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Decentralized Charging Control for Large Populations of Plug-in Electric Vehicles

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***Abstract*— The paper develops a novel decentralized charging control strategy for large populations of plug-in electric vehicles (PEVs). We consider the situation where PEV agents are rational and weakly coupled via their operation costs. At an established Nash equilibrium, each of the PEV agents reacts optimally with respect to the average charging strategy of all the PEV agents. Each of the average charging strategies can be approximated by an infinite population limit which is the solution of a fixed point problem. The control objective is to minimize electricity generation costs by establishing a PEV charging schedule that fills the overnight demand valley. The paper shows that under certain mild conditions, there exists a unique Nash equilibrium that almost satisfies that goal. Moreover, the paper establishes a sufficient condition under which the system converges to the unique Nash equilibrium. The theoretical results are illustrated through various numerical examples.**

***Keywords:* Plug-in electric vehicles (PEVs); Decentralized control; Nash equilibrium; 'Valley-filling' charging strategy.**

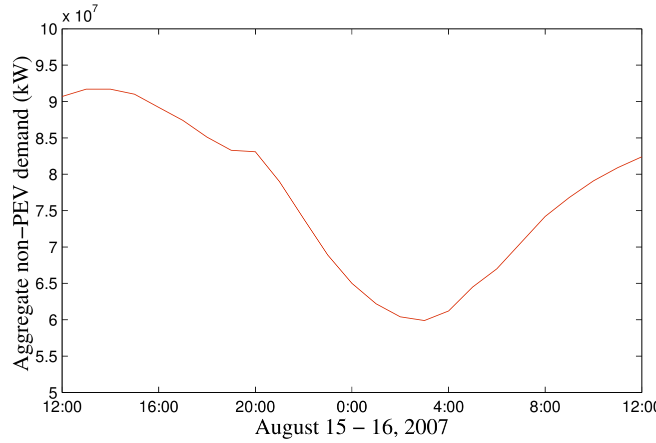

Fig. 1. A typical summer aggregate non-PEV base demand for the region managed by the Midwest Independent System Operator (MISO).

I. Introduction

Plug-in electric vehicles (PEVs) are beginning to compete with conventional petroleum-combustion vehicles, and they may achieve significant market penetration over the next few decades. While PEVs will reduce consumption of exhaustible petroleum resources and may reduce green-house gas emissions, their impact on the electrical power grid could be significant.

A number of studies have been undertaken recently to explore the potential impacts of high penetrations of PEVs on the power grid [1], [2], [3], [4]. In [5], we studied centralized optimal charging control, for large populations of homogeneous PEVs. This work rigorously explored the conditions under which the socially optimal (i.e. generation cost minimizing) control strategy results in *valley filling*. Referring to Figure 1, this strategy ensures that PEV demand fills the overnight load valley, such that the aggregate PEV load together with non-PEV demand remain constant during the charging period.

A practical process for achieving the valley-filling demand pattern is not straightforward, however. For large populations of PEVs, centralized charging control by a utility or system operator would require significant communications and computational capability. Furthermore, the centralized approach may be unrealizable due to a reluctance among PEV owners to allow third parties to directly control vehicle charging rates. Instead, in this paper, we allow each PEV to choose and implement its own local charging control.

We will assume that the (electricity) charging price, seen by all PEVs, is a function of the total demand on the grid, which is the summation of the inelastic non-PEV base demand together with the aggregated charging of the whole population of PEVs. Because of the coupling through this common price signal, each PEV agent effectively interacts with the average charging strategy of the rest of the PEV population. As the population grows, the influence of each individual PEV on that average charging strategy becomes negligible. Accordingly, in a large population, each PEV will observe the same average strategy in the set of all PEVs not including itself. In this situation, a collection of local charging controls is a Nash equilibrium (NE), if

(i) Each of the local controls is optimal with respect to one commonly observed charging trajectory, and
(ii) The average of these local optimal charging controls is equal to the common trajectory, i.e. the average charging strategy is collectively reproduced by the local optimal control laws.

This result is closely related to the Nash certainty equivalence (NCE) principle, as proposed by Huang *et al.* [6], [7]. This framework also has connections with mean-field game models that were studied by Lasry and Lions [8], [9], and

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TABLE I

LIST OF SYMBOLS

x^n	state of charge of PEV n
N	PEV population size
$[0, T]$	charging interval (identical for all PEVs)
β^n	battery size of PEV n
α^n	charging efficiency of PEV n
u^n	local charging rate of PEV n
$\mathcal{U}^n$	set of local charging controls of PEV n
$\mathbf{u}$	$\equiv \{u^n; 1 \leq n \leq N\}$, the collection of local controls for the entire PEV population
$\text{avg}(\mathbf{u})$	average value of collection of local controls $\mathbf{u}$
$\mathbf{u}^{-n}$	collection of all PEV controls except n
p	real-time electricity retail price
d	(inelastic) non-PEV base demand
$\lvert\mathbf{x}\rvert_1$	l_1-norm of the vector $\mathbf{x}$

close connections with the notion of oblivious equilibrium proposed by Weintraub, Benkard, and Van Roy [10] via a mean-field approximation.

