Decentralized Charging Control of Large Populations of Plug-in Electric Vehicles

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***Abstract*—This paper develops a strategy to coordinate the charging of autonomous plug-in electric vehicles (PEVs) using concepts from non-cooperative games. The foundation of the paper is a model that assumes PEVs are cost-minimizing and weakly coupled via a common electricity price. At a Nash equilibrium, each PEV reacts optimally with respect to a commonly observed charging trajectory that is the average of all PEV strategies. This average is given by the solution of a fixed point problem in the limit of infinite population size. The ideal solution minimizes electricity generation costs by scheduling PEV demand to fill the overnight non-PEV demand "valley". The paper's central theoretical result is a proof of the existence of a unique Nash equilibrium that almost satisfies that ideal. This result is accompanied by a decentralized computational algorithm and a proof that the algorithm converges to the Nash equilibrium in the infinite system limit. Several numerical examples are used to illustrate the performance of the solution strategy for finite populations. The examples demonstrate that convergence to the Nash equilibrium occurs very quickly over a broad range of parameters, and suggest this method could be useful in situations where frequent communication with PEVs is not possible. The method is useful in applications where fully centralized control is not possible, but where optimal or near-optimal charging patterns are essential to system operation.**

***Index Terms*—Decentralized control, Nash equilibrium, non-cooperative games, optimal charging control, plug-in electric vehicles (PEVs), plug-in hybrid electric vehicles (PHEVs).**

I. Introduction

VEHICLES that obtain some or all of their energy from the electricity grid (including pure electric vehicles and plug-in hybrids) may achieve significant market penetration over the next few years. Such vehicles, which we generically refer to as plug-in electric vehicles (PEVs), will reduce consumption of exhaustible petroleum resources and may reduce pollutant emissions including greenhouse gases. As their population grows, however, electric power system operation will become more challenging. For example, if a large number of PEVs began charging around the time most people finish their evening commute, a new demand peak could result, possibly requiring substantial new generation capacity and ramping capability [1].

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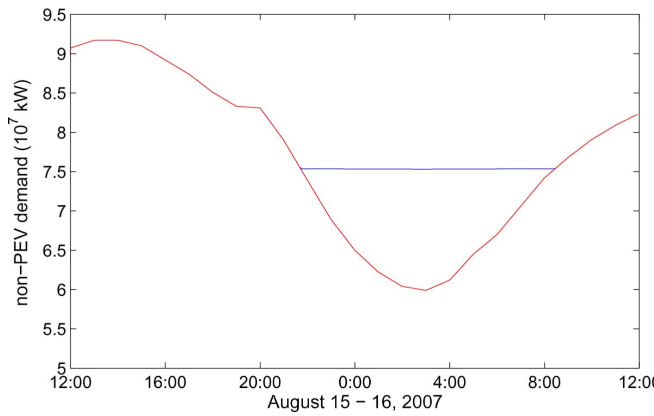

Fig. 1. Typical (non-PEV) base demand in summer for the region managed by the MISO. An ideal valley-filling profile is also shown.

In the United States, the average vehicle is driven about 28 miles per day, and the average one-way commute time is around 25 min [2]. This implies PEVs will spend a significant amount of time parked and available to charge. On the other hand, charging times for electric vehicles are likely to be 8 h or less, well under the time that most PEVs will actually be available for charging. This suggests that there could be useful flexibility with respect to when and how fast vehicles charge. If vehicle charging can be coordinated, it could be possible to construct aggregated charge profiles that avoid detrimental power system impacts and minimize system-wide costs.

A number of recent studies have explored the potential impacts of high penetrations of PEVs on the power grid [3]–[6]. In general, these studies assume that PEV charging patterns "fill the valley" of night-time demand. For example, the overnight dip in Fig. 1 would be replaced by a total demand that remained relatively constant during the charging period. However, these studies do not address the issue of how to coordinate PEV charging patterns. Possible coordination strategies can be divided into the following two categories.

1) In *centralized strategies*, a central operator dictates precisely when and at what rate every individual PEV will charge. Decisions could be made on the basis of system-level considerations only, or they could factor vehicle-level preferences, for example desired charging

interval, final state of charge, and budget. These strategies could be further distinguished by whether they attempt to identify charge patterns that are in some way optimal, or instead follow rules-of-thumb that seek to achieve aggregate charging patterns that are reasonably close to optimal.

2) *Decentralized or distributed strategies* allow individual PEVs to determine their own charging pattern. Vehicle charging decisions could, for example, be made on the basis of time-of-day or electricity price. The outcome of a decentralized approach may or may not be optimal, depending on the information and methods used to determine local charging patterns. Care must be taken to ensure charging strategies cannot inadvertently synchronize the responses of large numbers of PEVs, as the resulting abrupt changes in aggregate demand could potentially destabilize grid operations [7].

Though the valley-filling charging pattern is conceptually straightforward, the process of achieving it in a practical manner with a large population of independent PEVs presents a number of challenges. For example, a centralized approach may not be palatable to consumers, who are accustomed to having complete decision-making authority over their electricity consumption patterns. On the other hand, a decentralized strategy may preserve individual authority. However, as we will discuss below, the challenge of coordinating many autonomous agents to achieve an optimal or near-optimal outcome is non-trivial.

This paper explores the potential of decentralized strategies. Because time-based and fixed schedule price-based strategies have difficulty effectively filling the night-time valley [7], we instead focus on methods that utilize real-time marginal electricity price information. We allow each PEV to choose and implement its own local charging control strategy, with the aim of minimizing its individual charging cost.

The rest of the paper is organized as follows. Section II frames this work in the context of existing research, especially in the area of game theory. In Section III we formulate charging control problems for large populations of PEVs. Section IV develops a decentralized control strategy for optimally charging an infinite population of PEVs, and establishes existence and uniqueness properties of the resulting Nash equilibrium. Section V presents the control strategy as an algorithm that also applies to finite systems. The valley filling property of the Nash equilibrium is verified in Section VI. Finite population numerical examples are presented in Section VII. In Section VIII, we provide conclusions and suggest various extensions.

A summary of the notation used throughout the paper is provided in Table I.

TABLE I
LIST OF KEY SYMBOLS

$\mathcal{N}$	PEV populations
$\mathcal{T}$	PEV charging interval
x_{nt}	State of charge of PEV n at instant t
u_{nt}	Charging rate of PEV n at instant t
β_n	Battery capacity of PEV n
α_n	Charging efficiency of PEV n
$\mathbf{u}$	Collection of charging controls of PEV population
$\Sigma(\mathbf{u}_n)$	Total charging energy delivered to PEV n
$\overline{\mathbf{u}}_t$	Average charging rate of PEV population at instant t
$\mathbf{u}_{-n}$	Collection of local charging controls except PEV n
$\mathcal{U}_n$	Set of admissible charging controls of PEV n
d_t	(Inelastic) non-PEV demand at instant t
$\mathbf{u}_n^*(\mathbf{z})$	Optimal charging control of PEV n w.r.t. $\mathbf{z}$

II. Relationship to Control and Game-Theoretic Literature

The decentralized PEV charging control problem studied in this paper is a form of non-cooperative game, where a large number of selfish PEVs share electricity resources on a finite collection of charging intervals. This charging game falls within the class of *potential games* identified by Monderer and Shapley [8]. It is mathematically equivalent to routing and flow control games in telecommunications, where networks of parallel links are congested [9]. From this point of view, PEV charging games are conceptually similar to network games [10].

Substantial work has been presented in the literature on the computation of Nash equilibria, or ε-Nash equilibria, for potential games, especially in relation to network games. Research on *centralized mechanisms* includes Christoudoulou *et al.* [11], who consider two classes of potential games, selfish routing games and cut games, and Even-Dar *et al.* [12] who study the number of steps required to reach a Nash equilibrium in load balancing games. Research on *decentralized or distributed mechanisms* includes Berenbrink [13], who propose a strongly distributed setting for load balancing games such that all agents update their strategy simultaneously. Also, Even-Dar *et al.* [14] present convergence results for an approximate ε-Nash equilibrium under a non-centralized setting in routing games, and Fischer *et al.* [15], [16] propose a distributed and concurrent process for convergence to Wardrop equilibria [17] in adaptive routing problems.

We note that non-cooperative game theory is widely used to study the supply side of electricity markets, especially in the context of imperfect competition (see, for example, [18]–[21]). Some game theoretic work has been done to understand demand-side behavior in the face of dynamic pricing tariffs, primarily in the context of demand aggregators [22], [23]. Though this paper also applies game-theoretic principles to understand outcomes in electricity markets, to our knowledge it is unique in that: 1) we are examining multi-period demand-side behavior with a local energy constraint applied to the total energy consumed across all periods and 2) we are analyzing the problem from a decentralized perspective.

