reactive power flow from bus 1 to bus 2. Because this line is now lossy, this change in reactive power flow would cause a change in losses, and hence a change in real power flow. This would be reflected in the real power flow from bus 3 to bus 1, and hence also in the reactive power generated by bus 3. So variation of τ influences bus 3 reactive power, and hence can be (theoretically) used to drive Q_3 to its limit. The coupling would only be small though, and this would reflect through the zero diagonal being filled in by a small number. The system could easily be ill-conditioned.

If however bus 1 was a PQ bus, a much stronger coupling would exist between τ and Q_3. During the elimination process, a much larger value would fill in the zero diagonal.

In implementing the corrector algorithm, it is necessary to take account of these requirements on the factorization order. The simplest approach would be to order the limiting bus last. Then, if any ordering scheme could achieve fill-in of the diagonal, this one would. However this ordering strategy may be sub-optimal, resulting in unnecessary extra fill-in. Alternatively, often the 'natural' optimal ordering scheme will produce an order that satisfies the need for fill-in of the zero diagonal. As mentioned earlier, the number of connections to the limiting bus can be high, so it would be placed near the end of the factorization order anyway. However, it is still a good idea (and one that is not all that difficult to implement) to build into the optimal ordering process the requirement that the limiting bus is not allowed to be ordered until an appropriate load bus and path of connecting buses have been ordered.

As noted above, if a PV bus is one of the buses on the path, the filled-in diagonal may be small. If possible, it is best to find a path that dodges PV buses. Also, the smaller the impedance between the limiting bus and a varying load bus, i.e. a load bus with nonzero μ_i, the larger will be the filled-in diagonal. So a good choice for a load bus to be eliminated early is one that is local to the limiting bus. Also, the more load buses with nonzero μ_i that are eliminated (together with appropriate paths in each case) before the limiting bus, the greater will be the fill-in of the zero diagonal. (Ordering the limiting bus last clearly maximizes this criterion.) One further point to note is that the connecting path between the load bus and the limiting bus should not contain the slack bus. If it did, the slack bus would totally decouple the variation of τ from the variation of reactive power at the limit bus.

An alternative formulation of the limit point corrector can be obtained as a modification of the corrector of the continuation method given in Reference 9. As in the previous formulation, the limiting bus is converted to a PQ bus, with reactive power given by (20). However, in this case, an extra equation of the form (17) is added to constrain the voltage at the limiting bus. Also, the limiting bus voltage is kept as a variable. Considering the example of Figure 4, the Jacobian for this formulation would be

$$\begin{bmatrix} dp_1 \\ dq_1 \\ dp_2 \\ dq_2 \\ dp_3 \\ dq_3 \\ dV_3 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} H_{11} & N_{11} \\ J_{11} & L_{11} \end{bmatrix} & \begin{bmatrix} H_{12} & N_{12} \\ J_{12} & L_{12} \end{bmatrix} & \begin{bmatrix} H_{13} & N_{13} \\ J_{13} & L_{13} \end{bmatrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{bmatrix} H_{21} & N_{21} \\ J_{21} & L_{21} \end{bmatrix} & \begin{bmatrix} H_{22} & N_{22} \\ J_{22} & L_{22} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{matrix} \mu_3 \\ \mu_4 \end{matrix} \\ \begin{bmatrix} H_{31} & N_{31} \\ J_{31} & L_{31} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} H_{33} & N_{33} \\ J_{33} & L_{33} \end{bmatrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 \end{matrix} & \begin{matrix} 0 & 1 \end{matrix} & 0 \end{bmatrix} \times \begin{bmatrix} d\alpha_1 \\ dV_1 \\ d\alpha_2 \\ dV_2 \\ d\alpha_3 \\ dV_3 \\ d\tau \end{bmatrix} \quad (25)$$

This compares with the Jacobian for the earlier formulation, which was given by (24).

It can be seen from (25) that the Jacobian again has a zero on the diagonal. Conditions for the fill-in of that zero diagonal are effectively the same as for the earlier formulation. It is normal for the extra voltage equation to be ordered last. When that is the case, the conditions for fill-in of the zero diagonal are the same as those for the earlier formulation with the limiting bus ordered last.