Implementation of the decentralized control is achieved through a charging negotiation procedure, developed in the paper, which takes place at some time prior to the actual charging interval. Under certain mild conditions, the decentralized charging control drives the system to a unique Nash equilibrium that is nearly socially optimal (or 'valley-filling'). In the case of homogeneous PEV populations, the Nash equilibrium degenerates to a perfect 'valley-filling' charging strategy.

The paper is organized as follows. Section II establishes a class of decentralized charging control problems for large populations of PEVs. Section III develops existence, uniqueness and social optimality properties of the Nash equilibrium. These results are explored with illustrative examples in Section IV, and conclusions are presented in Section V.

II. FORMULATION OF DECENTRALIZED CHARGING CONTROL FOR LARGE PEV POPULATIONS

A. Decentralized charging control problems

We consider charging control for a significant penetration of PEVs with a population size equal to N. For individual PEV n, we adopt the notation of Table I. The charging dynamics can be written,

$$x_{t+1}^n = x_t^n + \frac{\alpha^n}{\beta^n} u_t^n, \quad \text{with } t = 0, \cdots, T-1, \tag{1}$$

with an initial state-of-charge (SOC) value of x_0^n. It is straightforward to verify that the terminal SOC value x_T^n equals 1 when subject to a charging control u^n that satisfies $\mathcal{E}(u^n) = \frac{\beta^n}{\alpha^n}(1 - x_0^n)$ where $\mathcal{E}(u^n) \triangleq \sum_{t=0}^{T-1} u_t^n$. In this paper we consider *the set of feasible full charging controls*,

$$\mathcal{U}^n \triangleq \Big\{ u^n \equiv \big(u_0^n, \cdots, u_{T-1}^n\big);\ \text{s.t. } u_t^n \geq 0,\ x_T^n = 1 \Big\}, \tag{2}$$

where the final constraint on x_T^n requires that all PEVs are charged completely by the end of the interval.

Considering a collection of local controls of the PEV population $\mathbf{u} = \{u^n; 1 \leq n \leq N\}$, we denote $\text{avg}(\mathbf{u}) \equiv \big\{\text{avg}(\mathbf{u}_t); t = 0, \cdots, T-1\big\}$ as the average charging control with respect to $\mathbf{u}$, where

$$\text{avg}(\mathbf{u}_t) \triangleq \frac{1}{N} \sum_{n=1}^{N} u_t^n. \tag{3}$$

In order to formally establish properties of the Nash equilibrium, it will be assumed that the population is infinite. Therefore, as $N \to \infty$,

(i) The total electric generation capacity, denoted by C, is proportional to the PEV population size N, i.e. $C/N = c$, with some positive constant c.

(ii) The aggregate non-PEV base demand at instant t, denoted by D_t, is unrelated to the PEV charging controls. It is, however, proportional to the grid generation capacity C, and hence to the PEV population size N, i.e. $D_t/N = d_t$ where d is the inelastic normalized non-PEV base demand.

We specify an (electricity) retail price function, denoted by $p(.)$, that depends on the ratio of the total aggregate demand in the grid and the generation capacity. By (3), (i) and (ii), for a collection of charging controls $\mathbf{u}$, we have

$$p(\cdot) \equiv p\Big(\frac{D_t + \sum_{n=1}^{N} u_t^n}{C}\Big) = p\Big(\frac{d_t + \text{avg}(\mathbf{u}_t)}{c}\Big).$$

Note that this formulation assumes that electricity price depends only on the current demand level, which implicitly neglects intertemporal constraints such as generator ramping and minimum run times. It will be seen in Section III, that the shape of the electricity retail price function $p(r)$ has a major influence on the performance of the decentralized charging control strategy.

Coordinated control of PEV charging often assumes a centralized control framework, where the utility controls the charging rates for all PEVs. The objective is to implement a collection of PEV charging rates that achieve the dual objectives, 1) the aggregated PEV load fills the overnight valley of the non-PEV base load, and 2) every PEV is fully charged at the end of its charging interval. In contrast, this paper proposes a charging control scenario where individual PEVs minimize their own operating cost by implementing a charging strategy that takes into account the collection of charging strategies adopted by other agents.

More specifically, subject to a collection of charging strategies $\mathbf{u}$, we suppose that the cost function of agent n, denoted by $J^n(\mathbf{u})$, is specified as,

$$J^n(\mathbf{u}) \triangleq \sum_{t=0}^{T-1} \Big\{ p(r_t) u_t^n + \delta \big(u_t^n - \text{avg}(\mathbf{u}_t)\big)^2 \Big\} \tag{4}$$

where $r_t \equiv \frac{d_t + \text{avg}(\mathbf{u}_t)}{c}$ and the tracking parameter δ is a non-negative constant. It follows from (4) that each agent's optimal charging strategy must achieve a trade-off between the total electricity cost $p(.)u^n$ and the cost incurred in deviating from the average behavior of the PEV population $(u^n - \text{avg}(\mathbf{u}))^2$. The examples in Section IV illustrate that the small tracking costs are more than compensated by cost savings that arise from valley filling.

