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Systematic Tuning of Nonlinear Power System Controllers

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Abstract

Non-smooth nonlinearities have a significant influence within many power system controllers, and hence affect overall system dynamic behaviour. This paper focusses on the output limiters of power system stabilizers (PSSs). Even though these limits play an important role in the response of generators to large disturbances, only *ad hoc* tuning procedures have been available. Therefore a systematic, optimization-based approach is proposed in this paper. Dynamic behaviour is improved by minimizing the deviation of generator angles and terminal voltages from their post-disturbance values. The algorithm builds on techniques for efficiently computing trajectory sensitivities for hybrid systems.

1 Introduction

Many power system controllers are influenced by hard (non-smooth) nonlinearities, such as deadbands, hysteresis and saturation limits. Some devices, for example braking resistors, behave in an inherently non-smooth fashion, whilst others may have non-smooth control regimes, e.g., bang-bang control of FACTS devices. Often there is scope for tuning parameters related to these hard nonlinearities to improve system dynamic performance. However tuning procedures have not been straightforward, with *ad hoc* approaches often adopted.

Generator excitation systems play a fundamental role in power system control. Figure 1 provides a (slightly) simplified block diagram representation of a typical excitation system. Subsystems include automatic voltage regulator (AVR), exciter and power system stabilizer (PSS) [1]. Two important hard nonlinearities are apparent:

- non-windup limits on the exciter output, constraining the generator field voltage E_{fd},
- windup limits on the PSS output V_{PSS}.

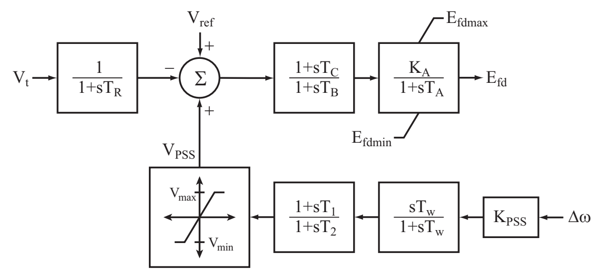

Figure 1: PSS/AVR block representation.

The former represent saturation limits of the exciter amplifier. They are physical limits, and so are not tunable. The PSS output limits, on the other hand, are tunable. This paper focusses on tuning these latter limits to optimize post-disturbance behaviour. The performance criteria that form the basis for this optimization are presented later.

The primary role of PSSs is to provide damping in the presence of small disturbances. Controller tuning focusses on determining the gain and phase shift that are appropriate under those conditions. Therefore PSSs often have a detrimental affect on stability immediately following large disturbances. Typically, during the transient period, PSSs act to force the field voltage E_{fd} in an unhelpful direction [2]. There is also the concern that the PSS may cause an unacceptably high generator terminal voltage [3]. PSS output limiters therefore seek to reduce the influence of PSSs during the transient phase, whilst allowing them to actively damp subsequent oscillations.

Various rules of thumb have been established for choosing maximum and minimum limit values that achieve the desired goals [2, 3]. However no systematic tuning process exists. This paper addresses that need by establishing a procedure for optimally tuning the limits. In fact, it will be shown that transient performance can also be improved by appropriate choice of limits. A caveat is required though. Due to the nonlinear nature of power system dynamics, values that are optimal for one disturbance scenario may be suboptimal for others. However the procedure presented in the paper

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at least provides a systematic approach for addressing each case. It then becomes a matter of qualitatively assessing the outcomes of the various cases to achieve overall robustness.