IV. Example

Figures 1 and 2 show the nose curve, limit points, and predictor/corrector sequence for an example which is based on the system in Figure 5. This system originates

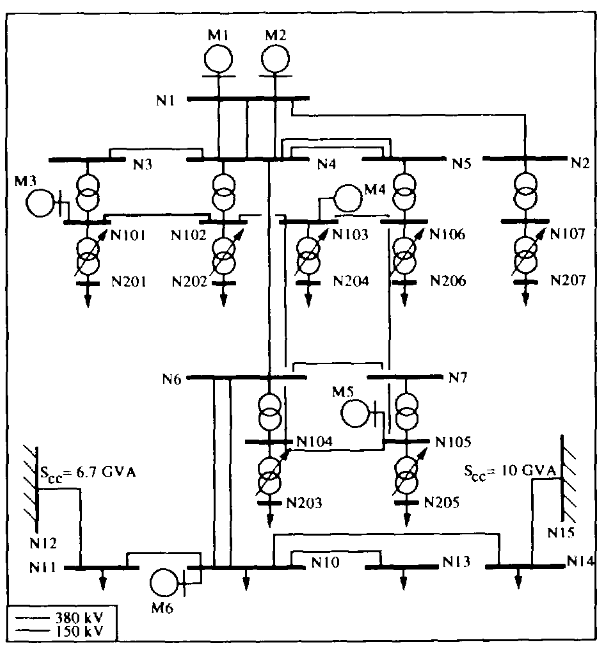

Figure 5. CIGRE 31-bus system

Table 1. Comparison of predictor and corrector results

Limit point		Predictor	Corrector
B	Q_{N205} (p.u.)	2.971	2.820
	V_{N205} (p.u.)	0.9431	0.9437
	V_{N206} (p.u.)	0.9923	0.9923
	V_{N105} (p.u.)	1.0013	1.0015
C	Q_{N205}	4.879	4.302
	V_{N205}	0.7719	0.7781
	V_{N206}	0.9259	0.9258
	V_{N105}	0.9080	0.9102
D	Q_{N205}	4.545	4.343
	V_{N205}	0.6774	0.6855
	V_{N206}	0.8606	0.8600
	V_{N105}	0.8423	0.8451

from the CIGRE Task Force that investigated voltage collapse indices[16]. In this example, the reactive power load at bus N205 is increased. Figures 1 and 2 show the variation of voltage at N205 as the reactive power load is varied. This load was chosen as it clearly illustrated the predictor/corrector process. Other loads, or combinations of real and/or reactive power loads, could have been chosen.

At the initial point A, the generators M3, M4 and M5 are already on their maximum reactive power limits. As the load at N205 increases, the reactive power limit of generator M2 is encountered. This occurs at limit point B. Note that the slope of the curve is nearly the same on both sides of point B. This is because when the M2 limit is met, some voltage support is lost, but M1 still provides strong support. Further increase in the load at N205 results in generator M1 encountering its limit at point C. The slope of the nose curve changes dramatically at this point. After M1 encounters its limit, there is no longer any voltage support in the upper section of the system. Therefore voltages fall much more rapidly as the load increases. Further increase in load results in the point of collapse being quickly reached. Notice that generator M6 encounters its limit at point D on the lower section of the curve.

The predictor/corrector process for finding points B, C and D is illustrated in Figure 2. Predicted and corrected values of the load and some bus voltages are given in Table 1. From the figure it can be seen that the predictor, which estimates points B′, C′ and D′, is always tangent to the nose curve at the point from which the prediction is made. This occurs because the predictor is based on linearization of the power flow equations. The results also indicate that the predictor provides a good estimate of the voltages, but always tends to overestimate the load parameter.

At all three limit points, the corrector took only two iterations to solve to a tolerance of 0.0001 p.u. This was despite the fact that both limit points C and D are very close to the point of collapse.

Figure 3 was produced using the same system, and the same load variation as Figures 1 and 2. However in this case, the reactive power limit at machine M6 was reduced from 10 p.u. to 8 p.u. Even though point D is now the point of collapse, i.e. the point of maximum

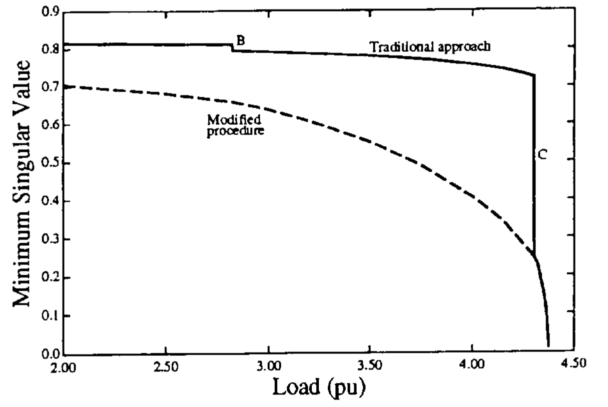

Figure 6. Singular value variation

loadability, the corrector again converged in two iterations.