The underlying decentralized PEV charging control scheme is a form of *finite-horizon noncooperative dynamic game*. Each PEV agent shares with other PEV agents the (limited, valuable and divisible) electricity resources beyond the inelastic non-PEV demand base d, and also tracks the average charging strategy of the whole population. A collection of charging controls $\mathbf{u}$ is a Nash equilibrium if, for all n, u^n is a charging strategy that minimizes the cost (4) with respect to $\mathbf{u}^{-n}$.

So far we have formulated the decentralized charging control problem as a class of dynamic games. Such problems are, however, generally computationally intractable for large population size N, see [11] and the references therein. We address this issue in the next section.

B. Implementation of decentralized PEV charging control

At an established Nash equilibrium, each agent reacts optimally with respect to its local state and the collectively average trajectory of all other agents. These trajectories are approximated by an identical deterministic infinite population limit (associated with the mean field or ensemble statistics of the random agents) which is shown later, in Theorem 3.1, to be the solution of a fixed point problem.

Before proceeding to develop the control strategy, it is important to confirm that the decentralized charging control formulation of Section II-A, with the SOC dynamics given by (1) and the cost functions of individual PEV agents specified by (4), gives rise to a Nash equilibrium. The requirements can be formalized as the following theorem, which we state without proof.

Theorem 2.1: Consider the charging control for an infinite population of PEVs. The collection of charging controls $\{u^n; 1 \le n < \infty\}$ is a *Nash equilibrium (NE)*, if

(i) Every $u^n \in \mathcal{U}^n$ is a local control infimizing the cost function,
$$J^n(u^n;\overline{u}) = \sum_{t=0}^{T-1}\Big\{p\big(\frac{d_t+\overline{u}_t}{c}\big)u_t^n + \delta(u_t^n-\overline{u}_t)^2\Big\} \quad (5)$$
with respect to a fixed $\overline{u}$; and

(ii) $\lim_{N\to\infty} \operatorname{avg}(\mathbf{u}_t) = \overline{u}_t$, i.e. $\overline{u}$ can be collectively reproduced subject to the local optimal controls of all individual systems.

For any finite population size N, however, this methodology gives the weaker result that $\{u^n; 1 \le n \le N\}$ is an ε-NE, for some positive ε. All the $\operatorname{avg}(\mathbf{u}^{-n})$ are only approximately equal in this case, because of the finite population size N. Nevertheless the value of the error ε tends to zero as the population size N grows.

Implementation of the decentralized control is achieved through a *charging negotiation procedure*, which takes place at some time prior to the actual charging interval:

(S1) The utility broadcasts the prediction of non-PEV base demand d to all the PEV agents.

(S2) Each of the PEVs proposes a charging control that minimizes its charging cost with respect to a common aggregate PEV demand broadcast by the utility.

(S3) The utility collects all the individual optimal charging strategies proposed in (S2), and updates the aggregate PEV demand corresponding to the proposed charging strategies. This updated aggregate PEV demand is rebroadcast to all of the PEVs.

(S4) Repeat (S2) and (S3) until the optimal strategies proposed by the agents no longer change.

At convergence (assuming it occurs), the collection of proposed individual charging strategies is an NE. Some time later, when the actual charging start time is reached, each PEV implements its optimal strategy obtained from (S1)-(S4).

In the negotiation procedure (S1)-(S4), each of the PEVs independently updates its own optimal feedback charging strategy with respect to the one-dimensional average value of all PEV agent strategies. The local computational complexity of the underlying decentralized control strategy is therefore independent of the PEV population size N.

III. Performance of the Decentralized Charging Control Problems

In this section, we first specify the optimal charging control that infimizing the cost function (5) with respect to fixed parameter $\overline{u}$. Based upon the properties of the optimal charging strategy, and under certain mild conditions on retail price function $p(r)$ and tracking parameter δ, we state the existence, uniqueness, and social optimality (or 'valley-filling' property) of the decentralized charging control problems. Rigorous and complete proofs for these results are presented in [12], while this paper provides proof outlines rather than the full technical details.

Define a charging control $u^n(\overline{u}, A)$ for agent n with respect to $\overline{u}$
$$u_t^n(\overline{u}, A) = \frac{1}{2\delta}\max\Big\{0,\ A - \Big(p\big(\frac{d_t+\overline{u}_t}{c}\big) - 2\delta\overline{u}_t\Big)\Big\}, \quad (6)$$
where the associated scalar $A \equiv A^n(\overline{u})$ is chosen to ensure
$$\sum_{t=0}^{T-1} u_t^n = \frac{\beta^n}{\alpha^n}(1-x_0^n). \quad (7)$$
Figure 2 illustrates the charging strategy $u_t^n(\overline{u}, A)$. It can be verified that the A is uniquely determined by $\overline{u}$ for any agent n.