In this paper, PEVs are coupled through a common price signal which is determined by the average charging strategy of the PEV population. Therefore each PEV effectively interacts with the average charging strategy of the rest of the PEV population. As the population grows substantially, the influence of each individual PEV on that average charging strategy becomes negligible. Accordingly, in the infinite population limit, all PEVs will observe the same average strategy as they calculate their optimal local strategy. In this situation, a collection of local charging controls is a Nash equilibrium, if the following are met:

1) each PEV's charging strategy is optimal with respect to a single commonly observed charging trajectory;
2) the average of all the local optimal charging strategies is equal to that common trajectory.

This paper presents a novel procedure that identifies a Nash equilibrium by simultaneously updating each PEV's best individual strategy with respect to the average charging strategy of the whole population. It is proposed that this procedure would be undertaken prior to the actual charging interval. The computation time of this algorithm is unrelated to the number of PEVs, since they simultaneously and independently update their charging strategy. Under certain mild conditions, the proposed decentralized charging control procedure drives the system asymptotically to a unique Nash equilibrium that is nearly globally optimal (valley filling). In the case of homogeneous populations, where all vehicles have identical parameters, this unique Nash equilibrium becomes a perfect valley-filling charging strategy.

The proposed algorithm converges to an ε-Nash equilibrium for a finite population, and ε tends to zero as the population size approaches infinity. In this limiting case, the Nash equilibrium corresponds to a Wardrop equilibrium [17]. This method also has connections with the Nash certainty equivalence principle (or mean-field games), proposed by Huang *et al.* [24], [25] in the context of large-scale games for sets of weakly coupled linear stochastic control systems.

III. Charging Control of Large PEV Populations

In this section we introduce the basic PEV charging dynamics, and optimization models that are relevant for both the centralized and decentralized framework. In the decentralized case, we will also formally define the conditions for a Nash equilibrium.

A. Model and Notation

We consider charging control of a significant PEV population of size N over charging horizon $\mathcal{T} \triangleq \{0, \ldots, T-1\}$ where T denotes the terminal charging instant. The population of PEVs is denoted $\mathcal{N} \triangleq \{1, \ldots, N\}$. For an individual PEV n, we adopt the notation of Table I. The state of charge (SOC) of PEV n at instant t is given by $0 \leq x_{nt} \leq 1$, and the SOC dynamics are described by the simplified model

$$x_{n,t+1} = x_{nt} + \frac{\alpha_n}{\beta_n} u_{nt}, \quad t \in \mathcal{T} \tag{1}$$

with initial SOC x_{n0}, battery size $\beta_n > 0$ and charger efficiency $\alpha_n \in (0, 1]$.

The charging control trajectory of PEV n, denoted $\mathbf{u}_n \equiv (u_{nt}; t \in \mathcal{T})$, is an *admissible charging control*, if it belongs to the set

$$\mathcal{U}_n(\gamma_n) \triangleq \{\mathbf{u}_n : u_{nt} \geq 0 \qquad \Sigma(\mathbf{u}_n) = \gamma_n\} \tag{2}$$

where

$$\Sigma(\mathbf{u}_n) \triangleq \sum_{t\in\mathcal{T}} u_{nt}, \qquad \gamma_n \triangleq \frac{\beta_n}{\alpha_n}(1 - x_{n0}). \tag{3}$$

The set of admissible charging controls for the entire PEV population is denoted by $\mathcal{U}$. It follows from the SOC dynamics described in (1) that $x_{nT} = 1$ for any admissible control $\mathbf{u}_n \in \mathcal{U}_n(\gamma_n)$.

Subject to an admissible charging control $\mathbf{u} \in \mathcal{U}$, the cost associated with delivering the total system demand is given by

$$\mathbb{J}(\mathbf{u}) = \sum_{t\in\mathcal{T}} p(r_t^N(\mathbf{u}_t)) \left(D_t^N + \sum_{n\in\mathcal{N}} u_{nt} \right) \tag{4}$$

where $\mathbf{u}_t \equiv (u_{nt}; n \in \mathcal{N})$ denotes the collection of PEV charging rates at time t, $p(r_t^N(\mathbf{u}_t))$ is the electricity charging price at instant t, and D_t^N is the total inelastic non-PEV demand at instant t. We assume the electricity charging price $p(r_t^N(\mathbf{u}_t))$ is determined by the ratio between the total demand and the total generation capacity, so

$$r_t^N(\mathbf{u}_t) \triangleq \frac{1}{C^N} \left(D_t^N + \sum_{n\in\mathcal{N}} u_{nt} \right) \tag{5}$$

where C^N denotes the total generation capacity. The importance of the dependence of C and D_t on N is discussed later in this section.

Note that this definition of price differs from typical retail electricity tariffs, which are either constant or change according to a fixed (demand-independent) schedule in time. Instead, price varies in proportion to total demand; this is how wholesale electricity market prices (and therefore the real-time marginal price of electricity) vary. In this formulation, we assume for simplicity that electricity price is a function only of instantaneous demand. We are aware that in practice, price in any given hour will be influenced by demand in all hours. This is because the mix of units available in a given hour is determined by the unit commitment process [26], which is a function of demand over the entire commitment interval. We note that this price definition does not consider transmission congestion or losses, under the assumption that they are negligible at night.

This paper studies systems where the number of PEVs is sufficiently large that the action of each individual PEV on the system is negligible, but the action of the aggregation of PEVs may be significant. For this reason we will examine asymptotic properties of the system in the large N limit. To ensure that key properties are preserved at that limit, we assume that non-PEV demand and total generation capacity vary with the number of PEVs and make the following asymptotic assumptions as PEV population size approaches infinity

$$\lim_{N\to\infty} \frac{D_t^N}{N} = d_t, \qquad \lim_{N\to\infty} \frac{C^N}{N} = c. \tag{6}$$

The implication inherent in (6) is that larger power systems, with greater capacity and base demand, are required to support large numbers of PEVs. Direct substitution into (5) gives

$$\lim_{N\to\infty} r_t^N(\mathbf{u}_t) = \frac{1}{c}(d_t + \overline{\mathbf{u}}_t) \triangleq r_t(\overline{\mathbf{u}}_t) \tag{7}$$

where

$$\overline{\mathbf{u}}_t \triangleq \lim_{N\to\infty} \frac{1}{N} \sum_{n\in\mathcal{N}} u_{nt}. \tag{8}$$

Definition of Valley-Filling

We define valley-filling charging as follows:

$$\overline{\mathbf{u}}_t^{\mathrm{vf}} = \max\{0,\ \vartheta - d_t\} \tag{9}$$

for some constant ϑ. In words, in hours when $\vartheta > d_t$ this strategy chooses total PEV demand such that system-wide demand is equal to ϑ; otherwise PEV demand is zero. The value of ϑ uniquely determines the total energy supplied for charging. This is illustrated in Fig. 1, where ϑ corresponds to the level of the horizontal line.

The following lemma establishes sufficient conditions for optimal centrally-determined charging strategies to be valley-filling.

Lemma 3.1: If $p(r_t(\overline{\mathbf{u}}_t))$ is convex and increasing on $\overline{\mathbf{u}}_t$, then given non-PEV demand d_t, the optimal charging strategy $(\overline{\mathbf{u}}_t^*; t \in \mathcal{T})$ is valley-filling.

Proof: Define a Lagrangian

$$\mathcal{L}(\mathbf{u}, \boldsymbol{\lambda}, \nu) = \mathbb{J}(\mathbf{u}) - \sum_{t\in\mathcal{T}} \lambda_t \overline{\mathbf{u}}_t + \nu \left(\frac{1}{N} \sum_n \gamma_n - \sum_t \overline{\mathbf{u}}_t \right)$$

where $\lambda_t \geq 0$ and ν are Lagrange multipliers. Since this is a convex optimization problem the Karush-Kuhn-Tucker conditions ensure the following optimality conditions:

$$p'\left(\frac{1}{c}(d_t+\overline{\mathbf{u}}_t^*)\right)\frac{1}{c}(d_t+\overline{\mathbf{u}}_t^*)+p\left(\frac{1}{c}(d_t+\overline{\mathbf{u}}_t^*)\right)-\lambda_t^* = \nu^* \quad \forall t \in \mathcal{T} \tag{10}$$

$$\overline{\mathbf{u}}_t^* \geq 0 \qquad \forall t \in \mathcal{T} \tag{11}$$

$$\lambda_t^* \overline{\mathbf{u}}_t^* = 0 \quad \forall t \in \mathcal{T} \tag{12}$$

$$\frac{1}{N}\sum_n \gamma_n^* = \sum_t \overline{\mathbf{u}}_t^* \tag{13}$$

where (12) holds with complementary slackness. The right-hand side of (10) is independent of t, and complementary slackness requires that $\lambda_t^* = 0$ when $\overline{\mathbf{u}}_t^* > 0$. Therefore, because $p(\cdot)$ is convex and increasing, $d_t + \overline{\mathbf{u}}_t^*$ must be constant for all t when $\overline{\mathbf{u}}_t^* > 0$. It follows that (9) must hold. ■

It will be shown in Section VI that in the case of a homogeneous population of PEVs, the proposed decentralized control process achieves this same minimum-cost strategy.