2 Modelling

The discussion of Section 1 indicates that power systems commonly exhibit a mix of continuous time dynamics, discrete-time and discrete-event dynamics, switching action and jump phenomena. It is shown in [4, 5] that such systems, known generically as *hybrid systems*, can be modelled by a set of differential-algebraic equations, adapted to incorporate impulsive (state reset) action and switching of the algebraic equations. This *DA Impulsive Switched* (DAIS) model can be written in the form,

$$\dot{x} = f(x,y) + \sum_{j=1}^{e} \delta(y_{e,j})\Big(h_j(x,y) - x\Big) \tag{1}$$

$$0 = g(x,y) \equiv g^{(0)}(x,y) + \sum_{i=1}^{d} g^{(i)}(x,y) \tag{2}$$

where

$$g^{(i)}(x,y) = \begin{cases} g^{(i-)}(x,y) & y_{d,i} < 0 \\ g^{(i+)}(x,y) & y_{d,i} > 0 \end{cases} \quad i = 1, ..., d \tag{3}$$

and

- $\delta(.)$ is the Dirac delta. Each impulse term of the summation in (1) can be expressed in the alternative state reset form

  $$x^+ = h_j(x^-, y^-) \quad \text{when } y_{e,j} = 0 \tag{4}$$

  where the notation x^+ denotes the value of x just after the reset event, whilst x^- and y^- refer to the values of x and y just prior to the event. This form motivates a generalization to an implicit mapping $h'_j(x^+, x^-, y^-) = 0$.

- $x \in \mathbb{R}^n$ includes continuous dynamic states, for example generator angles, velocities and fluxes; discrete dynamic states, such as transformer tap positions and protection relay logic states; and parameters λ such as generator reactances, controller gains and limiter values[1].

- $y \in \mathbb{R}^m$ are algebraic states, e.g., load bus voltage magnitudes and angles.

- y_d, y_e are selected elements of y that trigger algebraic switching and state reset (impulsive) events respectively; y_d and y_e may share common elements.

- $f, h_j : \mathbb{R}^{n+m} \to \mathbb{R}^n$.

- $g^{(0)}, g^{(i\pm)} : \mathbb{R}^{n+m} \to \mathbb{R}^m$. Some elements of each $g^{(\cdot)}$ will usually be identically zero, but no elements of the composite g should be identically zero. The $g^{(i\pm)}$ are defined like g in (2), allowing a recursive structure for g.

Equations (1)-(4) are a reformulation (and slight generalization) of the model proposed in [4].

Away from events, system dynamics evolve smoothly according to the familiar differential-algebraic model

$$\dot{x} = f(x,y) \tag{5}$$
$$0 = g(x,y) \tag{6}$$

where g is composed of $g^{(0)}$ together with appropriate choices of $g^{(i-)}$ or $g^{(i+)}$, depending on the signs of the corresponding elements of y_d. At switching events (3), some component equations of g change. To satisfy the new $g = 0$ constraints, algebraic variables y may undergo a step change. Impulse events (1) (alternatively reset events (4)) force a discrete change in the elements of x that correspond to discrete states. Algebraic variables may again step to ensure $g = 0$ is always satisfied.

The *flows* of x and y are defined as

$$x(t) = \phi_x(x_0, t) \tag{7}$$
$$y(t) = \phi_y(x_0, t) \tag{8}$$

where $x(t)$ and $y(t)$ satisfy (1)-(3), along with initial conditions,

$$\phi_x(x_0, t_0) = x_0 \tag{9}$$
$$g\big(x_0, \phi_y(x_0, t_0)\big) = 0. \tag{10}$$

3 Optimal tuning

3.1 Objective

Optimal controller tuning, the focus of this paper, is one application of optimization in the analysis of power system dynamics. Numerous other applications arise naturally, for example determining the optimal location, amount and switching times for load shedding [6, 7]. Most problems can be formulated using a Bolza form of objective function

$$\min_{\lambda, t_f} \mathcal{J}(x, y, \lambda, t_f) \tag{11}$$

[1] Incorporating parameters λ into the state x allows a convenient development of trajectory sensitivities, discussed later. To ensure parameters remain fixed at their initial values, the corresponding differential equations (1) are defined as $\dot{\lambda} = 0$.