Lemma 3.1: Considering a fixed $\overline{u}$, $u^n(\overline{u}, A) \in \mathcal{U}^n$ satisfying (6) and (7) is the unique charging control infimizing the cost function (5).

Outline of the proof of Lemma 3.1.

The infimization problem of $J^n(u^n;\overline{u})$ subject to $\mathcal{U}^n$ is a constrained optimization problem. Lemma 3.1 can be shown by applying the method of Lagrange multipliers. □

In order to establish properties of the optimal charging strategy, some further notation is required. Denote $\widetilde{u}^n(\overline{u})$ and $\widetilde{u}^n(\overline{v})$ as the charging controls infimizing the cost function

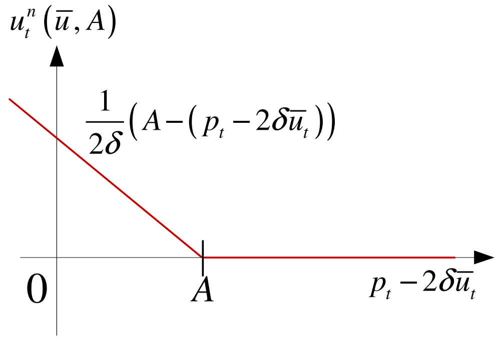

Fig. 2. Agent charging strategy given in (6).

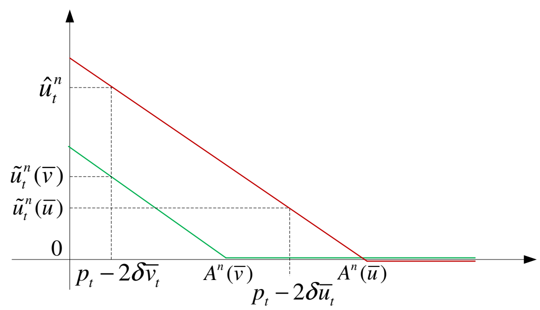

Fig. 3. Illustrations of agent charging controls $\widetilde{u}^n(\overline{u})$, $\widetilde{u}^n(\overline{v})$ and u^n.

(5) with respect to $\overline{u}$ and $\overline{v}$ respectively, subject to the set of full charging controls $\mathcal{U}^n$. By Lemma 3.1,

$$\widetilde{u}^n(\overline{u}) = u^n(\overline{u}, A^n(\overline{u}))$$
$$\widetilde{u}^n(\overline{v}) = u^n(\overline{v}, A^n(\overline{v}))$$

where $A^n(\overline{u})$ and $A^n(\overline{v})$ uniquely depend upon $\overline{u}$ and $\overline{v}$ respectively, and

$$\widetilde{\mathbf{u}}(\overline{u}) = \big\{\widetilde{u}^n(\overline{u}); 1 \le n < \infty\big\}$$
$$\widetilde{\mathbf{u}}(\overline{v}) = \big\{\widetilde{u}^n(\overline{v}); 1 \le n < \infty\big\}.$$

Consider a charging control $\widehat{u}^n \equiv u^n(\overline{v}, A^n(\overline{u}))$, i.e. $\widehat{u}^n$ represents the charging control of agent n satisfying (6) with respect to $\overline{v}$ and $A^n(\overline{u})$. Charging controls $\widetilde{u}^n(\overline{u})$, $\widetilde{u}^n(\overline{v})$, and $\widehat{u}^n$ are illustrated in Figure 3.

Lemma 3.2: For any positive tracking parameter δ, the following inequalities hold,

$$\begin{aligned}&|\widetilde{u}_t^n(\overline{u}) - \widehat{u}_t^n| \\ &\quad\le \left|(\overline{u}_t - \overline{v}_t) - \frac{1}{2\delta}\Big(p\big(\frac{d_t + \overline{u}_t}{c}\big) - p\big(\frac{d_t + \overline{v}_t}{c}\big)\Big)\right| \end{aligned} \quad (8)$$

and

$$|\widetilde{u}^n(\overline{u}) - \widetilde{u}^n(\overline{v})|_1 \le 2|\widetilde{u}^n(\overline{u}) - \widehat{u}^n|_1 \quad (9)$$

where $|\mathbf{x}|_1$ denotes the l_1 norm of the vector $\mathbf{x} \in \mathbb{R}^T$, i.e. $|\mathbf{x}|_1 \triangleq \sum_{l=1}^{T} |x_l|$.

Proof of Lemma 3.2 is not included in this paper but can be found in [12]. This lemma is used to establish existence and uniqueness properties of the Nash equilibrium.