B. Decentralized Charging Control of Large Populations of PEVs

In the rest of the paper we will study a decentralized game-based charging control strategy for large PEV populations. This subsection develops the mathematical framework for this analysis, and establishes sufficient conditions for the decentralized problem to achieve a Nash equilibrium.

Consider the local cost function $\mathcal{J}_n$ for an individual PEV n subject to a collection of charging controls $\mathbf{u}$

$$\mathcal{J}_n(\mathbf{u}) \triangleq \sum_{t\in\mathcal{T}} p\big(r_t(\overline{\mathbf{u}}_t)\big) u_{nt}. \tag{14}$$

The locally optimal charging control problem with respect to a fixed collection of controls $\mathbf{u}_{-n}$ is given by the minimization

$$\mathbf{u}_n^*(\mathbf{u}_{-n}) \triangleq \underset{\mathbf{u}_n \in \mathcal{U}_n(\gamma_n)}{\operatorname{argmin}}\ \mathcal{J}_n(\mathbf{u}_n; \mathbf{u}_{-n}) \tag{15}$$

where $\mathbf{u}_{-n} \triangleq \{\mathbf{u}_m; m \in \mathcal{N}, m \neq n\}$, in other words $\mathbf{u}_{-n}$ denotes the collection of control strategies of all PEVs except the nth. If a minimizing function exists, it will be referred to as an optimal control law for the local charging control problem.

We are now in a position to formalize the definition of a Nash equilibrium in the context of PEV charging strategies.

Definition 3.1: A collection of PEV strategies $\{\mathbf{u}_n^*; n \in \mathcal{N}\}$ is a *Nash equilibrium* if each PEV n cannot benefit by unilaterally deviating from its individual strategy $\mathbf{u}_n^*$, i.e.,

$$\mathcal{J}_n(\mathbf{u}_n^*; \mathbf{u}_{-n}^*) \leq \mathcal{J}_n(\mathbf{u}_n; \mathbf{u}_{-n}^*), \qquad \text{for all } \ \mathbf{u}_n \in \mathcal{U}_n, \ \text{ and all } \ n \in \mathcal{N}.$$

In Section V, we propose an iterative algorithm for obtaining the Nash equilibrium. At each iteration, every PEV optimizes its strategy relative to $\mathbf{u}$ determined in the previous iteration. As we will show in Section VII, specifically Fig. 4, charging intervals with high PEV demand at one iteration tend to induce low PEV demand at the following iteration, and vice versa. This occurs because PEVs move their charging requirements from expensive to inexpensive intervals; the resulting changes in demand reduce the marginal electricity price in the previously expensive intervals and raise the price in previously inexpensive intervals. This establishes an oscillatory pattern from one iteration to the next, preventing convergence to the Nash equilibrium.

To mitigate these oscillations, the local cost function (14) is modified to include a quadratic term that penalizes the deviation of an individual control strategy from the population average:

$$\mathcal{J}_n(\mathbf{u}) \triangleq \sum_{t\in\mathcal{T}} \left(p\left(\frac{1}{c}(d_t + \overline{\mathbf{u}}_t)\right) u_{nt} + \delta(u_{nt} - \overline{\mathbf{u}}_t)^2 \right) \tag{16}$$

where δ determines the magnitude of the penalty for deviating from the mass average. It will be shown that the presence of the squared deviation term ensures convergence to a unique collection of locally optimal charging strategies that is a Nash equilibrium. Unfortunately, because of the penalty term, this Nash equilibrium only coincides with the globally optimal strategy (9) when all PEVs are identical (homogeneous). Nevertheless, we will see that the cost added due to this term can be quite small compared with the electricity price p.

Given the formal definition of a Nash equilibrium provided by Definition 3.1, and the local cost function (16) for each PEV, we can now establish the conditions governing a Nash equilibrium for an infinite population of PEVs.

Theorem 3.1: A collection of charging strategies $\mathbf{u} \in \mathcal{U}$ for an infinite population of PEVs is a Nash equilibrium, if the following occur:

1) for all $n \in \mathcal{N}$, $\mathbf{u}_n$ minimizes the cost function

$$J_n(\mathbf{u}_n; \mathbf{z}) = \sum_{t \in \mathcal{T}} \left(p\left(\frac{1}{c}(d_t + z_t)\right) u_{nt} + \delta(u_{nt} - z_t)^2 \right) \tag{17}$$

with respect to a fixed $\mathbf{z}$;

2) $z_t = \overline{\mathbf{u}}_t$, for all $t \in \mathcal{T}$, i.e., $\mathbf{z}$ can be reproduced by averaging the individual optimal charging trajectories of all PEVs.

Proof: Consider the collection of PEV charging strategies $\{\mathbf{u}_n^*; n \in \mathcal{N}\}$ where each $\mathbf{u}_n^* \in \mathcal{U}_n$ minimizes its corresponding cost function (17) with respect to the common, fixed trajectory $\mathbf{z}^*$, and $z_t^* = \overline{\mathbf{u}}_t^*$ for all $t \in \mathcal{T}$.

As the population size N approaches infinity, each individual PEV's charging strategy u_{nt} has negligible influence on the population average $\overline{\mathbf{u}}_t$. Therefore, for every $n \in \mathcal{N}$

$$\lim_{N \to \infty} \frac{1}{N} \left(u_{nt} + \sum_{m \in \mathcal{N} \setminus n} u_{mt}^* \right) = \lim_{N \to \infty} \frac{1}{N} \sum_{m \in \mathcal{N}} u_{mt}^* = \overline{\mathbf{u}}_t^* = z_t^*. \tag{18}$$

Using this relationship in (16) gives for every $n \in \mathcal{N}$

$$\begin{aligned} \mathcal{J}_n(\mathbf{u}_n; \mathbf{u}_{-n}^*) &= \sum_{t \in \mathcal{T}} \left(p\left(\frac{1}{c}(d_t + \overline{\mathbf{u}}_t^*)\right) u_{nt} + \delta(u_{nt} - \overline{\mathbf{u}}_t^*)^2 \right) \\ &= J_n(\mathbf{u}_n; \mathbf{z}^*). \end{aligned} \tag{19}$$

Each $\mathbf{u}_n^*$ minimizes $J_n(\mathbf{u}_n; \mathbf{z}^*)$, and so by (19) also minimizes $\mathcal{J}_n(\mathbf{u}_n; \mathbf{u}_{-n}^*)$. Hence, by Definition 3.1, $\{\mathbf{u}_n^*; n \in \mathcal{N}\}$ is a Nash equilibrium. ■

As mentioned earlier, PEV charging games are consistent with the Nash certainty equivalence principle (also known as mean-field games). The key similarity is that individual agents do not consider the behavior of other individuals. Instead individuals are influenced by the so-called "mass effect", i.e., the overall effect of the population on a given agent. In the case of PEV charging, the effect felt by all individuals is the electricity price, which we specify as a function of the mass average charging trajectory $\overline{\mathbf{u}}_t$.

IV. Implementation for Infinite Systems

In this section we study the existence and uniqueness of the Nash equilibrium for the decentralized charging optimization defined by (16). Section IV-A derives the local optimum with respect to an arbitrary mass average. Proofs of existence and uniqueness are presented in Section IV-B.

To obtain these results we will assume the PEV population size is infinite. Obviously, any implementation of the control strategy must work for finite PEV populations. We will discuss this issue further in Sections V and VII, and show that the infinite population results apply to finite systems.

A. Locally Optimal Charging Control

Lemma 4.1 determines the optimal charging trajectory $\mathbf{u}_n^*(\mathbf{z})$ for an individual PEV n when it is subjected to a fixed trajectory $\mathbf{z}$.

Lemma 4.1: Consider a fixed trajectory $\mathbf{z}$, and the control trajectory $\mathbf{u}_n(\mathbf{z}, A) \in \mathcal{U}_n(\gamma_n)$ defined by

$$u_{nt}(\mathbf{z}, A) = \frac{1}{2\delta} \max\left\{0, A - p\left(\frac{1}{c}(d_t + z_t)\right) + 2\delta z_t\right\}, \quad \text{for all } t \in \mathcal{T}. \tag{20}$$

For a particular value of A, uniquely dependent upon $\mathbf{z}$ and denoted $A^*(\mathbf{z})$, the trajectory (20) provides the unique optimal control minimizing the cost function $J_n(\mathbf{u}_n; \mathbf{z})$ given in (17), subject to the admissible control requirement $\mathbf{u}_n \in \mathcal{U}_n(\gamma_n)$ defined in (2).