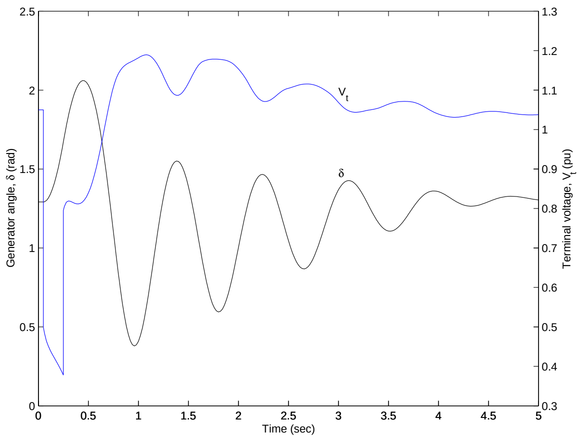

Figure 2: Generator angle and terminal voltage.

where

$$\mathcal{J} = \varphi\big(x(t_f), y(t_f), \lambda, t_f\big) + \int_{t_0}^{t_f} \psi\big(x(t), y(t), \lambda, t\big)dt,$$

also λ are the design parameters, i.e., the parameters adjusted to achieve the objective, and t_f is the final time.

The objective of controller tuning is to force the system to recover to the post-disturbance stable operating point as quickly as possible. Generator angles δ typically enable a good assessment of that recovery. An illustration is provided by Figure 2. During the transient period following a large disturbance, significant angle deviation is an indicator of marginal system stability. After the initial transients have subsided, prolonged angle fluctuations indicate poor damping. Therefore controller parameters that minimize angle deviation effectively maximize system recovery.

Angle regulation is achieved through control of the generator field voltage E_{fd}. However variation of E_{fd} also directly affects the generator terminal voltage V_t, as shown in Figure 2. Often improved angle regulation is achieved at the expense of degraded voltage regulation. The conflicting requirements of improved angle behaviour without voltage degradation can be achieved via the minimization

$$\min_{\lambda} \mathcal{J}(\lambda) \tag{12}$$

where

$$\mathcal{J}(\lambda) = \int_{t_0}^{t_f} \left[\begin{array}{c} \delta(\lambda,t) - \delta_s \\ V_t(\lambda,t) - V_{t,s} \end{array} \right]^t W \left[\begin{array}{c} \delta(\lambda,t) - \delta_s \\ V_t(\lambda,t) - V_{t,s} \end{array} \right] dt. \tag{13}$$

Note that:

- For the design situation considered in this paper, the optimization parameters λ are the upper and lower limits on the PSS output, V_{max} and V_{min} respectively in Figure 1.
- The dependence of generator angles $\delta(\lambda, t)$ and terminal voltages $V_t(\lambda, t)$ on parameters λ is provided by the flows (7),(8).
- The post-fault steady state values of δ and V_t are given by δ_s and $V_{t,s}$ respectively.
- The diagonal matrix of weighting factors W accounts for the different scaling in δ and V_t. It also provides a mechanism for balancing the conflicting requirements of angle and voltage regulation.

3.2 Events

The solution of (11) for hybrid systems may be complicated by discontinuous behaviour at events. However these complications largely disappear under the assumption that the order of events does not change as λ and t_f vary. This assumption is common throughout the literature, though it is expressed in various ways: transversal crossings of triggering hypersurfaces are assumed in [8], existence of trajectory sensitivities (defined in the following section) is assumed in [9], and [10] assumes all flows have the same *history*. All statements are equivalent.

Under that assumption, and other mild assumptions, [10] concludes that if $\mathcal{J}$ is continuous in its arguments then a solution to (11) exists. Further, [9] shows that if $\mathcal{J}$ is a smooth function of its arguments, then it is continuously differentiable in terms of λ and t_f. The minimization can therefore be solved using gradient-based methods. Trajectory sensitivities, the subject of the following section, underlie the gradient information.

If the event ordering assumption is not satisfied, $\mathcal{J}$ may be discontinuous. The optimization problem then takes on a combinatorial nature, as each continuous section of $\mathcal{J}$ must be searched for a local minimum.