Theorem 3.1: (Existence of Nash equilibrium)

Assume that the retail price $p(r)$ is continuous on $r \in [0,1]$. Then there exists a Nash equilibrium for the decentralized charging control problem.

Outline of the proof of Theorem 3.1.

By Lemma 3.2, we can show $\mathrm{avg}\big(\widetilde{\mathbf{u}}(\overline{u})\big)$ is continuous on $\overline{u}$ because the retail price p is continuous on $r \in [0,1]$. By the *Brouwer fixed point theorem* [13], there exists a *fixed point* $\overline{u}^*$ such that $\mathrm{avg}\big(\widetilde{\mathbf{u}}(\overline{u}^*)\big) = \overline{u}^*$, and by Theorem 2.1 $\widetilde{\mathbf{u}}(\overline{u}^*)$ is a Nash equilibrium for the decentralized charging problem. □

In preparation for Theorem 3.2, we define

$$r_{min} \triangleq \min\Big\{\frac{d_t}{c};\ t = 0, \cdots, T-1\Big\}.$$

Theorem 3.2: (A sufficient condition on the uniqueness and convergence of Nash equilibrium)

Assume the retail price $p(r)$ is continuously differentiable and increasing on r, such that

$$\sup_{r\in[r_{min},1]} \frac{dp}{dr} < 2 \inf_{r\in[r_{min},1]} \frac{dp}{dr}, \quad (10)$$

and the tracking parameter δ satisfies

$$\frac{1}{2c} \sup_{r\in[r_{min},1]} \frac{dp}{dr} \le \delta \le \frac{a}{c} \inf_{r\in[r_{min},1]} \frac{dp}{dr},$$

for some a, with $\frac{1}{2} < a < 1$. Then the system converges to a *unique Nash equilibrium* for the decentralized charging problem.

Outline of the proof of Theorem 3.2.

By Lemma 3.2, and under the inequality constraint for p given in the statement of the theorem, it can be shown that

$$\big|\mathrm{avg}(\widetilde{\mathbf{u}}(\overline{u})) - \mathrm{avg}(\widetilde{\mathbf{u}}(\overline{v}))\big|_1 \le (2 - \frac{1}{a})\big|\overline{u} - \overline{v}\big|_1.$$

By hypothesis on a we have $0 < 2 - \frac{1}{a} < 1$, and hence that $\mathrm{avg}(\widetilde{\mathbf{u}}(\overline{u}))$ is a contraction map/operator with respect to $\overline{u}$. By applying the *contraction mapping theorem* [14], iterations converge to a unique fixed point $\overline{u}$ such that $\mathrm{avg}(\widetilde{\mathbf{u}}(\overline{u})) = \overline{u}$. By Theorem 2.1, the collection of agent strategies $\mathbf{u}$ is a Nash equilibrium for the decentralized charging problem. □

Having established existence, uniqueness and convergence properties, we are now in a position to consider the 'valley-filling' property of the underlying Nash equilibrium.

Theorem 3.3: (Social optimality, or 'valley-filling' property, of Nash equilibrium)

Suppose that a collection of charging strategies $\widetilde{\mathbf{u}}$ is a Nash equilibrium for the decentralized charging problem, and assume that the retail price $p(r)$ is strictly increasing on $r \in [0,1]$. Then $\widetilde{\mathbf{u}}$ satisfies the properties:

$$\overline{u}_t \ge \overline{u}_s, \qquad \text{when } d_t \le d_s \quad (11a)$$
$$d_t + \overline{u}_t \le d_s + \overline{u}_s, \qquad \text{when } d_t \le d_s \quad (11b)$$
$$d_t + \overline{u}_t = B, \qquad t \in [\widehat{t}_0, \widehat{t}_s],\ \text{for some } B \quad (11c)$$

where $\overline{u} = \text{avg}(\widetilde{\mathbf{u}})$, and $[\widehat{t}_0, \widehat{t}_s]$ represents a sub-interval of the charging period $[0, T]$, such that $\widetilde{u}_t^n > 0$ for all n and all $t \in [\widehat{t}_0, \widehat{t}_s]$. Furthermore,

$$\widetilde{u}_t^n \geq \widetilde{u}_s^n, \qquad \text{when } d_t \leq d_s \tag{12a}$$

$$d_t + \widetilde{u}_t^n = B', \qquad t \in [\widehat{t}_0, \widehat{t}_s], \text{ for some } B'. \tag{12b}$$

An outline of the proof of Theorem 3.3 is given in the appendix. The theorem characterizes the 'valley-filling' property of the Nash equilibrium:

- For any pair of charging instants, the one with the smaller non-PEV base demand is assigned a larger individual charging rate and a larger average charging rate, and possesses a lower total aggregated demand.
- The total aggregated/individual demand, consisting of aggregated/individual PEV charging load together with non-PEV demand, is constant during charging sub-intervals where all the PEV charging rates are strictly positive.