Proof: We define a Lagrangian

$$L_n(\mathbf{u}_n; \mathbf{z}) \triangleq J_n(\mathbf{u}_n; \mathbf{z}) + A\big(\gamma_n - \Sigma(\mathbf{u}_n)\big)$$

where A is the Lagrange multiplier. Since $J_n(\mathbf{u}_n; \mathbf{z})$ is convex with respect to $\mathbf{u}_n$, the local charging control that minimizes $J_n(\mathbf{u}_n; \mathbf{z})$, subject to $\mathbf{u}_n \in \mathcal{U}_n(\gamma_n)$, must satisfy the following:

(i) $\partial L_n / \partial A = 0$;

(ii) $\partial L_n / \partial u_{nt} \leq 0, u_{nt} \geq 0$, with complementary slackness.

Condition (i) recovers the constraint $\Sigma(\mathbf{u}_n) = \gamma_n$. We can derive from condition (ii)

$$p\left(\frac{1}{c}(d_t + z_t)\right) + 2\delta(u_{nt} - z_t) - A \begin{cases} = 0, & \text{when } u_{nt} > 0 \\ < 0, & \text{otherwise} \end{cases} \tag{21}$$

which is equivalent to (20).

The form of the dependence of $u_{nt}(\mathbf{z}, A)$ on A expressed in (20) ensures that, for any fixed $\mathbf{z}$.

- There exists an A_- such that for $A \leq A_-$, $\Sigma(\mathbf{u}_n(\mathbf{z}, A)) = 0$.
- For $A > A_-$, $\Sigma(\mathbf{u}_n(\mathbf{z}, A))$ is strictly increasing with A, with the relationship continuous, though not smooth. Hence, $\Sigma(\mathbf{u}_n(\mathbf{z}, A))$ is invertible over this domain.

Therefore a constraint $\Sigma(\mathbf{u}_n(\mathbf{z}, A)) = K > 0$ defines a unique $A > A_-$ for each fixed $\mathbf{z}$, which may be written $A(\mathbf{z})$. The particular value of A that ensures satisfaction of the constraint $\Sigma(\mathbf{u}_n) = \gamma_n$ shall be denoted $A^*(\mathbf{z})$. The resulting control trajectory can be written $\mathbf{u}_n(\mathbf{z}, A^*(\mathbf{z})) = \mathbf{u}_n^*(\mathbf{z})$.

Since $J_n(\mathbf{u}_n; \mathbf{z})$ is convex with respect to $\mathbf{u}_n$, the minimizing control defined by (20) must be unique for a given $\mathcal{U}_n(\gamma_n)$. ■

We now turn to identifying conditions that guarantee existence and uniqueness of the Nash equilibrium identified in Section III-C. Before doing so we will introduce additional notation. We denote by

$$\mathbf{u}_n^*(\mathbf{z}) \triangleq \underset{\mathbf{u}_n \in \mathcal{U}_n(\gamma_n)}{\operatorname{argmin}} \; J_n(\mathbf{u}_n; \mathbf{z})$$

the charging strategy that minimizes the local cost function (17) with respect to a fixed $\mathbf{z}$. By Lemma 4.1, we have $\mathbf{u}_n^*(\mathbf{z}) = \mathbf{u}_n(\mathbf{z}, A^*(\mathbf{z}))$. We define another local control strategy $\mathbf{v}_n(\mathbf{z}, \widehat{\mathbf{z}}) \equiv \mathbf{u}_n(\widehat{\mathbf{z}}, A^*(\mathbf{z}))$ for PEV n. This $\mathbf{v}_n(\mathbf{z}, \widehat{\mathbf{z}})$

describes a local charging control satisfying (20) with respect to $\widehat{\mathbf{z}}$ and $A^*(\mathbf{z})$. There is no guarantee that $\Sigma(\mathbf{v}_n(\mathbf{z},\widehat{\mathbf{z}})) = \gamma_n$.

B. Existence and Uniqueness of the Nash Equilibrium

Lemma A.1, presented in Appendix A, establishes several important properties of the control trajectories $\mathbf{u}_n^*(\mathbf{z})$, $\mathbf{u}_n^*(\widehat{\mathbf{z}})$ and $\mathbf{v}_n(\mathbf{z},\widehat{\mathbf{z}})$. In this section, we will apply this key technical lemma to show existence and uniqueness of the Nash equilibrium.

Theorem 4.1: Assume $p(r)$ is continuous on r. Then there exists a Nash equilibrium for the infinite population charging control system.

Proof: From Lemma A.1, we have

$$\begin{aligned}|\mathbf{u}_n^*(\mathbf{z}) - \mathbf{u}_n^*(\widehat{\mathbf{z}})|_1 &\le 2|\mathbf{u}_n^*(\mathbf{z}) - \mathbf{v}_n(\mathbf{z},\widehat{\mathbf{z}})|_1 \\ &\le 2\sum_{t\in\mathcal{T}}\left|(z_t - \widehat{z}_t) - \frac{1}{2\delta}(p(r_t) - p(\widehat{r}_t))\right|\end{aligned}$$

where $r_t = (1/c)(d_t + z_t)$ and $\widehat{r}_t = (1/c)(d_t + \widehat{z}_t)$. This implies that $\mathbf{u}_n^*(\mathbf{z})$ is continuous in $\mathbf{z}$ if $p(r)$ is continuous in r. It follows that $\overline{\mathbf{u}}^*(\mathbf{z})$ is continuous in $\mathbf{z}$, since the average of a group of continuous functions is also continuous.

We define a convex compact set

$$\mathcal{Y} \triangleq \left\{\mathbf{y} \equiv (y_t; t\in\mathcal{T});\ \text{ such that }\ 0 \le y_t \le \max_{n\in\mathcal{N}}\{\gamma_n\}\right\}.$$

By the specifications of admissible controls given in (2), we have $\mathcal{U}_n(\gamma_n) \subset \mathcal{Y}$ and so by extension $\overline{\mathbf{u}}^*(\mathbf{z}) \in \mathcal{Y}$. Therefore, for any $\mathbf{z} \in \mathcal{Y}$, we have $\overline{\mathbf{u}}^*(\mathbf{z}) \in \mathcal{Y}$, so $\overline{\mathbf{u}}^*(\cdot)$ maps a convex compact set to itself. Consequently, by the Brouwer fixed point theorem [27], there must be a fixed point $\mathbf{z} \in \mathcal{Y}$ such that $\overline{\mathbf{u}}^*(\mathbf{z}) = \mathbf{z}$. Since $\{\mathbf{u}_n^*(\mathbf{z}); n\in\mathcal{N}\}$ is the set of locally optimal charging strategies, by Theorem 3.1 the fixed point $\overline{\mathbf{u}}^*(\mathbf{z}) = \mathbf{z}$ is a Nash equilibrium. ■

Theorem 4.2: The infinite population charging control system possesses a unique Nash equilibrium if $p(r)$ is continuously differentiable and strictly increasing on r, and

$$\frac{1}{2c}\max_{r\in[r_{\min},r_{\max}]}\frac{dp(r)}{dr} \le \delta \le \frac{a}{c}\min_{r\in[r_{\min},r_{\max}]}\frac{dp(r)}{dr} \tag{22}$$

for some a in the range $1/2 < a < 1$, where $r_{\min}$ and $r_{\max}$ denote, respectively, the minimum and maximum possible r over the charging interval $\mathcal{T}$, subject to the admissible charging control set $\mathcal{U}_n(\gamma_n)$.

The proof is provided in Appendix B.

Theorem 4.2 establishes a sufficient condition for a range of values of δ for which the system will converge to a unique Nash equilibrium. It may be difficult to satisfy this condition over a large demand range $[r_{\min}, r_{\max}]$, especially if the higher demand value approaches the capacity limits of the system (the supply curve is usually very steep there). However, for overnight charging this is less likely to be a binding factor. Moreover, as we will show using a numerical example in Section VII, convergence is still possible even when condition (22) is slightly violated.

V. Decentralized Computational Algorithm

Assuming the technical conditions underpinning Theorem 4.2 are satisfied, $\overline{\mathbf{u}}^*(\mathbf{z})$ is a contraction mapping with respect to $\mathbf{z}$. This result motivates an iterative algorithm for computing the unique Nash equilibrium associated with the decentralized charging control system.

(S1) The utility broadcasts the prediction of non-PEV base demand $(d_t; t\in\mathcal{T})$ to all the PEVs.

(S2) Each of the PEVs proposes an optimal charging strategy minimizing its charging cost with respect to a common aggregate PEV demand broadcast by the utility.

(S3) The utility collects all the optimal charging strategies proposed in (S2), and updates the aggregate PEV demand. This updated aggregate PEV demand is rebroadcast to all PEVs.

(S4) Repeat (S2) and (S3) until the optimal strategies proposed by all PEVs no longer change.

A more formal expression of this procedure is given by Algorithm 1. At convergence, the collection of optimal charging strategies is a Nash equilibrium. Some time later, when the actual charging period occurs, each PEV implements its optimal strategy.