3.3 Trajectory sensitivities

Trajectory sensitivities provide a way of quantifying the variation of a trajectory resulting from (small) changes to parameters and/or initial conditions [4, 11]. To obtain the sensitivity of the flows ϕ_x and ϕ_y to initial conditions x_0, the Taylor series expansions of (7),(8) are formed. Neglecting higher order terms gives

$$\Delta x(t) = \frac{\partial \phi_x(t)}{\partial x_0}\Delta x_0 \equiv \Phi_x(t)\Delta x_0 \tag{14}$$

$$\Delta y(t) = \frac{\partial \phi_y(t)}{\partial x_0}\Delta x_0 \equiv \Phi_y(t)\Delta x_0. \tag{15}$$

Recall that x_0 incorporates parameters λ, so sensitivity to initial conditions x_0 includes parameter sensitivity. Equations (14)-(15) describe the changes $\Delta x(t)$

and $\Delta y(t)$ in a trajectory, at time t along the trajectory, for a given (small) change in initial conditions Δx_0. The time-varying partial derivatives Φ_x and Φ_y are known as *trajectory sensitivities*. The variational equations describing the evolution of these sensitivities are developed in [4], with a summary provided in [12].

Along smooth sections of the trajectory, the trajectory sensitivities evolve according to a linear time-varying differential-algebraic system. For large systems, these equations have high dimension. However the computational burden is minimal when an implicit numerical integration technique such as trapezoidal integration is used to generate the trajectory [4, 13].

3.4 Implementation

Implementation of the minimization (12),(13) is quite straightforward, even though the cost is obtained by integrating over the system flow (trajectory). The simplest way of obtaining $\mathcal{J}$ is to introduce a new state variable x_{cost}, with $\dot{x}_{cost}$ equal to the integrand of (13). Then $x_{cost}(t_f) = \mathcal{J}$, and the trajectory sensitivities, calculated with respect to λ, directly provide the gradient

$$\nabla \mathcal{J} = \Phi_{x_{cost}}(t_f). \tag{16}$$

Note that through appropriate implementation of trajectory sensitivities, as described in [4, 13], negligible extra computation is required in determining (16).

Efficient computation of $\nabla \mathcal{J}$ ensures that numerous gradient-based minimization algorithms are available for solving (12). Steepest descent [14] is the simplest to implement, thought is often slow to converge. The cost function (13) has the form of a continuous-time nonlinear least-squares problem. A corresponding continuous-time adaptation of the Gauss-Newton algorithm [14] is also appropriate, and can be simply and efficiently implemented.

4 Example

4.1 System description

The single machine infinite bus power system of Figure 3 will be used to illustrate the optimal tuning process developed in Section 3. Even though this example utilizes a simple network structure, the tuning algorithm is applicable to arbitrarily large and complicated systems. More importantly, the generator is accurately represented by a sixth order machine model, viz., a two axis model with two windings in each axis [15], and the generator excitation system is modelled according to Figure 1.

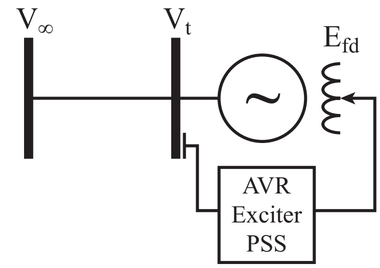

Figure 3: Single machine infinite bus system.

Table 1: Optimization results.

Case	Values		
	V_{max}	V_{min}	Cost, $\mathcal{J}$
Initial values	0.10	-0.10	6.2026
Optimal values	0.11	-0.33	5.2746

4.2 Results

A single phase fault was applied at the generator terminal bus at 0.05 sec. The fault was cleared, without line tripping, at 0.25 sec. The initial values of the PSS limits are given in Table 1. The corresponding generator angle and terminal voltage trajectories are shown in Figure 2.