Note that these conditions do not guarantee a perfect 'valley-fill' because there may be intervals in which not all PEVs charge. However, in the case of a homogeneous PEV population, the individual agent strategies u^n are coincident with the aggregate strategy $\overline{u}$. It follows that (11) and (12) together are equivalent to,

$$\overline{u}_t = \begin{cases} B - d_t, & \text{when } d_t \leq B \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

for some $B > 0$, such that $\sum_{t=0}^{T-1} \overline{u}_t = \frac{\beta^n}{\alpha^n}(1 - x_0)$. In other words, in the optimal case of a homogeneous PEV population, the Nash equilibrium degenerates into a purely 'valley-filling' charging strategy.

IV. Numerical examples

This section presents a number of numerical examples that explore the convergence properties and valley-filling performance of the decentralized charging control process. The examples are based on the aggregate non-PEV base demand D shown in Figure 1. This curve is typical for a summer day in the region managed by the Midwest Independent System Operator (MISO). The normalized non-PEV demand d has exactly the same shape as Figure 1, but is scaled by the PEV population size N. The following numerical examples use a PEV population of 10^7, which is approximately 30% of all the vehicles in the MISO region.

The examples assume the normalized generation capacity c in the MISO region is about 10 kW. It is further assumed that all PEVs have an initial SOC of 15%, and identical charging efficiency α of 85%. We consider a 24-hour charging interval from noon on one day to noon on the next, with one hour time steps. Other parameters, such as PEV battery size β^n and the tracking-cost parameter δ, will be specified for each of the examples.

Consider a retail price function $p(r) = 0.15r^2$, on $r \in [0, 1]$. Recall that r is the ratio between the total demand and the generation capacity. It follows from Figure 1 that $r_{min} \approx 0.6$. The requirement (10) from Theorem 3.2 becomes,

$$\sup_{r \in [r_{min}, 1]} \frac{dp}{dr} = 0.3 \; < \; 2 \times \inf_{r \in [r_{min}, 1]} \frac{dp}{dr} = 0.36.$$

By applying Theorem 3.2, the decentralized negotiation procedure is guaranteed to converge to a unique Nash equilibrium if the tracking parameter δ satisfies,

$$\frac{1}{2c} \sup_{r \in [r_{min}, 1]} \frac{dp}{dr} = 0.015 \leq \delta \leq \frac{a}{c} \inf_{r \in [r_{min}, 1]} \frac{dp}{dr} = 0.018a,$$

for some a with $\frac{1}{2} < a < 1$.

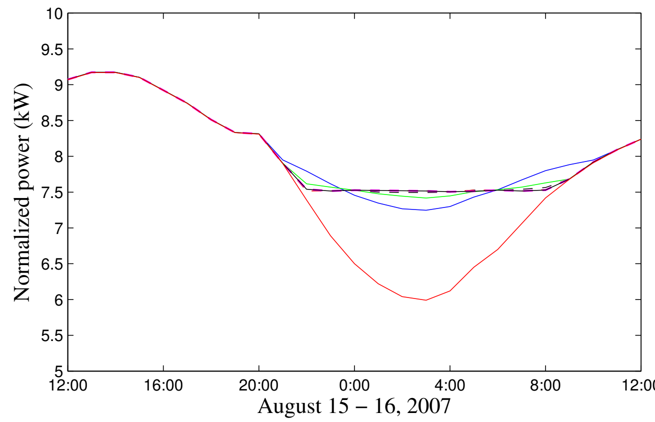

Fig. 4. Convergence of decentralized charging process for a homogeneous PEV population with $\delta = 0.015$.

Figure 4 shows a PEV's best charging strategy for each iteration of the negotiation process (S1)-(S4) described in Section II-B. For this case, the PEV population was homogeneous, with all PEVs having identical battery size of 10 kWh and maximum charging rate of 3 kW. The tracking parameter δ was set to 0.015. Notice that the charging system converges to the Nash equilibrium in a few negotiation cycles, and that the equilibrium is valley-filling.

The conditions established in Theorem 3.2 are sufficient to ensure uniqueness of the Nash equilibrium resulting from the decentralized process, and convergence to that equilibrium. However they are not necessary. Indeed, the convergent process shown in Figure 5 was obtained with tracking parameter $\delta = 0.007$, which does not satisfy the conditions specified in Theorem 3.2. The process converges to the same Nash equilibrium as in Figure 4.

Note though that the tracking parameter δ must be non-negligible for reasonable expectation of convergence. The choice of $\delta = 0.003$, for example, results in the non-convergent process shown in Figure 6.