Algorithm 1: Implementation of decentralized charging control.

Initialize a positive ϵ, and define a tolerance ϵ_{stop} required to terminate iterations.

Provide an initial average charging control forecast $\mathbf{z}^{(1)}$, and set $i = 1$.

while $\epsilon > \epsilon_{\text{stop}}$ **do**

Obtain optimal charging control $\mathbf{u}_n^{*(i)}$, w.r.t. $\mathbf{z}^{(i)}$, for all n;

Set $\mathbf{z}^{(i+1)}$ equal to $\overline{\mathbf{u}}^{*(i)}$, where $\mathbf{u}^{*(i)} \equiv (\mathbf{u}_n^{*(i)}; n\in\mathcal{N})$;

Update $\epsilon = |\mathbf{z}^{(i+1)} - \mathbf{z}^{(i)}|_1$;

$i = i + 1$;

end

During iterations, the optimal charging trajectories proposed by PEVs may result in large average demand $\overline{\mathbf{u}}_t$ at some charging instants. Consequently, the demand $d_t + \overline{\mathbf{u}}_t$ at those instants may exceed generation capacity c, giving $r_t > 1$. This would, however, only occur as the population iterated towards the Nash equilibrium. It will be shown in Section VI that the Nash equilibrium corresponds to a charging strategy that is almost valley-filling, implying that large excursions in demand are not likely at the equilibrium.

Implementation of the charging strategy must, of course, work for finite groups of PEVs. To understand the consequences of a finite population N, we refer to Theorem 3.1. The infinite population limit is required in (18) to establish equality in (19) between $\mathcal{J}_n(\mathbf{u}_n; \mathbf{u}_{-n}^*)$, which quantifies the Nash equilibrium concept in Definition 3.1, and $J_n(\mathbf{u}_n; \mathbf{z}^*)$, which underpins Algorithm 1. For finite N, the equality (18) reverts to the approximation

$$\frac{1}{N}\mathbf{u}_n + \frac{N-1}{N}\overline{\mathbf{u}}_{-n}^* \approx \overline{\mathbf{u}}^*.$$

Nevertheless, for large N, this approximation is sufficiently accurate.

VI. Valley-Filling Property of the Nash Equilibrium

Having proven existence, uniqueness and convergence for the Nash equilibrium obtained from the PEV charging control process of Algorithm 1, this section establishes that the Nash equilibrium is *valley filling*. In its simplest form, the valley filling property appears as in Fig. 1. However there are several special cases to consider. The following key points capture the various cases that are formalized in Theorem 6.1.

(i) For any pair of charging instants, the one with the smaller non-PEV base demand is assigned a larger charging rate (for individual PEVs as well as for the average over all PEVs), and possesses an equal or lower total aggregate demand.

(ii) The total demand, consisting of aggregate PEV charging load together with non-PEV demand, is constant during charging subintervals when all PEV charging rates are strictly positive. This is also true of the demand obtained by summing non-PEV demand with the charging load of any individual PEV.

For a homogeneous population of PEVs, this second outcome guarantees perfect valley filling. That is not the case, however, for heterogeneous populations because there may be charging subintervals when the charging rate for some PEVs is zero. The examples of Section VII illustrate these outcomes.

Theorem 6.1: Suppose that the collection of charging trajectories $\mathbf{u}^* \equiv \{\mathbf{u}_n^*; n \in \mathcal{N}\}$ is a Nash equilibrium, and that $p(r)$ is strictly increasing on r. Then $\mathbf{u}_n^*$ and the average $\mathbf{z} = \overline{\mathbf{u}}^*$ satisfy the following valley filling properties for all $\delta > 0$:

$$\text{(i)}\quad z_t \ge z_s, \quad d_t + z_t \le d_s + z_s, \quad u_{nt}^* \ge u_{ns}^*, \quad \text{when } d_t \le d_s, \text{ with } t, s \in \mathcal{T} \tag{23a}$$

$$\text{(ii)}\quad d_t + z_t = \theta, \quad d_t + u_{nt}^* = \theta_n \quad \text{for some } \theta, \theta_n > 0, \text{ with } t \in \widehat{\mathcal{T}} \tag{23b}$$

where $\widehat{\mathcal{T}} \equiv \{t \in \mathcal{T}; u_{nt}^* > 0 \text{ for all } n \in \mathcal{N}\}$.

The proof is provided in Appendix C.

In case of homogeneous PEV populations, each of the individual optimal strategies $\mathbf{u}_n^*$ is coincident with their average strategy $\overline{\mathbf{u}}^*$. It follows that the properties of the Nash equilibrium specified in (23) are equivalent to

$$\overline{\mathbf{u}}_t^* = \max\{0, \theta - d_t\}, \quad \text{for some } \theta > 0 \tag{24}$$

which is the normalized form of the optimal valley-filling strategy given in (9). In other words, in the case of a homogeneous PEV population, the Nash equilibrium coincides with the charging strategy given by centralized control, and is therefore globally optimal.

VII. Numerical Examples

A. Background

A range of examples will be used in this section to illustrate the main results of the paper. In particular, we will consider the conditions for convergence of Algorithm 1, and explore the nature of valley filling.

The examples use the non-PEV demand profile of Fig. 1, which shows the load of the Midwest ISO region for a typical summer day during 2007. It is assumed that the total generation capacity is 1.2×10^8 kW. Furthermore, the simulations are based on assuming $N = 10^7$, which corresponds to roughly 30% of vehicles in the Midwest Independent System Operator (MISO) footprint. This gives $c^N \triangleq (1/N)C^N = 12$ kW, where the superscript N indicates finite population size. Also, we define $d_t^N \triangleq (1/N)D_t^N$, with D_t^N displayed in Fig. 1.

The following PEV population parameters will be used for all the examples. All PEVs have an initial SOC of 15%, i.e., $x_{n0} = 0.15$ for all n, and 85% charging efficiency, i.e., $\alpha_n = 0.85$ for all n. The charging interval $\mathcal{T}$ covers the 12-hour period from 8:00 pm on one day to 8:00 am on the next. The continuously differentiable and strictly increasing price function

$$p(r) = 0.15 r^{1.5} \, \frac{\$}{\text{kWh}} \tag{25}$$

is used in all cases.

Other parameters, such as PEV battery size β_n and the tracking cost parameter δ, are specified within each of the examples.

B. Computation of Nash Equilibrium for Homogeneous PEV Population

This section considers the computation of the Nash equilibrium for a homogeneous population of PEVs, each of which possesses an identical battery size of $\beta^n = 10$ kWh. First, it may be verified from Fig. 1 that

$$r_{\min} = \min_{t \in \mathcal{T}} \frac{\{d_t^N\}}{c^N} \approx 0.5.$$

To determine $r_{\max}$, we assume the entire energy requirement γ_n is delivered over a single time step, so

$$r_{\max} = \frac{\left(\max_{t \in \mathcal{T}} \{d_t^N\} + \gamma_n\right)}{c^N} \approx 1.5.$$

Referring to (25), this gives

$$\frac{1}{2c^N} \max_{[r_{\min}, r_{\max}]} \frac{dp(r)}{dr} = 0.012 \le \frac{a}{c^N} \min_{[r_{\min}, r_{\max}]} \frac{dp(r)}{dr} = 0.013a$$

which can be satisfied for some a in the range $1/2 < a < 1$. Therefore a tracking parameter δ exists such that condition (22) of Theorem 4.2 holds.

Fig. 2 provides simulation results for the decentralized computation algorithm of Section IV, for the homogeneous PEV population of this section. The tracking parameter $\delta = 0.012$ was used for this case. Each line in the figure corresponds to an iterate of the algorithm. We observe that convergence to the Nash equilibrium (shown by the solid flat curve) is achieved in a few cycles. This Nash equilibrium is clearly the globally optimal valley-filling strategy.

The condition on δ established in Theorem 4.2 is sufficient, but not necessary. Fig. 3 confirms this. Here,

$$\delta = 0.007 < \frac{1}{2c^N} \max_{[r_{\min}, r_{\max}]} \frac{dp(r)}{dr} = 0.012$$

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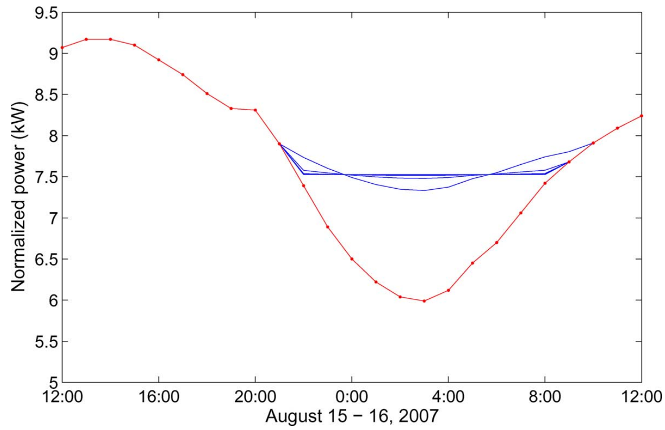

Fig. 2. Convergence of Algorithm 1 for a homogenous PEV population, with $\delta = 0.012$.