The optimal tuning process described in Section 3 was used to determine PSS output limit values that minimized angle deviation without degrading terminal voltage response. The optimal values are given in Table 1. It can be seen that V_{max} has changed very little, but V_{min} moved quite significantly. The effect of this optimal tuning was rather dramatic. Figures 4 and 5 show that damping improvement *and* voltage regulation have been achieved. Note that a lowering of V_{min} is quite counter-intuitive; manual tuning would likely not even search in that direction for improved response.

For completeness, the behaviour of the PSS output V_{PSS} and field voltage E_{fd} are shown in Figures 6 and 7 respectively. The effects of the change in PSS limits is quite evident.

5 Conclusions

Non-smooth nonlinearities often have a significant influence on power system dynamic behaviour. This paper has addressed the systematic tuning of parameters related to hard nonlinearities. The focus has been on power system stabilizer (PSS) output limits.

The objective of controller tuning is to force the system to recover to the post-disturbance stable operating point as quickly as possible. Generator angles typically provide a good indication of that recovery. Therefore

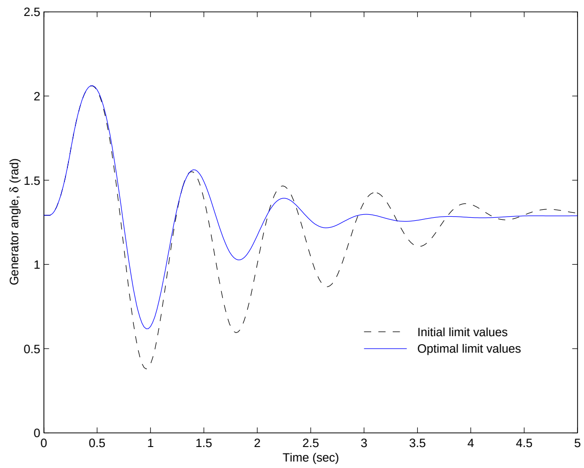

Figure 4: Generator angle response.

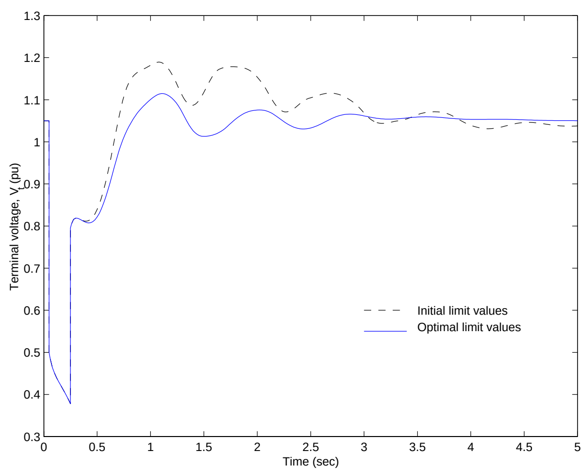

Figure 5: Generator terminal voltage response.

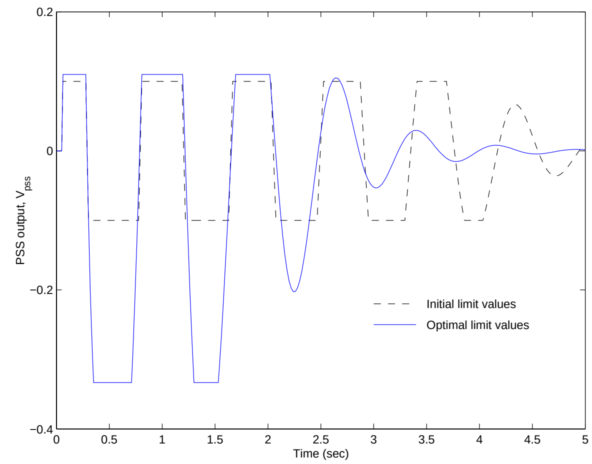

Figure 6: PSS output response.