The previous examples considered a homogeneous population. In this final case, the population is heterogeneous in battery size (half of the PEVs have a 20kWh battery and half have a 10kWh battery; all other parameters are as before). Figure 7 shows the converged Nash equilibrium of the decentralized scheme with a tracking parameter of $\delta = 0.015$. The result is not a perfect valley-fill because some

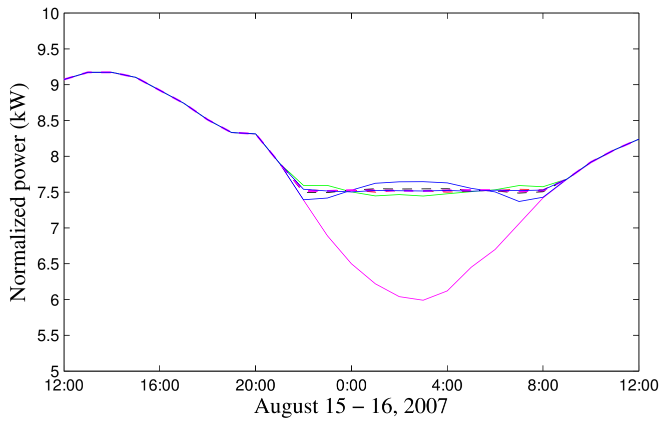

Fig. 5. Simulation results with $\delta = 0.007$. Although the tracking parameter does not satisfy the conditions of Theorem 3.2, convergence still occurs.

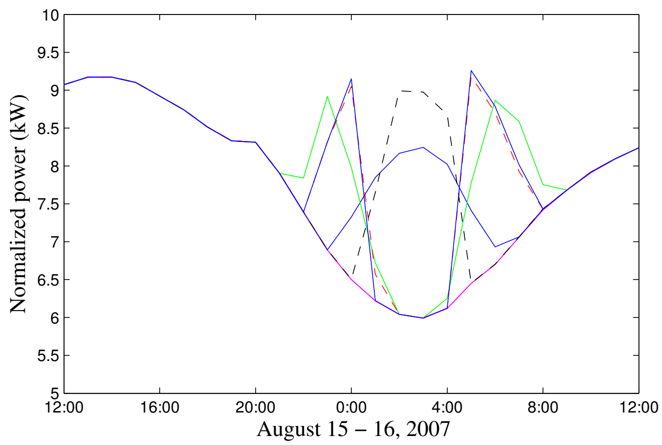

Fig. 6. Simulation results with $\delta = 0.003$. At this point the tracking parameter is small enough that the negotiation process does not converge.

PEVs charge for less time than others. As a consequence, total demand ramps down at the beginning of the charging interval, and ramps up at the end.

V. Conclusions

In this paper, decentralized charging control of large populations of PEVs is formulated as a class of finite-horizon dynamic games. The decentralized approach works by solving a relatively simple local problem and iterates quickly to a global Nash equilibrium. This strategy does not require significant central computing resources or communications infrastructure.

The paper establishes, under certain mild conditions, existence, uniqueness and social optimality of the Nash equilibrium attained through decentralized control. A negotiation procedure is proposed that converges to a charging strategy that is nearly optimal. In fact, for a homogeneous PEV population, the charging strategy degenerates to a purely social optimal 'valley-filling' strategy. The results are illustrated with various numerical examples.

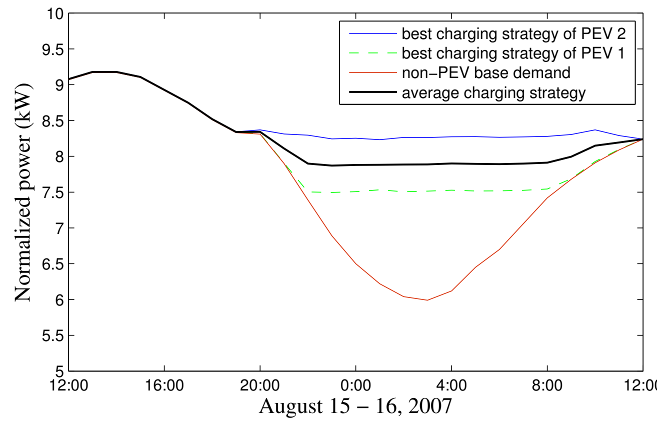

Fig. 7. Converged Nash equilibrium for a heterogeneous population of PEVs with $\delta = 0.015$.

Appendix

The proof of Theorem 3.3 proceeds by considering, without loss of generality, adjacent time instants t and $s = t+1$. Local charging controls $(\widetilde{u}_t^n, \widetilde{u}_{t+1}^n)$, that are optimal with respect to $\overline{u}$ and x_t^n, can be decomposed as $\widetilde{u}_t^n = b^{n,*} - a^{n,*}$ and $\widetilde{u}_{t+1}^n = b^{n,*} + a^{n,*}$ respectively. It is possible to show that

$$a^{n,*} = \operatorname*{arginf}_{a^n \in \mathbb{S}_{b^{n,*}}} \Big\{ \Big(a^n - \frac{1}{2}(\overline{u}_{t+1} - \overline{u}_t) + \frac{1}{4\delta}\big(p(d_{t+1} + \overline{u}_{t+1}) - p(d_t + \overline{u}_t)\big)\Big)^2 \Big\}$$

with $\mathbb{S}_{b^{n,*}} \triangleq \{a^n; -b^{n,*} \le a^n \le b^{n,*}\}$.