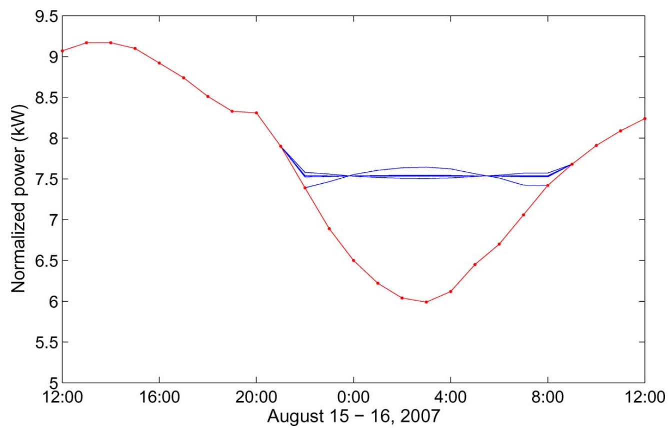

Fig. 3. Convergence of Algorithm 1 for a homogenous PEV population, with $\delta = 0.007$. This violates the condition given in Theorem 4.2.

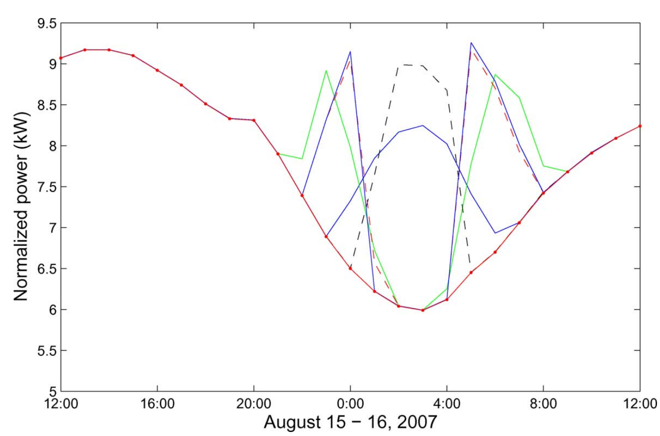

Fig. 4. Non-convergence of Algorithm 1 for a homogenous PEV population, with $\delta = 0.003$. This significantly violates the condition given in Theorem 4.2.

yet the system still converges to the same valley-filling solution as in Fig. 2. As δ decreases, however, the process eventually ceases to converge. This can be observed in Fig. 4, which shows the iterations when $\delta = 0.003$. In order to avoid an unreasonably high charging rate during the non-PEV demand valley, we have constrained the charging rate to a maximum of 3 kW. This constraint does not effect the convergence property of the algorithm.

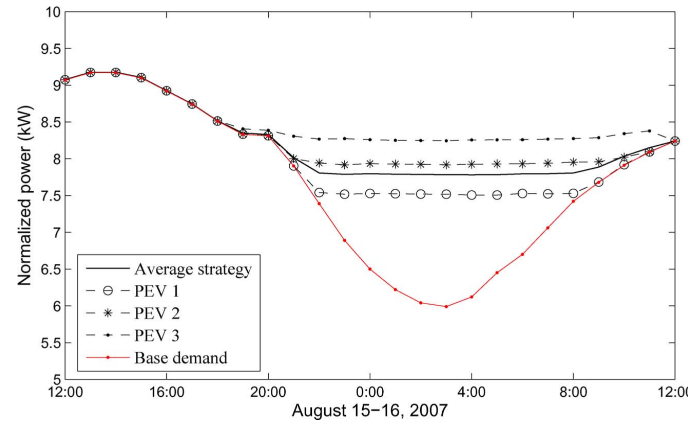

Fig. 5. Convergence of Algorithm 1 for a heterogeneous PEV population.

C. Computation of Nash Equilibrium for Heterogeneous PEV Populations

PEV populations are heterogeneous if vehicles do not have identical charging requirements. To examine optimal charging outcomes for heterogeneous populations, we constructed a simplified case with PEVs having one of three charging energy requirements: 10, 15, or 20 kWh. We further assumed that the number of PEVs in each group accounted for about 50%, 30% and 20% of the population, respectively. We can verify that

$$r_{\min} = \min_{t \in \mathcal{T}} \frac{\{d_t^N\}}{c^N} \approx 0.5, \quad r_{\max} = \frac{\left(\max\limits_{t \in \mathcal{T}} \{d_t^N\} + \overline{\gamma}\right)}{c^N} \approx 1.8$$

where $\overline{\gamma} = 13.5$ kWh denotes the energy delivery requirement of each PEV averaged across the entire population. It follows that

$$\begin{aligned} \frac{1}{2c^N} \max_{[r_{\min}, r_{\max}]} \frac{dp(r)}{dr} &= 0.0125 \\ &\leq \frac{a}{c^N} \min_{[r_{\min}, r_{\max}]} \frac{dp(r)}{dr} = 0.013a \end{aligned}$$

which can be satisfied for some a in the range $1/2 < a < 1$. Fig. 5 shows the results for this heterogeneous case, with tracking parameter $\delta = 0.0125$. In particular the dashed curves with marks show the optimal charging strategies for the first, second, and third class of PEVs, and the solid curve provides the average demand value across the entire population. Notice that this curve of average demand is flat between 10 pm and 8 am, where all PEVs are charging, in accordance with Theorem 6.1.

VIII. Conclusion and Future Research

This paper introduces a class of decentralized charging control problems for large populations of PEVs. These problems are formulated as large-population games on a finite charging interval. We study the existence, uniqueness and optimality of the Nash equilibrium of the charging problems. In particular, following the decentralized computational mechanism established in the paper, we show that, under certain mild conditions, the large-population charging games will converge to a unique Nash

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equilibrium which is either globally optimal (for homogeneous populations) or nearly globally optimal (for the heterogeneous case). These results are demonstrated with illustrative examples.

These examples demonstrate that convergence to the Nash equilibrium occurs very quickly over a broad range of parameters. Therefore, the method may be particularly useful in applications where fully centralized control is not possible, yet optimal or near-optimal charging patterns are essential to system operation. The results in this paper will be important when PEV market penetration becomes sufficiently large that electricity demand patterns change significantly with PEV charging. This is because the algorithm allows users to choose their own locally optimal charging pattern while still achieving near-optimal global conditions. The strategy may improve PEV market penetration, especially relative to centralized strategies that could deter consumers who wish to independently determine their charging strategy.

The paper has only included two user-specific preferences in the constrained optimization problem, namely to minimize local electricity costs and to fully charge. This work should be extended to include other local considerations such as time constraints, a willingness to trade off gasoline versus electricity costs (in plug-in hybrid vehicles), and battery state of health concerns. This is the subject of ongoing research.

We have assumed that supply and non-PEV demand are deterministic and predictable. This is clearly not the case in practice, as demand is stochastic, conventional generators experience forced outages, and wind and solar generation is variable and difficult to predict. Therefore a natural extension to this work would incorporate stochastic supply and demand forecasting models into the optimization process. We have begun work in this direction [28]. Along a similar line, PEVs may ultimately have "vehicle-to-grid" (V2G) capability, whereby they could act as loads at certain times and sources at other times. This function could be used to counterbalance load and generator supply forecast errors.

Appendix A
Statement and Proof of Lemma A.1

Lemma A.1: Control trajectories $\mathbf{u}_n^*(\mathbf{z})$, $\mathbf{u}_n^*(\widehat{\mathbf{z}})$ and $\mathbf{v}_n(\mathbf{z},\widehat{\mathbf{z}})$ satisfy the following inequalities for all $\delta > 0$:

$$|u_{nt}^*(\mathbf{z}) - v_{nt}(\mathbf{z},\widehat{\mathbf{z}})| \le \left|(z_t - \widehat{z}_t) - \frac{1}{2\delta}\big(p(r_t) - p(\widehat{r}_t)\big)\right|, \quad \text{for all } t \in \mathcal{T} \tag{26}$$

$$|\mathbf{u}_n^*(\mathbf{z}) - \mathbf{u}_n^*(\widehat{\mathbf{z}})|_1 \le 2|\mathbf{u}_n^*(\mathbf{z}) - \mathbf{v}_n(\mathbf{z},\widehat{\mathbf{z}})|_1 \tag{27}$$

where $r_t = (1/c)(d_t + z_t)$, $\widehat{r}_t = (1/c)(d_t + \widehat{z}_t)$, and $|\cdot|_1$ denotes the l_1 norm of the associated vector.

Proof: For notational simplicity, we will use $\mathbf{v_n} \equiv \mathbf{v_n}(\mathbf{z},\widehat{\mathbf{z}})$ throughout the proof.