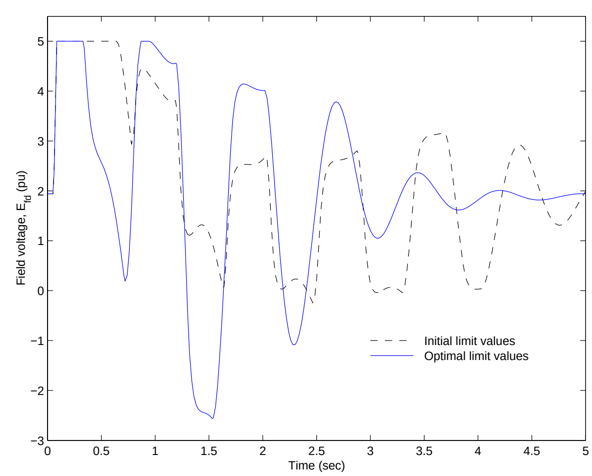

Figure 7: Generator field voltage response.

the tuning process is based on minimizing angle deviation over the post-fault period. Often improved angle response is achieved at the expense of degraded voltage regulation. A cost function that captures these conflicting requirements has been established. A simple yet illustrative example has highlighted the benefits of systematic tuning.

References

[1] IEEE Std 421.5-1992, *IEEE Recommended Practice for Excitation System Models for Power System Stability Studies*, Institute of Electrical and Electronics Engineers, Inc., New York, 1992.

[2] E.V. Larsen and D.A. Swann, "Applying power system stabilizers, Part I: General concepts, Part II: Performance objectives and tuning concepts, Part III: Practical considerations," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-100, pp. 3017–3046, 1981.

[3] P. Kundur, *Power System Stability and Control*, EPRI Power System Engineering Series, McGraw Hill, 1994.

[4] I.A. Hiskens and M.A. Pai, "Trajectory sensitivity analysis of hybrid systems," *IEEE Transactions on Circuits and Systems I*, vol. 47, no. 2, pp. 204–220, February 2000.

[5] I.A. Hiskens, "Power system modeling for inverse problems," *IEEE Transactions on Circuits and Systems I*, Submitted.

[6] C. Moors and T. Van Cutsem, "Determination of optimal load shedding against voltage instability," in *Proceedings of the 13th Power Systems Computation Conference*, Trondheim, Norway, June 1999.

[7] W. Tangmunarunkit and I.A. Hiskens, "Optimization of load shedding against voltage instability," in *Proceedings of 33rd North American Power Symposium*, College Station, TX, October 2001.

[8] M.S. Branicky, V.S. Borkar, and S.K. Mitter, "A

unified framework for hybrid control: Model and optimal control theory," *IEEE Transactions on Automatic Control*, vol. 43, no. 1, pp. 31–45, January 1998.

[9] S. Galán and P.I. Barton, "Dynamic optimization of hybrid systems," *Computers chem. Engng*, vol. 22, Suppl., pp. S183–S190, 1998.

[10] B. Piccoli, "Hybrid systems and optimal control," in *Proceedings of the 37th IEEE Conference on Decision and Control*, Tampa, FL, December 1998.

[11] P.M. Frank, *Introduction to System Sensitivity Theory*, Academic Press, New York, 1978.

[12] I.A. Hiskens, "Nonlinear dynamic model evaluation from disturbance measurements," *IEEE Transactions on Power Systems*, vol. 16, no. 4, pp. 702–710, November 2001.

[13] W.F. Feehery, J.E. Tolsma, and P.I. Barton, "Efficient sensitivity analysis of large-scale differential-algebraic systems," *Applied Numerical Mathematics*, vol. 25, pp. 41–54, 1997.

[14] J.E. Dennis and R.B. Schnabel, *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1983.

[15] P.W. Sauer and M.A. Pai, *Power System Dynamics and Stability*, Prentice Hall, Upper Saddle River, NJ, 1998.