Relationship (11a) can be established by contradiction. If (11a) were not true, then it can be shown that $a^{n,*} < \frac{1}{2}(\overline{u}_{t+1} - \overline{u}_t)$, implying that $\widetilde{u}_{t+1}^n - \widetilde{u}_t^n < \overline{u}_{t+1} - \overline{u}_t$ for all n, and hence that

$$\mathrm{avg}(\widetilde{\mathbf{u}}_{t+1}) - \mathrm{avg}(\widetilde{\mathbf{u}}_t) < \overline{u}_{t+1} - \overline{u}_t$$

where $\widetilde{\mathbf{u}} \equiv \big\{\widetilde{u}^n; 1 \le n < \infty\big\}$. This, however, conflicts with the fact that $\{\widetilde{u}^n; n < \infty\}$ is a Nash equilibrium with respect to $\overline{u}$, see Theorem 2.1. Hence a contradiction.

Relationship (11b) is also proved by contradiction, and follows a similar argument as the proof of (11a). In this case though, it is determined that

$$\mathrm{avg}(\widetilde{\mathbf{u}}_{t+1}) - \mathrm{avg}(\widetilde{\mathbf{u}}_t) > \overline{u}_{t+1} - \overline{u}_t,$$

which conflicts with $\{\widetilde{u}^n; n < \infty\}$ being a Nash equilibrium with respect to $\overline{u}$.

Proof by contradiction is also used to establish (11c). Assume there are adjacent times $t, t+1 \in [\widehat{t}_0, \widehat{t}_s]$, such that

$$d_{t+1} + \overline{u}_{t+1} = d_t + \overline{u}_t + B, \tag{14}$$

where $B > 0$ without loss of generality. Then there will also exist an n and $C \ge B$ such that

$$d_{t+1} + \widetilde{u}_{t+1}^n = d_t + \widetilde{u}_t^n + C.$$

The theorem states that $\widetilde{u}_s^n > 0$ for all n and all $s \in [\widehat{t}_0, \widehat{t}_s]$, so there always exists a sufficiently small $\varepsilon > 0$ such that

$\widetilde{u}^n_{t+1} - \varepsilon > 0$. A revised control law $u^{n,\varepsilon}$ is established which exactly duplicates $\widetilde{u}^n$, except $u^{n,\varepsilon}_t = \widetilde{u}^n_t + \varepsilon$ and $u^{n,\varepsilon}_{t+1} = \widetilde{u}^n_{t+1} - \varepsilon$.

Using the assumption $d_t + \overline{u}_t < d_{t+1} + \overline{u}_{t+1}$ from (14), the proof proceeds to show that for sufficiently small $\varepsilon > 0$,

$$J^n(u^{n,\varepsilon}; \overline{u}) < J^n(\widetilde{u}^n; \overline{u}).$$

But $\widetilde{u}^n$ infimizes J^n with respect to $\overline{u}$, hence a contradiction, so $B = 0$ in (14). The desired result follows.

To prove (12a), suppose that $d_{t+1} \geq d_t$. Then (11a) and (11b) give

$$\frac{1}{2}(\overline{u}_{t+1} - \overline{u}_t) - \frac{1}{4\delta}(p(d_{t+1} + \overline{u}_{t+1}) - p(d_t + \overline{u}_t)) \leq 0.$$

This inequality together with (14) gives

$$\widetilde{u}^n_{t+1} - \widetilde{u}^n_t = 2a^{n,*} \leq 0,$$

for all n, which establishes the result.

To prove (12b), consider a subset of local charging controls $\mathcal{U}'^n(U)$, such that each control $u^n \in \mathcal{U}'^n(U)$ is feasible and satisfies, (i) u^n_t is fixed for $t \notin [\widehat{t}_0, \widehat{t}_s]$, (ii) $u^n_t > 0$ for all $t \in [\widehat{t}_0, \widehat{t}_s]$, and (iii) $\sum_{t=\widehat{t}_0}^{\widehat{t}_s} u^n_t = U$.

It is then possible to manipulate the local cost function, giving for all $u^n \in \mathcal{U}'^n(U)$,

$$J^n(u^n; \overline{u}) = p\left(\frac{B}{c}\right) U + \delta \sum_{t \in [\widehat{t}_0, \widehat{t}_s]} (u^n_t - B + d_t)^2 + F$$

where B is given in (11c), F gives the cost incurred over the period $t \notin [\widehat{t}_0, \widehat{t}_s]$, and is constant since u^n_t is assumed fixed on this interval. It follows that the infimum of $J^n(u^n; \overline{u})$ for $u^n \in \mathcal{U}'^n(U)$ is obtained when $d_t + \widetilde{u}^n_t$ is constant for all $t \in [\widehat{t}_0, \widehat{t}_s]$. The desired result follows. □

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