(26) Proof: There are four cases to consider:

(i) $v_{nt} = u_{nt}^*(\mathbf{z}) = 0$. It follows immediately that $v_{nt} - u_{nt}^*(\mathbf{z}) = 0$.

(ii) $v_{nt} > 0$ and $u_{nt}^*(\mathbf{z}) = 0$. By (20), $v_{nt} > 0$ implies $v_{nt} = (1/2\delta)(A^*(\mathbf{z}) - p(\widehat{r}_t) + 2\delta\widehat{z}_t)$, and $u_{nt}^*(\mathbf{z}) = 0$ implies $A^*(\mathbf{z}) - p(r_t) + 2\delta z_t \le 0$. Together these give

$$\begin{aligned}0 &< v_{nt} - u_{nt}^*(\mathbf{z})\\ &\le \frac{1}{2\delta}(A^*(\mathbf{z}) - p(\widehat{r}_t) + 2\delta\widehat{z}_t) - \frac{1}{2\delta}(A^*(\mathbf{z}) - p(r_t) + 2\delta z_t)\end{aligned}$$

with the last term equal to $(\widehat{z}_t - z_t) - (1/2\delta)(p(\widehat{r}_t) - p(r_t))$.

(iii) $v_{nt} = 0$ and $u_{nt}^*(\mathbf{z}) > 0$. Similarly to (ii), we can derive

$$0 < u_{nt}^*(\mathbf{z}) - v_{nt} \le (z_t - \widehat{z}_t) - \frac{1}{2\delta}(p(r_t) - p(\widehat{r}_t)).$$

(iv) $v_{nt} > 0$ and $u_{nt}^*(\mathbf{z}) > 0$. By (20), we have directly

$$v_{nt} - u_{nt}^*(\mathbf{z}) = (\widehat{z}_t - z_t) - \frac{1}{2\delta}(p(\widehat{r}_t) - p(r_t)).$$

(27) Proof: There are the following three cases to consider.

(i) $\Sigma(\mathbf{v}_n) = \Sigma(\mathbf{u}_n^*(\mathbf{z}))$. This equality ensures $\mathbf{v}_n \in \mathcal{U}_n(\gamma_n)$. Also, charging control $\mathbf{v}_n$ has the form (20) with $A = A^*(\mathbf{z})$. Therefore, by Lemma 4.1, $\mathbf{v}_n$ is the local optimal control with respect to $\widehat{\mathbf{z}}$, and hence $\mathbf{u}_n^*(\widehat{\mathbf{z}}) = \mathbf{v}_n$. It follows that

$$|\mathbf{u}_n^*(\widehat{\mathbf{z}}) - \mathbf{u}_n^*(\mathbf{z})|_1 = |\mathbf{v}_n(\mathbf{z},\widehat{\mathbf{z}}) - \mathbf{u}_n^*(\mathbf{z})|_1 \le 2|\mathbf{v}_n(\mathbf{z},\widehat{\mathbf{z}}) - \mathbf{u}_n^*(\mathbf{z})|_1.$$

(ii) $\Sigma(\mathbf{v}_n) > \Sigma(\mathbf{u}_n^*(\mathbf{z}))$. By (2) we have $\Sigma(\mathbf{u}_n^*(\widehat{\mathbf{z}})) = \Sigma(\mathbf{u}_n^*(\mathbf{z})) = \gamma_n$. Therefore $\Sigma(\mathbf{u}_n^*(\widehat{\mathbf{z}})) < \Sigma(\mathbf{v}_n)$ which, together with (20) and the definitions of $\mathbf{u}_n^*(\widehat{\mathbf{z}})$ and $\mathbf{v}_n$, implies,

$$A^*(\widehat{\mathbf{z}}) < A^*(\mathbf{z}), \qquad u_{nt}^*(\widehat{\mathbf{z}}) \le v_{nt}, \quad \text{for all } t.$$

Hence

$$\begin{aligned}0 \le |\mathbf{v}_n - \mathbf{u}_n^*(\widehat{\mathbf{z}})|_1 &= \Sigma(\mathbf{v}_n) - \Sigma(\mathbf{u}_n^*(\widehat{\mathbf{z}}))\\ &= \Sigma(\mathbf{v}_n) - \Sigma(\mathbf{u}_n^*(\mathbf{z})) \le |\mathbf{v}_n - \mathbf{u}_n^*(\mathbf{z})|_1\end{aligned}$$

with the last line a consequence of the triangle inequality for norms, taking into account that $\Sigma(\cdot) = |\cdot|_1$ for all valid control trajectories. Then

$$\begin{aligned}|\mathbf{u}_n^*(\widehat{\mathbf{z}}) - \mathbf{u}_n^*(\mathbf{z})|_1 &\le |\mathbf{v}_n - \mathbf{u}_n^*(\mathbf{z})|_1 + |\mathbf{v}_n - \mathbf{u}_n^*(\widehat{\mathbf{z}})|_1\\ &\le 2|\mathbf{v}_n - \mathbf{u}_n^*(\mathbf{z})|_1.\end{aligned} \tag{28}$$

(iii) $\Sigma(\mathbf{v}_n) < \Sigma(\mathbf{u}_n^*(\mathbf{z}))$. A similar argument to (ii) can be used to show that (28) holds in this case. ■

Appendix B
Proof of Theorem 4.2

Proof of Theorem 4.2: First notice that $r_t - \widehat{r}_t = (1/c)(z_t - \widehat{z}_t)$. Therefore

$$\begin{aligned}|p(r_t) - p(\widehat{r}_t)| &\le \max_{r\in[r_{\min},r_{\max}]} \frac{dp(r)}{dr} \times |r_t - \widehat{r}_t|\\ &= \max_{r\in[r_{\min},r_{\max}]} \frac{dp(r)}{dr} \times \frac{1}{c}|z_t - \widehat{z}_t|\\ &\le 2\delta|z_t - \widehat{z}_t|\end{aligned}$$

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where the final inequality follows directly from (22). This result, together with a similar argument in terms of $\min_{r\in[r_{\min},r_{\max}]} dp(r)/dr$ gives

$$\frac{1}{2a}|z_t-\widehat{z}_t| \le \frac{1}{2\delta}\big|p(r_t)-p(\widehat{r}_t)\big| \le |z_t-\widehat{z}_t|. \tag{29}$$

Manipulation of (29) results in

$$\left(1-\frac{1}{2a}\right)|z_t-\widehat{z}_t| \ge |z_t-\widehat{z}_t| - \frac{1}{2\delta}\big|p(r_t)-p(\widehat{r}_t)\big| \ge 0. \tag{30}$$

Because $p(r_t)$ is strictly increasing with $r_t=(1/c)(d_t+z_t)$, (30) can be rewritten

$$\left(1-\frac{1}{2a}\right)|z_t-\widehat{z}_t| \ge \left|(z_t-\widehat{z}_t)-\frac{1}{2\delta}\big(p(r_t)-p(\widehat{r}_t)\big)\right| \ge 0.$$

This inequality, in conjunction with (26) and (27) of Lemma A.1, gives

$$|\mathbf{u}_n^*(\mathbf{z})-\mathbf{u}_n^*(\widehat{\mathbf{z}})|_1 \le \left(2-\frac{1}{a}\right)|\mathbf{z}-\widehat{\mathbf{z}}|_1$$

and hence

$$|\overline{\mathbf{u}}^*(\mathbf{z})-\overline{\mathbf{u}}^*(\widehat{\mathbf{z}})|_1 \le \left(2-\frac{1}{a}\right)|\mathbf{z}-\widehat{\mathbf{z}}|_1.$$

Since $1/2<a<1$, it follows that $\overline{\mathbf{u}}^*(\mathbf{z})$ is a contraction mapping with respect to $\mathbf{z}$. It may be concluded from the contraction mapping theorem [29] that the infinite population of PEVs possesses a unique fixed point which is the unique Nash equilibrium for the infinite population charging control system. ■

Appendix C
Proof of Theorem 6.1

Proof of Theorem 6.1: Consider any pair of time instants $t,s\in\mathcal{T}$, and denote by $\mathcal{U}_n(\{t,s\})$ the set of charging controls u_{nt} and u_{ns} that satisfy $u_{nt},u_{ns}\ge 0$ and $u_{nt}+u_{ns}\le\gamma_n$. Let

$$a=\frac{u_{ns}-u_{nt}}{2},\qquad b=\frac{u_{ns}+u_{nt}}{2} \tag{31}$$

so that $u_{nt}=b-a$, and $u_{ns}=b+a$. It follows that $\mathcal{U}_n(\{t,s\})$ is equivalent to $\mathcal{S}\triangleq\big\{(a,b);\ \text{s.t. } a\in[-b,b],\ b\in[0,\gamma_n/2]\big\}$.