V. Use of limit points in voltage collapse indices

In realistic power systems, limit points will usually be encountered on the upper portion of the nose curve. In that case, a knowledge of the limit points can assist in determining the proximity to voltage collapse. This section discusses two possibilities.

V.1 *Point of collapse estimation*

In assessing vulnerability of systems to load increases, we are usually only interested in the limit points which occur on the upper portion of the nose curve, e.g. points B and C in Figure 1. However it is often possible to find at least one limit point on the lower section too, e.g. point D. In such cases, we have a power flow solution both sides of the point of collapse. It is then possible to use some form of interpolation technique to obtain an estimate of system conditions at the point of collapse. This is explored further in Reference 17. Using these ideas, it is possible to obtain a good estimate of the point of collapse robustly, and the computational burden is equivalent to just a few power flows.

V.2 *Improvement of eigenvalue and singular value indices*

Voltage collapse indices have been proposed which use the minimum eigenvalue or minimum singular value of the power flow Jacobian as a measure of how close the Jacobian is to singularity, and hence how close the operating point is to the point of collapse, see for example Reference 18. Unfortunately these indices are affected significantly by reactive power sources encountering limits. The solid line of Figure 6 illustrates this effect. Whenever a limit is met, the change of the reactive power source from a PV bus to a PQ bus effectively introduces a new equation and variable. This change in the size of the Jacobian matrix causes a step change in the minimum eigenvalue and singular value.

However, a knowledge of which reactive power sources are on limits at the point of collapse can be used to overcome this problem. The troublesome steps are due to buses changing type. Therefore the steps can be removed by ensuring that no bus type changes occur. This can be achieved by always treating buses that are on limits at the point of collapse as PQ buses when the eigenvalues and/or singular values are being calculated.

Assume for now that the reactive power sources that are on limits at the point of collapse are known. If we are interested in determining the minimum eigenvalue or singular value for a particular value of loading parameter, then the first step is just the normal solution of the power flow problem. For this power flow solution, reactive power sources are treated in the normal manner. However when it comes to building the Jacobian for evaluation of the eigenvalues or singular values, reactive power sources that are on limits at the point of collapse are treated as PQ buses, independent of whether they are on limits at the current point. For these sources, the current value of reactive power being generated (which may or may not be the limit value) is treated as a reactive load. In this way, the Jacobian is always built with the same number of PQ buses, so no steps occur in the eigenvalues or singular values as the load parameter varies.

Figure 6 enables this procedure to be illustrated. The curves correspond to the same example used to produce Figure 1. The unbroken line refers to the normal singular value calculation, whilst the dashed line was obtained using the modification considered in this section. Step B was caused by generator M2 encountering a limit, and step C by generator M1 limiting. Hence in producing the dashed line, both generators M1 and M2 were treated as PQ buses at all points. Notice that for values of load above step C, the two curves coincide. This is because above step C, both generators are on limits and would traditionally be treated as PQ buses anyway. Figure 6 illustrates the benefits of the proposed procedure. Consider a load of say 4.0 p.u. Using the traditional approach, an eigenvalue or singular value based index would indicate that the system was in a comparatively strong position. There would be no way of knowing that for a small increase in load, a limit would be met and the system would be in a very marginal state. However it can be seen from the dashed curve that the proposed procedure gives a much better indication that the system is nearing its loadability limit.

This alteration to eigenvalue/singular value indices relies on a knowledge of the reactive power sources that are on limits at the point of collapse. The predictor/corrector procedure can be used to provide that information. Note though that the exact limit points are not required, just a knowledge of which sources have hit limits. Therefore, often it will not be necessary to use the predictor/corrector procedure every time the voltage collapse index is to be calculated. Consider an operating environment. The predictor/corrector procedure could be run every so often to confirm which sources encountered limits as the load was increased. The index could be calculated much more regularly, using that information.

VI. Conclusions

A knowledge of the points where reactive power sources encounter limits is important when assessing the vulnerability of a power system to voltage collapse. This paper presents a predictor/corrector technique that robustly determines such points. The predictor is based on linearization of the power flow equations about the point from which the prediction is to be made. It provides an estimate of limit points. The corrector is a slight modification of the standard power flow. It uses the estimated limit point given by the predictor as the starting point for solution. The corrector generally takes around two iterations to converge. Because the corrector does not use exactly the same constraints as the standard power flow, it has no problem converging near the point of collapse.

Having found the limit points, it is possible to use interpolation to provide a fast estimate of the point of collapse. Also, a knowledge of the sources that encounter limits as the load is increased can be used to improve the usefulness of voltage collapse indices which are based on eigenvalues and singular values.

VII. Acknowledgements

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