We proceed by writing the minimum of the cost function (17) as a Bellman equation [30]. To do so, we define

$$V_n(\gamma',\mathcal{T}')=\min_{u_{nk};k\in\mathcal{T}'}\left\{\sum_{k\in\mathcal{T}'}\left(p(r_k)u_{nk}+\delta(u_{nk}-z_k)^2\right)\right\}$$
$$\text{s.t.}\quad u_{nk}\le 0\quad \text{for all } k\in\mathcal{T}'$$
$$\sum_{k\in\mathcal{T}'}u_{nk}=\gamma'.$$

The minimum over the entire charging period $\mathcal{T}$ can then be written

$$V_n(\gamma_n,\mathcal{T})=\min_{u_{nt},u_{ns}\in\mathcal{U}_n(\{t,s\})}\left\{\sum_{k\in\{t,s\}}\left(p(r_k)u_{nk}+\delta(u_{nk}-z_k)^2\right)+V_n\big(\gamma_n-(u_{nt}+u_{ns}),\mathcal{T}\setminus\{t,s\}\big)\right\}. \tag{32}$$

In terms of a and b defined at (31), this becomes

$$V_n(\gamma_n,\mathcal{T})=\min_{(a,b)\in\mathcal{S}}\left\{2\delta\left(a-\frac{1}{2}(z_s-z_t)+\frac{1}{4\delta}(p(r_s)-p(r_t))\right)^2+g(b)\right\} \tag{33}$$

where $g(b)$ is an expression in b that is unrelated to a. Let a_n^* and b_n^* denote the values of a and b associated with the optimal controls u_{nt}^* and u_{ns}^*. Then by (33), a_n^* is a function of b_n^* that satisfies

$$a_n^*(b_n^*)=\operatorname*{argmin}_{a\in[-b_n^*,b_n^*]}\left\{(a-\zeta)^2\right\} \tag{34}$$

with

$$\zeta\equiv\frac{1}{2}(z_s-z_t)-\frac{1}{4\delta}(p(r_s)-p(r_t)). \tag{35}$$

It follows from (34) that

$$0<a_n^*\le\zeta\qquad \text{if } \zeta>0 \tag{36a}$$
$$a_n^*=0\quad \text{if } \zeta=0 \tag{36b}$$
$$\zeta\le a_n^*<0\quad \text{if } \zeta<0. \tag{36c}$$

First Part of (23a): We prove this result by establishing a contradiction. Suppose there exist two time instants t and s, such that $d_t\le d_s$ and $z_t<z_s$, which implies $r_t<r_s$. Since $p(r)$ is strictly increasing on r, $p(r_t)<p(r_s)$, and so from (35), $\zeta<(1/2)(z_s-z_t)$. It follows from (36) that $a_n^*<(1/2)(z_s-z_t)$. Hence, $u_{ns}^*(\mathbf{z})-u_{nt}^*(\mathbf{z})=2a_n^*<z_s-z_t$, for all $n\in\mathcal{N}$, which implies $\overline{\mathbf{u}}_s^*(\mathbf{z})-\overline{\mathbf{u}}_t^*(\mathbf{z})<z_s-z_t$. However $\mathbf{u}^*$ is a Nash equilibrium, so $\overline{\mathbf{u}}^*(\mathbf{z})=\mathbf{z}$, hence a contradiction.

Second Part of (23a): Proof by contradiction is again used. Suppose there exist two time instants t and s, such that $d_t+z_t>d_s+z_s$ when $d_t\le d_s$. It follows that $p(r_t)>p(r_s)$, and so from (35), $\zeta>(1/2)(z_s-z_t)$. But from (i.1), $z_s-z_t\le 0$, so it follows from (36) that $u_{ns}^*(\mathbf{z})-u_{nt}^*(\mathbf{z})=2a_n^*>z_s-z_t$, for all $n\in\mathcal{N}$, which implies $\overline{\mathbf{u}}_s^*(\mathbf{z})-\overline{\mathbf{u}}_t^*(\mathbf{z})>z_s-z_t$. However $\mathbf{u}^*$ is a Nash equilibrium, so $\overline{\mathbf{u}}^*(\mathbf{z})=\mathbf{z}$, hence a contradiction.

Third Part of (23a): Again consider two time instants t and s, where $d_t\le d_s$. From (i.1) and (i.2), we have $z_t\ge z_s$ and $p(r_t)\le p(r_s)$ respectively. Therefore (35) implies $\zeta\le 0$, so we may conclude from (36) that $u_{ns}^*-u_{nt}^*=2a_n^*\le 0$. Hence $u_{nt}^*\ge u_{ns}^*$ as desired.

(ii.1), First Part of (23b): Proof by contradiction will again be used to establish this result. Suppose there exist two time instants $t,s\in\widehat{\mathcal{T}}$ such that $d_s+z_s\ne d_t+z_t$. Without lose of generality, assume $d_s+z_s=d_t+z_t+\eta$, for some $\eta>0$. Then there exist n and $\widehat{\eta}\ge\eta$, such that $d_s+u_{ns}^*=d_t+u_{nt}^*+\widehat{\eta}$. By the definition of $\widehat{\mathcal{T}}$, $u_{ns}^*>0$ for $s\in\widehat{\mathcal{T}}$ and all $n\in\mathcal{N}$. Therefore there exists a sufficiently small $\varepsilon>0$ such that $u_{ns}^*-\varepsilon>0$.

Consider a revised charging strategy $\mathbf{u}_n^\varepsilon$, with

$$u_{nt}^\varepsilon=u_{nt}^*+\varepsilon$$
$$u_{ns}^\varepsilon=u_{ns}^*-\varepsilon$$
$$u_{nk}^\varepsilon=u_{nk}^*\quad \text{for } k\in\mathcal{T}\setminus\{t,s\}.$$

For the cost function $J_n(\mathbf{u}_n;\mathbf{z})$ defined at (17), it follows that

$$J_n(\mathbf{u}_n^{\varepsilon};\mathbf{z}) - J_n(\mathbf{u}_n^{*};\mathbf{z}) \\ = \varepsilon(p(r_t) - p(r_s)) + 2\delta\varepsilon((u_{nt}^* - u_{ns}^*) - (z_t - z_s)) + 2\delta\varepsilon^2.$$

Notice that $(u_{nt}^* - u_{ns}^*) - (z_t - z_s) = \eta - \widehat{\eta} \le 0$. Also, because $d_t + z_t < d_s + z_s$ and $p(r)$ is strictly increasing, $p(r_t) - p(r_s) < 0$. Therefore $J_n(\mathbf{u}_n^{\varepsilon};\mathbf{z}) < J_n(\mathbf{u}_n^{*};\mathbf{z})$ for sufficiently small $\varepsilon > 0$. However, $\mathbf{u}^*$ is a Nash equilibrium, and therefore minimizes $J_n(\mathbf{u}_n;\mathbf{z})$. Hence a contradiction.

(ii.2), Second Part of (23b): The total energy delivered to the nth PEV over the period $\widehat{\mathcal{T}}$ by the optimal charging strategy $\mathbf{u}^*$ is given by $\Sigma_{k\in\widehat{\mathcal{T}}} u_{nk}^* = \omega_n^* > 0$, for every $n \in \mathcal{N}$. Provided fixed energy ω_n^* is delivered over $\widehat{\mathcal{T}}$, variation of the trajectory $\{u_{nk}; k \in \widehat{\mathcal{T}}\}$ has no influence on the cost over the balance of the charging period $\mathcal{T} \setminus \widehat{\mathcal{T}}$. The optimal choice for $\{u_{nk}; k \in \widehat{\mathcal{T}}\}$ is therefore given by

$$\min_{u_{nk}, k\in\widehat{\mathcal{T}}} \sum_{k\in\widehat{\mathcal{T}}} \left(p(r_k)u_{nk} + \delta(u_{nk} - z_k)^2\right) \tag{37}$$

$$\text{s.t.} \quad \Sigma_{k\in\widehat{\mathcal{T}}} u_{nk} = \omega_n^*. \tag{38}$$

According to (ii.1), $d_t + z_t = d_s + z_s$, for all $t, s \in \widehat{\mathcal{T}}$. Therefore the electricity charging price $p(r_k)$, with $r_k = (1/c)(d_k + z_k)$, is a constant p for all $k \in \widehat{\mathcal{T}}$. This allows the cost function (37) to be rewritten as $p\omega + \delta\sum_{k\in\widehat{\mathcal{T}}}(u_{nk} - z_k)^2$, so the minimum cost can be found from

$$\min_{u_{nk}, k\in\widehat{\mathcal{T}}} \sum_{k\in\widehat{\mathcal{T}}} (u_{nk} - z_k)^2$$

subject to (38). Using Lagrange multipliers, optimality is achieved when all $u_{nk}^* - z_k, k \in \widehat{\mathcal{T}}$ are equal. In conjunction with (ii.1), this gives $d_k + u_{nk}^* = \theta_n$ for all $k \in \widehat{\mathcal{T}}$. ■

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